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GENERATION OF P-M INTERACTION CURVE FOR SECTION UNDER AXIAL COMPRESSION AND UNIAXIAL BENDING

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ABSTRACT

Understanding the correlation between axial force (P) and moment (M) in structural elements like beams and columns is crucial for designing reinforced concrete sections under biaxial loading. This correlation is depicted by the P-M interaction curve, also known as an interaction diagram, which aids engineers in assessing the impact of combined loads on member capacity. Safe load combinations lie on or below this curve. While recent research suggests that the IS:456-2000 code tends to be slightly unconservative under higher loads and conservative under lower axial loads, it still offers a simplified representation of this relationship. The generation of this curve, particularly for columns under uniaxial loading, is the objective of proposed methodologies.

Keyword Uniaxial bending; P-M Interaction curve; Strain compatibility; Equilibrium method; Stress-Strain Curves' Balance failure, Compression Failure; Tension Failure;

INTRODUCTION

Compression members play a vital role in the vertical framing systems of reinforced concrete buildings, enduring primarily axial compressive forces. These elements, encompassing columns, walls, struts, and various inclined or rigid frame members, are essential for structural stability. While "column" and "compression member" are often used interchangeably, in accordance with IS:456, a compression member qualifies as a column when its effective length surpasses three times its minimum lateral dimension; otherwise, it's termed a "pedestal."

Effective length (le) delineates the distance between inflection points of a buckled member, dictated by its end restraints. This parameter aids in distinguishing between columns and pedestals, with the latter having le not exceeding three times their minimum lateral dimension, constrained further by a maximum ratio of four times the least lateral dimension.

Columns, based on their unsupported length and end restraints, are further classified into different categories. Walls, another form of compression member, are subject to prescribed height-to-thickness and length-to-thickness ratios as per the code.

Understanding these specifications is critical for the efficient design and classification of compression members. Columns, for instance, are classified based on slenderness ratios and loadings:

Classification Based on Slenderness Ratios:

- Short Columns: Slenderness ratios fall below a defined threshold, less susceptible to buckling.
- Slender or Long Columns: Slenderness ratios exceed the threshold, making them prone to buckling.

Classification Based on Loadings:

- Columns Subjected to Axial Loads Only (Concentric): Experience pure axial compression.
- Columns Subjected to Combined Axial Load and Uniaxial Bending: Undergo axial compression alongside bending in one direction.

- Columns Subjected to Combined Axial Load and Biaxial Bending: Experience axial compression and bending in two perpendicular directions.

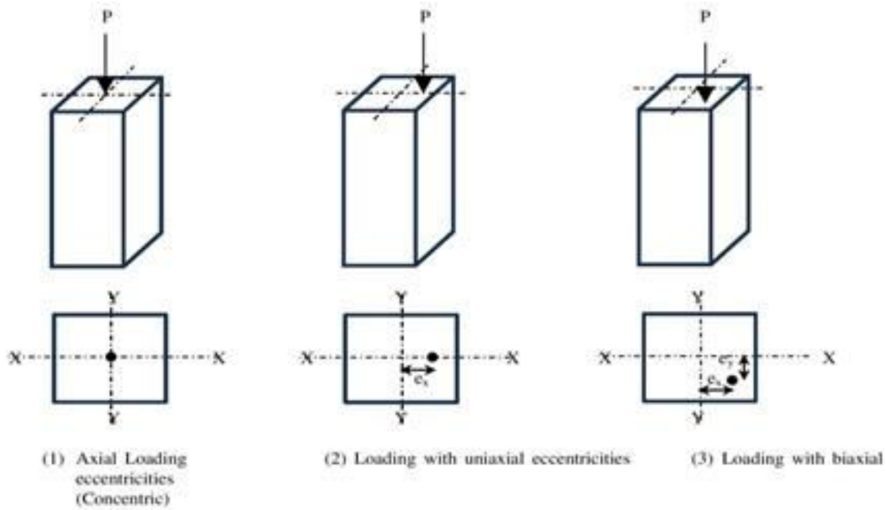


Fig. 1: Classification based on loading.

In practical situations, columns often endure axial compression along with bending and occasionally shear due to factors like rigid frame action and lateral loading. The combination of axial compression and bending is quantified by an eccentricity, denoted as e = M/P. Reinforced concrete columns, especially in framed buildings, frequently encounter biaxial eccentricities, particularly in corner columns.

During typical gravity loads in symmetric buildings, eccentricities are minimal and may be disregarded. However, under lateral loads such as wind or seismic forces, all columns, both internal and external, face significant bending.

Design considerations for columns vary depending on their loading conditions:

1. Axially Loaded Columns: Emphasize efficient resistance against axial compression.
2. Uniaxially Loaded Columns: Manage significant bending in one direction in addition to axial compression.
3. Biaxially Loaded Columns: Address bending moments in two perpendicular directions, necessitating a balanced approach to reinforcement and dimensions to ensure stability and load-carrying capacity.

1 Assumptions for the design of compression members by the Limit State of Collapse encompass various factors

The Limit State of Collapse encompass various factors such as reinforcement type, loading conditions, and slenderness ratios. Notably, these assumptions align consistently across different column classifications. The following assumptions, initially outlined for flexural member design, are equally applicable to compression member design (cl. 39.1 of IS 456):

1. Maximum Compressive Strain in Concrete: A maximum compressive strain of 0.002 is adopted in axial compression.
2. Compressive Strain Distribution: The maximum compressive strain at the highly compressed extreme fiber in concrete, subjected to both axial compression and bending without tension, is set as 0.0035 minus 0.75 times the strain at the least compressed extreme fiber.

Assumptions for Flexural Member Design Include:

1. **Plane Sections Remain Plane:** Sections normal to the axis retain their planarity after bending, ensuring that the cross-section of the member does not deform due to applied loads. This assumption implies that the strain at any point on the cross-section is directly proportional to its distance from the neutral axis.
2. **Maximum Concrete Strain in Bending:** The maximum strain in concrete at the outermost compression fiber is taken as 0.0035, beyond which concrete is considered to reach a state of collapse.
3. **Concrete Stress-Strain Curve:** The acceptable stress-strain curve of concrete is assumed to be parabolic.
4. **Ignoring Tensile Strength of Concrete:** Concrete's tensile strength is disregarded, and reinforcement is assumed to resist tensile stress. However, concrete's tensile strength is considered for checking deflection and crack widths in the limit state of serviceability.
5. **Reinforcement Design Stresses:** Design stresses of the reinforcement are derived from representative stress-strain curves, with a partial safety factor γ_m of 1.15.
6. **Limiting Tensile Strain in Reinforcement:** The maximum strain in the tension reinforcement at failure shall not be less than $f_y/(1.15\ Es) + 0.002$, ensuring ductile failure.

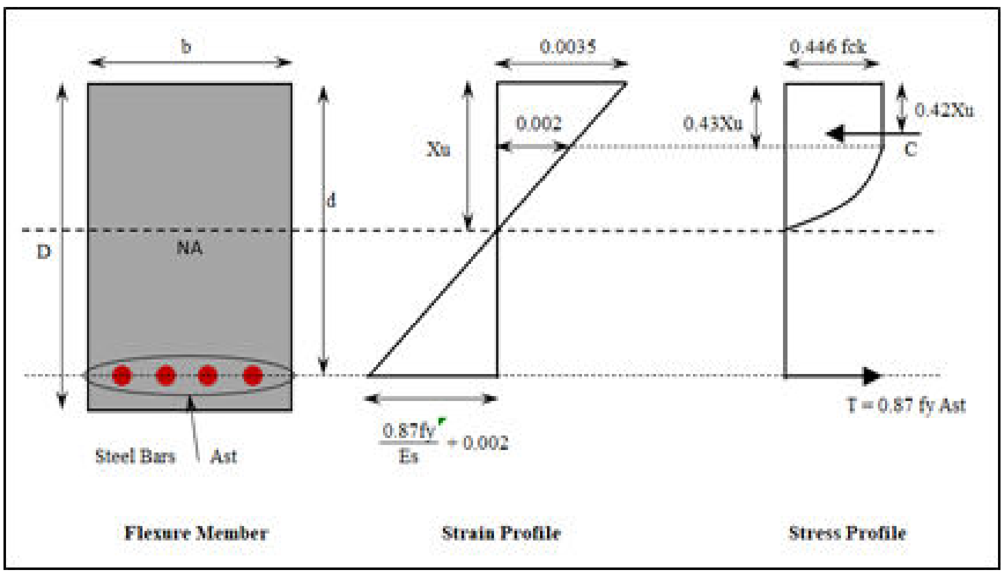


Fig. 2: Rectangular Section under flexure.

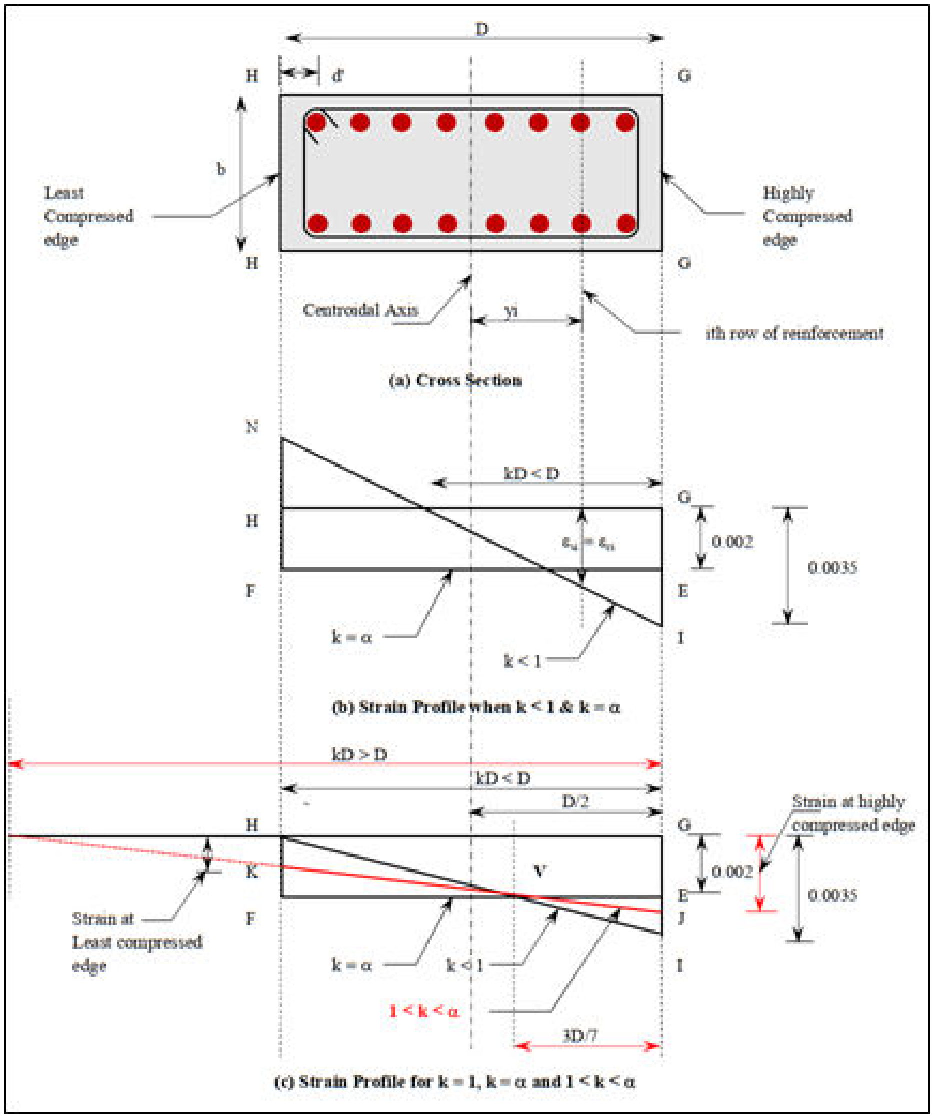


Fig. 3: Strain profile for different positions of neutral axis.

Additionally, a strain profile JK is depicted on Fig. 3 (c), passing through the fulcrum V, with its neutral axis outside the section. The maximum compressive strain GJ in this profile is related to the minimum compressive strain HK by the expression GJ = GI – IJ = GI – 0.75 HK. This relationship is established by considering two similar triangles JVI and HVK, where IJ/HK = VE/VF = 0.75. Thus, the maximum compressive strain GJ for profile JK is 0.0035 minus 0.75 times the strain HK on the least compressed edge, constituting assumption (ii) in this section (cl. 39.1b of IS 456).

2 Design Strength of Axially Loaded Short Columns

The Code specifies a maximum compressive strain of $\varepsilon c = 0.002$ at the limit state of collapse in compression (Cl.39.1a of IS:456-2000). For mild steel (Fe 250), the design strength at a strain of 0.002 is calculated as 0.87fy, where fy is the yield strength divided by 1.15. Cold worked deformed bars (Fe 415 and Fe 500) have design strengths obtained from Table A of SP-16, which provides stresses and corresponding strains. This table is preferred over reading from figures (Fig.23A of IS:456-2000) to avoid errors. Linear interpolation in Table A of SP-16 yields the design strengths at a strain of 0.002 as follows:

To meet equilibrium requirements, the resultant compressive force Cc+Cs within the section must equal the external load Po and act in the opposite direction, passing through the point of application [Fig.4]. If fcc and fsc represent the stresses in the concrete and longitudinal steel, respectively, corresponding to the uniform compressive strain εc, it follows that:

$Po=Cc+Cs=fccAc+fscAsc$

$Po=fccAg+(fsc-fcc)Asc$ (Eq.1)

Where:

Ag – Gross cross-section area = Ac+Asc

Asc – Total Area of longitudinal reinforcement = $\sum Asi$

Ac – Net area of concrete in the section = Ag–Asc

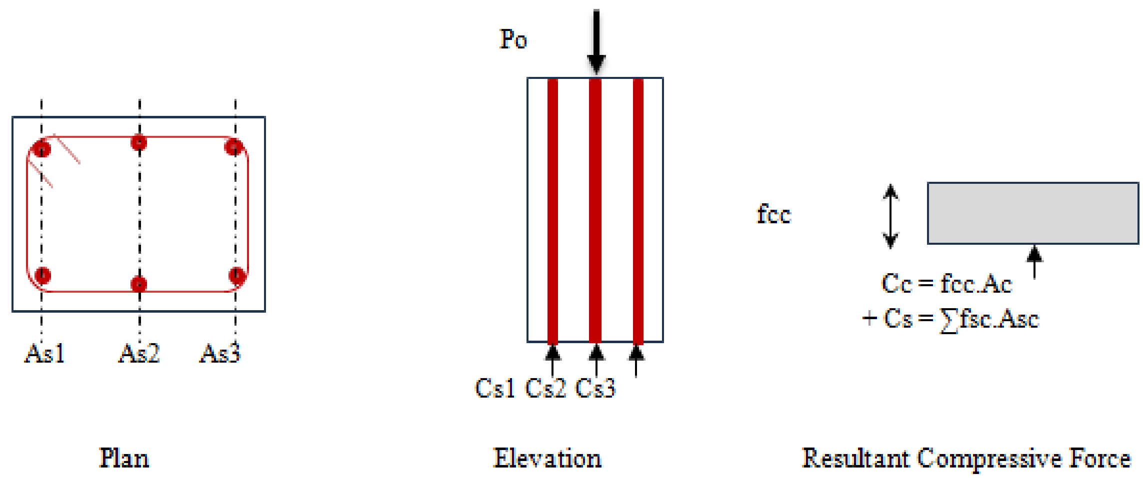


Fig. 4: Axial loading on short column

Short Column Design and Behavior

Design Strength Calculation
For steel grades Fe415 and Fe500:

$0.9fyd +0.05fyd (0.00241-0.001920.002-0.00192)=0.908fyd =0.789fy$ (Fe415)

$0.85fyd +0.05fyd (0.00226-0.001950.002-0.00195)=0.859fyd =0.746fy$(Fe500)

Under 'pure' axial loading, the design strength of a short column (Pu0) can be obtained from Equation 4:

$Pu0=0.447\, fck\, Ag+(fsc -0.447fck)Asc$

With:={0.870 fy for Fe 250 ; 0.789 fy for Fe 415; 0.746 fy for Fe 500}

However, due to the Code's requirement for columns to be designed for 'minimum eccentricities,' Equation 4 cannot be directly applied. When the calculated minimum eccentricity does not exceed 0.05 times the lateral dimension, the Code permits the use of a simplified formula (Equation 5), resulting in a conservative design. Additionally, the Code allows a 5 percent enhancement in load capacity when spiral reinforcement is provided (Equation 6).

To account for the minimum eccentricity, a further reduction is applied to each of the three values, resulting in design strengths of 0.783fy, 0.710fy, and 0.671fy for Fe250, Fe415, and Fe500, respectively. Clause 39.3 of IS

456 specifies 0.67fy as the design strength for all steel grades when designing short columns. Consequently, the assumed design strengths for concrete and steel in the design of short axially loaded columns are 0.4fck and 0.67fy, respectively.

For Short tied columns:

Pu0=0.4fck Ag+(0.67f y −0.4f ck)Asc (Equation 5)

For columns with helical reinforcement:

Pu0=1.05×[0.4fckAg+(0.67f y −0.4fck)Asc] (Equation 6)

3 Behavior of Short Column under Axial Load & Uniaxial Bending

Typically, side columns within a grid of beams and columns experience axial load (P) and uniaxial moment (Mx) causing bending about the major axis (M). Consider a symmetrically reinforced short rectangular column exposed to axial load Pu at an eccentricity (e), resulting in moment Mu leading to column failure.

In Fig. 3, strain profiles IN and EF are presented. For IN, the depth of the neutral axis (kD) is less than D, placing it within the section and causing a maximum compressive strain of 0.0035 on the right edge, with tensile strains on the left forming cracks. This column collapses under Pu and Mu, where IN is the strain profile. Reducing Pu's eccentricity to zero yields EF, resulting in a constant compressive strain of 0.002, another collapse load distinct from the IN scenario. For EF, the neutral axis is at infinity (k = α).

Fig. 3 (c) introduces EF along with two additional strain profiles, IH and JK, intersecting at the fulcrum point V. IH has a neutral axis depth (kD) equal to D, causing collapse due to the maximum compressive strain reaching 0.0035 on the right edge. JK has kD > D, with the maximum compressive strain at the right edge between 0.002 and 0.0035, causing collapse as it satisfies assumption (ii) of section 4.2.

The four strain profiles, IN, EF, JK, and IH from Fig. 4 (b and c), individually lead to the collapse of the same column under four distinct pairs of Pu and Mu. This demonstrates that the column may collapse due to a uniform constant strain throughout (0.002 by EF) or the maximum compressive strain at the right edge satisfying assumption (ii) of section 4.2, irrespective of the strain at the left edge (zero for IH and tensile for IN). The positions of the neutral axis and load eccentricities vary widely:

For EF, kD is infinity, and the eccentricity of the load is zero.

For JK, kD is outside the section (D < kD < α) with an appropriate eccentricity and compressive strain.

For IH, kD is just at the left edge of the section (kD = D), with appropriate eccentricity and compressive strains at both edges.

For IN, kD is within the section (kD < D), with appropriate eccentricity and tensile strains on the left and 0.0035 compressive strain on the right edge.

The gradual increase in the eccentricity of the load Pu from zero results in a sequential shift of strain profiles from EF to JK, IH, and ultimately to IN. It can be inferred that if we extend the eccentricity of the load to infinity, only the moment Mu will act on the column. Denoting the load causing collapse when acting alone as Po and the moment causing collapse when acting alone as Mo, these points are marked on Fig. 5. along the vertical and horizontal axes. They represent extreme points on the Pu versus Mu plot, where any point on this plot signifies a pair of Pu and Mu (of different magnitudes) leading to the collapse of the same column, whether the neutral axis is outside or within the column.

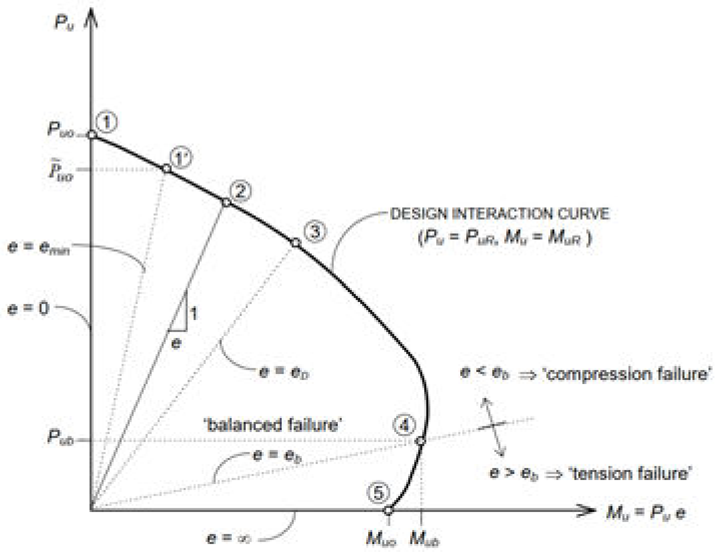


Fig. 5: Typical P-M Interaction Curve (Source ref. S. Unnikrishna Pillai and Devdas Menon et. al., 2003)

Strain Distribution of Column under Axial Load & Uniaxial Bending at Ultimate Limit State

In the special case of uniaxial eccentric compression, two limiting conditions are considered:

1. Zero Eccentricity (e = 0, Mu = 0): This corresponds to axial loading, where the strain across the section is uniform and limited to εcu = 0.002 at the limit state of collapse in compression, as per the Code.
2. Infinite Eccentricity (e = ∞, Pu = 0): This corresponds to 'pure' flexure, with linearly distributed strains across the section and εcu = 0.0035 at the limit state of collapse in flexure.

For general uniaxial eccentric compression (Mu ≠ 0, Pu ≠ 0), where 0 ≤ e < ∞, the strain profile is non-uniform, assumed to be linearly varying across the section. The maximum strain, εcu, in the highly compressed edge varies between 0.002 and 0.0035 at the ultimate limit state, as depicted in Fig. 4. Assumptions made in the ultimate limit state analysis for flexure, excluding the one related to minimum tensile strain εst* at the centroid of the tension steel, are applicable in eccentric compression.

The depth of the neutral axis (NA) with reference to the highly compressed edge can vary from a minimum value xu,min (corresponding to e = ∞) to the maximum value xu = ∞ (no neutral axis, corresponding to e = 0) in eccentric compression. The Code (Cl. 39.1) permits εcu = 0.0035 when the loading eccentricity is sufficiently high to induce tensile strain, coinciding with the edge farthest from the highly compressed edge (xu = D or e = eD). For relatively low loading eccentricities (entire section subjected to non-uniform compression, NA outside the section), the Code limits the strain in the highly compressed edge to a value between 0.002 and 0.0035 using Equation 7.

εcu=0.0035−0.75εc, min for xu≥D (Eq.7)

This equation satisfies limiting strain conditions corresponding to zero and infinite eccentricities, with the point of intersection occurring at 3D/7 from the highly compressed edge, serving as a pivot for strain profiles.

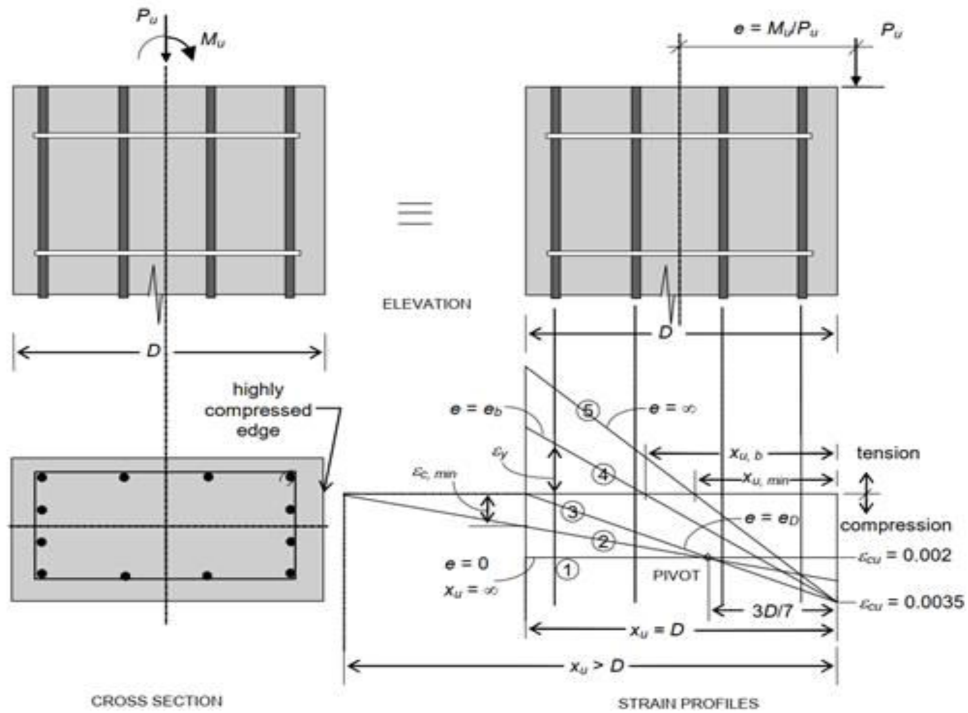


Fig.6: Possible strain profile under limit state in eccentric compression. (Source ref. S. Unnikrishna Pillai and Devdas Menon et. al., 2003)

Ecu = 0.002[1+ ((3d/7)/ (x_u -((3d/7))] for $x_u \geq D$

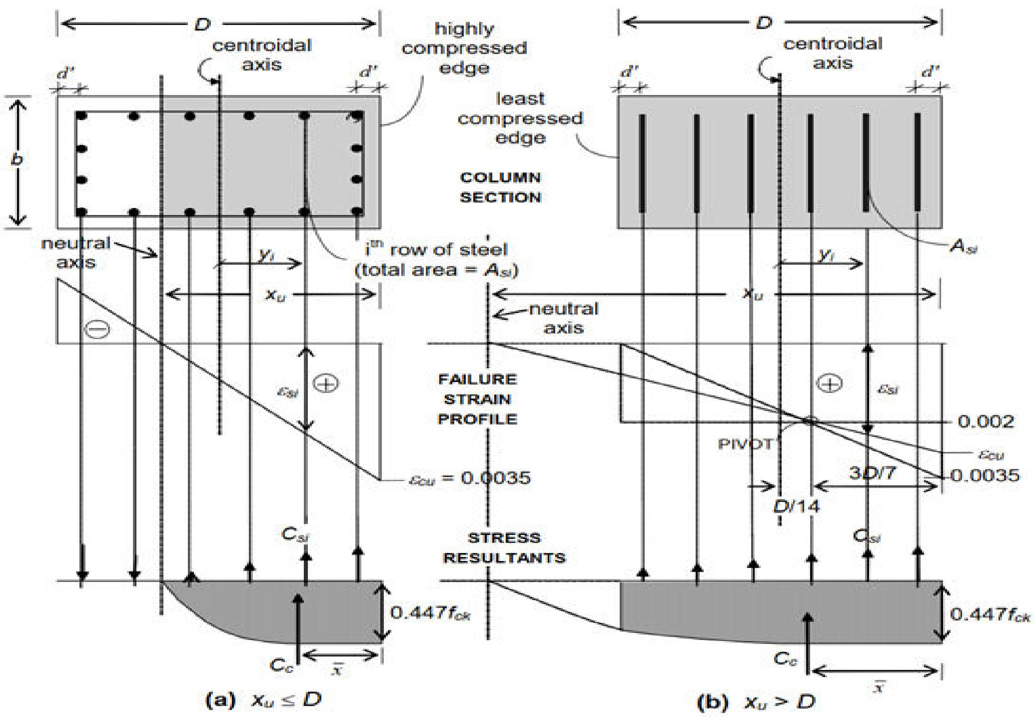


Fig. 10: Analysis of Design Strength of Rectangular Section under Eccentric Compression (Source ref. S. Unnikrishna Pillai and Devdas Menon et. al., 2003)

In Fig. 10, the analysis delves into the design strength of a rectangular section under eccentric compression, exploring two distinct cases:

a. Case 1: High Loading Eccentricity (Neutral Axis Inside Section)
In this scenario [Fig. 10 (a)], the loading eccentricity (e) exceeds the critical value (eD in Fig. 8), resulting in the neutral axis being located inside the column section (xu ≤ D). Equilibrium equations (Eq. 9 and 10) are applicable, but expressions for parameters such as Cc, Cs, Mc, and Ms differ from those in Case 2 due to the unique configuration of the section.

b. Case 2: Low Loading Eccentricity (Neutral Axis Outside Section)
Contrarily, in Case 2 [Fig. 10 (b)], the loading eccentricity is relatively low (e < eD in Fig. 8), causing the neutral axis to lie outside the section. Again, equilibrium equations remain valid, but expressions for Cc, Cs, Mc, and Ms are adjusted accordingly based on the section's characteristics.

$$P_{uR} = C_c + C_s \qquad \text{.............................Eq.9}$$

$$M_{uR} = M_c + M_s \qquad \text{.............................Eq.10}$$

Here, Mc and Ms represent the resultant moments due to Cc and Cs, respectively, with respect to the centroidal axis under consideration.

The equilibrium equations (Eq.9, 10) allow for the direct determination of PuR and MuR for a given neutral axis location (xu/D). However, when given an arbitrary e, determining the design strength (PuR or MuR = PuR e) using Eq.9 requires first locating the neutral axis. This can be achieved by considering moments of forces Cc and Cs about the eccentric line of action of PuR, applying static equilibrium. Unfortunately, the expressions for Cc and Cs in terms of xu are nonlinear, making it challenging to obtain a closed-form solution for xu in terms of e. As a result, a trial-and-error solution is needed.

$$C_c = a\, f_{ck}\, bD \qquad \text{.............................Eq.11}$$

$$M_c = C_c (D/2 - \bar{x}) \qquad \text{.............................Eq.12}$$

Where, a = stress block area factor

x = distance between highly compressed edge and the line of action of Cc

(i.e., centroid of stress block area)

By means of simple integration, it is possible to derive expression for a and x for the case (a): xu ≤ D and for the case (b): xu > D.

$$a = \begin{cases} 0.362\, x_u / D & \text{for } x_u \le D \\ 0.447(1 - 4g/21) & \text{for } x_u > D \end{cases} \qquad \text{.............................Eq.13}$$

$$\bar{x} = \begin{cases} 0.416 x_u & \text{for } x_u \le D \\ (0.5 - 8g/49)\{D/(1 - 4g/21)\} & \text{for } x_u > D \end{cases} \qquad \text{.............................Eq.14}$$

$$\text{where} \quad g = 16 / (7x_u/D - 3)^2 \qquad \text{.............................Eq.15}$$

Similarly, expressions for the resultant force in steel (C_s) and its moment (M_s) with respect to the centroidal axis of bending are obtained.

$$C_s = \sum_{i=1}^{n} \left(f_{si} - f_{ci}\right) A_{si}$$ ……………………….Eq.16

$$M_s = \sum_{i=1}^{n} \left(f_{si} - f_{ci}\right) A_{si}\ y_i$$ ……………………….Eq.17

Where,

A_{si} = area of steel in the ith row (of n rows) [refer Fig.10];

y_i = distance of ith row of steel from the centroidal axis, measured positive in the direction towards the highly compressed edge;

f_{si} = design stress in the ith row (corresponding to the strain ε_{si}) obtainable from design stress-strain curves for steel;

ε_{si} = strain in the ith row, obtainable from strain compatibility conditions (εsi and fsi are assumed to be positive if compressive, and negative if tensile);

f_{ci} = design compressive stress level in concrete, corresponding to the strain $\varepsilon_{ci} = \varepsilon_{si}$ adjoining the ith row of steel, obtainable from the design stress-strain curve for concrete [Note: f_{ci} = 0 if the strain is tensile]:

$$f_{ci} = \begin{cases} 0 & \text{if } \varepsilon_{si} \le 0 \\ 0.447 f_{ck} & \text{if } \varepsilon_{si} \ge 0.002 \\ 0.447 f_{ck}\left[2(\varepsilon_{si}/0.002) - (\varepsilon_{si}/0.002)^2\right] & \text{otherwise} \end{cases}$$ ……………………….Eq.18

Also, from Fig.10, it can be observed (applying similar triangles rule) that:

$$\varepsilon_{si} = \begin{cases} 0.0035\left[\dfrac{x_u - D/2 + y_i}{x_u}\right] & \text{for } x_u \le D \\ 0.002\left[1 + \dfrac{y_i - D/14}{x_u - 3D/7}\right] & \text{for } x_u > D \end{cases}$$ ……………………….Eq.18

It's worth noting that in the case of spiral columns, the Code allows for a 5 percent enhancement in the design strength (both PuR and MuR) specifically for short columns.

4 Preparation of Interaction Curve

To establish the coordinates of the 'design interaction curve,' representing MuR (on the x-axis) and PuR (on the y-axis) for an arbitrary neutral axis depth xu, the following approach is employed using Eqs.9 & 10. The initial value of xu corresponding to PuR = 0, denoted as xu,min (refer to Fig.8), is unknown and requires determination through a trial-and-error process. To initiate this process, an initial assumption of xu,min ≈ 0.15D can be made.

This is likely to yield a negative PuR value, indicating a condition of eccentric tension. By incrementing xu/D in suitable steps (e.g., 0.05 or less), the transition from PuR < 0 to PuR > 0 can be identified. The exact value of the ultimate moment of resistance Muo corresponding to 'pure bending' (PuR = 0) can be obtained through repeated trial-and-error using very fine increments of xu/D.

Once xu,min/D is approximately determined, the coordinates of the design interaction curve can be calculated using Eqs.9 & 10. These values can be tabulated for incremental xu/D values (e.g., increments of 0.05), and the process can be continued until PuR exceeds the maximum limit Puo allowed by the Code (refer to Eq.5). The procedure may also be extended to Puo (e = 0) as given by Eq.4. Given the repetitive nature of this process, it is more convenient to perform it using a computer software. The resulting coordinates (MuR, PuR) of the design interaction curve can then be tabulated and/or plotted. It is beneficial to include the ratios xu/D and e/D in the table for reference. The construction and utilization of a design interaction curve for a typical column section are exemplified and results presented below.

For Column shown in below fig.7 we have plotted the P-M interaction curve for axial compression with uniaxial bending about axis X-X. Also compared the results with SP-16 interaction charts for following load cases.

i) Pu = 2275 kN, Mux = 46.4 kN.m

ii) Pu = 1105 kN, Mux = 125 kN.m

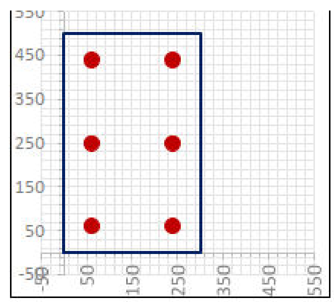
Given Data

Width of column (b)	=	300	mm
Depth of Column (D)	=	500	mm
Grade of concrete	fck =	35	Mpa
Grade of steel	fy =	415	Mpa

Rebars

Bars along width (b)	=	2	Nos	25	mm dia.
Bars along Depth (D)	=	3	Nos	25	mm dia.

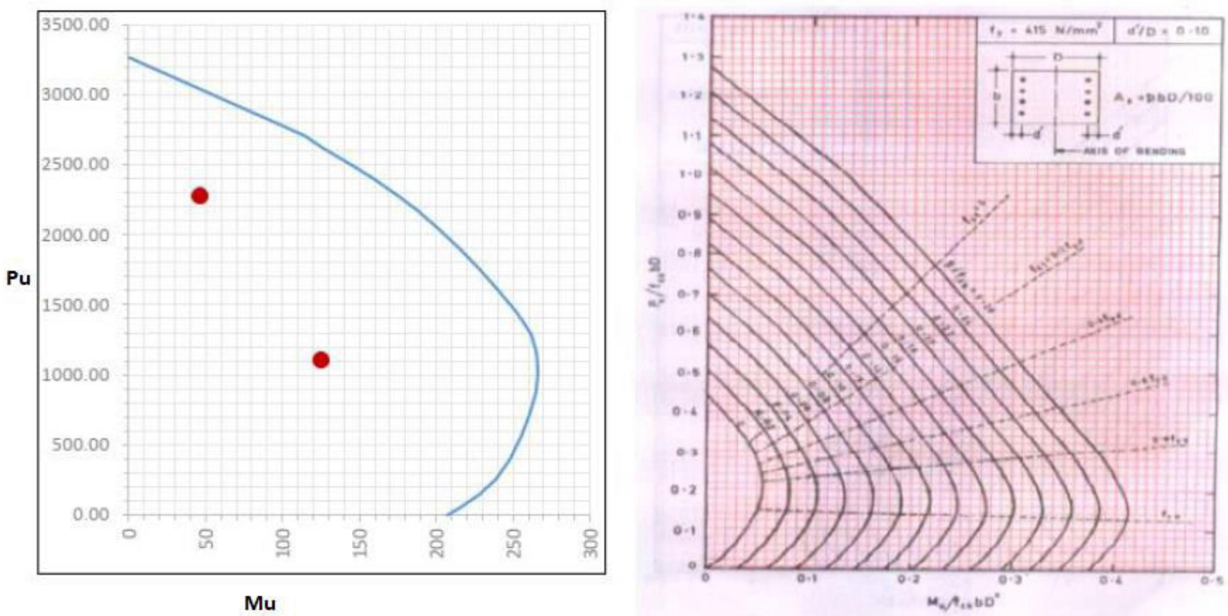
Stirrups	=	8	mm dia.
Clear cover	=	30	mm
effective cover	d' =	50.5	mm
	d'/D =	0.101	
Total steel	Asc =	2945.243	mm2
	pt =	1.963495	%
	pt/fck =	0.0561	



SECTION & REBARS DETAILS

Strain-Compatibility & Equillibrium			Validation with SP-16 chart					
						SP-16		
Pu =	3265.05	KN	Pu/fckbD	=	0.620	Pu =	3255.00	KN
Mu =	207.38	KN.m	Mu/fckbD	=	0.08025	Mu =	210.66	KN.m

(Results from P-M Interaction Chart generated for Uniaxial Bending)



P-M Interaction curve SP-16 Chart

Fig.7: Interaction curve for example under consideration.

CONCLUSION

Understanding the interplay between axial force (P) and moment (M) in structural elements like beams and columns is essential for designing reinforced concrete sections subjected to uniaxial and biaxial loading. The P-M interaction curve, or interaction diagram, provides engineers with a vital tool to assess the effects of combined loads on member capacity, enabling the determination of safe load combinations. Although the IS:456-2000 code simplifies this relationship, recent research suggests potential conservatism at different load levels, highlighting the importance of refining the P-M interaction curve, particularly for biaxially loaded columns. Proposed methods aim to enhance accuracy and reliability in predicting member capacity under varying loading conditions.

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