International Journal of Applied Engineering & Technology

STABILITY ANALYSIS FOR DISCRETIZED BRAYTON-MOSER EQUATIONS WITH IMPLICIT MODEL PREDICTIVE CONTROL

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ABSTRACT

The nonlinear dynamics of RLC network circuits can be described by Brayton-Moser equations. It has been shown that the continuous-time Brayton-Moser equation can be discretized as a discrete-time nonlinear implicit system. Model predictive control (MPC) is a kind of optimal feedback control in which the control performance over a finite future is optimized and its performance index has a moving initial time and a moving terminal time. Implicit MPC method has been proposed for discretized Brayton-Moser equations that belong to a class of discrete-time nonlinear implicit systems. This study focuses on the stability of implicit MPC for discretized Brayton-Moser equations. The objective of this study is to show the stability criterion for the closed-loop system with the implicit MPC for discretized Brayton-Moser equations.

Index Terms: Model Predictive Control, Brayton-Moser Equation, RLC Network, Nonlinear Dynamics

INTRODUCTION

In recent decades, the mathematical modeling of RLC network circuits has attracted much attention in the field of electrical networks. Recently, it has been shown in [1] that the dynamics of nonlinear RLC circuits including independent and controlled voltage or current sources can be described by the Brayton-Moser equations. In this study, the continuous-time Brayton-Moser equation is discretized as a discrete-time nonlinear implicit system. Thus, the discretized Brayton-Moser equation is introduced to consider the control problem of a class of discrete-time nonlinear implicit systems.

Model predictive control (MPC) is a well-established control method in which the current control input is obtained by solving a finite horizon open-loop optimal control problem using the current state of the system as the initial state. This procedure is repeated at each sampling instant. Thus, MPC is a kind of optimal feedback control in which the control performance over a finite future is optimized and its performance index has a moving initial time and a moving terminal time. MPC is known as one of the most successful control methodologies because it enables control performance to be optimized while taking into account constraints on state and control variables.

For the MPC problem of nonlinear explicit systems, the stationary conditions that must be satisfied for a performance index to be optimized are well-known as the Euler-Lagrange equations. Several numerical algorithms for solving the MPC control problems have been proposed in [2]-[5]. For nonlinear implicit systems, H^{∞} optimal control problems have been investigated in [6]-[7]. On the other hand, the MPC problem of nonlinear implicit systems has been considered in [8]. The stationary conditions called the generalized Euler-Lagrange equations for the optimal control problem of nonlinear implicit systems have been derived in [8]. Furthermore, the implicit MPC method has been proposed in [9] for discretized Brayton-Moser equations that belong to a class of discrete-time nonlinear implicit systems.

This paper examines the stability problem of the implicit MPC for discretized Brayton-Moser equations. The objective of this study is to establish the stability criterion for the closed-loop system with implicit MPC for discretized Brayton-Moser equations.

In Section 2, we introduce some notation and the system model. In Section 3, we derive the generalized Euler-Lagrange equations for the optimal control problem of the discretized Brayton-Moser equation. In Section 4, we derive the stability condition for the closed-loop system with implicit MPC of discretized Brayton-Moser equations. Finally, concluding remarks are described in Section 5.

NOTATION AND SYSTEM MODEL

Let $\mathbb{R}$ denote a set of real numbers. Let $\mathbb{R}_+$ and $\mathbb{Z}_+$ denote the sets of nonnegative real numbers and integers, respectively. Let $\mathbb{N}$ denote the sets of positive integers.

For a matrix $A \in \mathbb{R}^{n\times n}$, the transpose and the inverse of A are denoted by A^T and A^{-1}, respectively. The determinant and rank of a matrix A are denoted by $\det(A)$ and $\mathrm{rank}(A)$, respectively. Let I denote the identity matrix.

A function $\alpha: \mathbb{R}_+ \to \mathbb{R}_+$ is said to belong to class K if it is continuous, strictly increasing and $\alpha(0) = 0$. A function $\alpha: \mathbb{R}_+ \to \mathbb{R}_+$ is said to belong to class K$^\infty$ if $\alpha \in K$ and $\lim_{s\to\infty} \alpha(s) = \infty$.

For a scalar function $\phi(x): \mathbb{R}^n \to \mathbb{R}$ the differentiation of $\phi(x)$ with respect to $x \in \mathbb{R}^n$ is defined by

$$\frac{\partial\phi(x)}{\partial x} := \begin{bmatrix}\frac{\partial\phi(x)}{\partial x_1} & \frac{\partial\phi(x)}{\partial x_2} & \cdots & \frac{\partial\phi(x)}{\partial x_n}\end{bmatrix}.$$

The Jacobian matrix of a vector-valued function $F(x): \mathbb{R}^n \to \mathbb{R}^n$ is defined by

$$\frac{\partial F(x)}{\partial x} := \begin{bmatrix}\frac{\partial F_1(x)}{\partial x_1} & \cdots & \frac{\partial F_1(x)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_n(x)}{\partial x_1} & \cdots & \frac{\partial F_n(x)}{\partial x_n}\end{bmatrix}.$$

We consider RLC network circuits that can be described by the Brayton-Moser equations [1]. Let $n_L \in \mathbb{N}$ and $n_C \in \mathbb{N}$ denote the dimensions of circuit systems. Let $i_L \in \mathbb{R}^{n_L}$ and $v_C \in \mathbb{R}^{n_C}$ denote the inductor currents and capacitor voltages, respectively. Let $A(i_L(t)) \in \mathbb{R}^{n_L\times n_L}$ and $C(v_C(t)) \in \mathbb{R}^{n_C\times n_C}$ denote the inductance and capacitance matrices, respectively, both assumed to be positive definite. Let $W(i_L, v_C): \mathbb{R}^{n_L} \times \mathbb{R}^{n_C} \to \mathbb{R}$ be a scalar function called the mixed potential function. A general form of the continuous Brayton-Moser equation has been proposed in [1].

$$A(i_L(t))\frac{di_L(t)}{dt} = \frac{\partial W(i_L, v_C)}{\partial i_L}$$

$$C(v_C(t))\frac{dv_C(t)}{dt} = -\frac{\partial W(i_L, v_C)}{\partial v_C}$$

In this study, we consider the discretized Brayton-Moser equation with controlled voltage inputs. Let the state $x \in \mathbb{R}^n$ be defined by $x := [i_L^T \quad v_C^T]^T$, where $n = n_L + n_C$. Let $u \in \mathbb{R}^m$ denote the controlled voltage inputs. The discretized Brayton-Moser equation can be written as the following system model:

$$E(x(t))x(t+1) = E(x(t))x(t) + F(x(t), u(t)) \tag{1}$$

where $E(x(t))$ and $F(x(t), u(t))$ are given by

$$E(x(t)) = \begin{bmatrix}A(i_L(t)) & 0 \\ 0 & C(v_C(t))\end{bmatrix}$$

$$F(x(t), u(t)) = \begin{bmatrix}\Delta t\left(\frac{\partial W(i_L, v_C)}{\partial i_L}\right)^T \\ \Delta t\left(\frac{\partial W(i_L, v_C)}{\partial v_C}\right)^T + Bu\end{bmatrix}$$

Therein, Δt and B denote the sampling time and input coefficient, respectively.

For sufficiently large n, it is impossible to derive the Jacobian matrix of $E(x(t))^{-1}F(x(t),u(t))$, even if we utilize a symbolic math software such as Mathematica and Maple. In this case, to derive the generalized Euler-Lagrange equations is useful for solving this difficulty.

Next, we introduce some preliminary results.

Lemma 1 ([10]): Consider a system $x(t+1)=F(x(t))$, where $F(0)=0$. Suppose that there exist a Lyapunov function $V(x):\mathbb{R}^n\to\mathbb{R}_+$, class K^∞ functions α_1, α_2 and a positive definite function α_3 satisfying all the following conditions:

$$V(x)\geq\alpha_1(\|x\|)$$

$$V(x)\leq\alpha_2(\|x\|)$$

$$V(F(x))-V(x)\leq-\alpha_3(\|x\|)$$

Then, the origin $x=0$ is asymptotically stable.

Lemma 1 is well known as Lyapunov stability theory. The following lemma is well known as implicit function theorem.

Lemma 2 ([11]): Let $f(x,y):\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}^n$ be continuously differentiable function. For each point (x,y) of an open set $S\subset\mathbb{R}^n\times\mathbb{R}^m$, suppose that $f(x,y)=0$ and the Jacobian matrix $\partial f(x,y)/\partial y$ is nonsingular. Then, there exist neighborhoods $W\subset\mathbb{R}^n$ of x and $U\subset\mathbb{R}^m$ of y such that for each $x\in W$ the equation $f(x,y)=0$ has a unique solution $y\in U$. Moreover, this solution can be given as a continuously differentiable function $y=g(x)$.

MODEL PREDICTIVE CONTROL

In this section, we consider the model predictive control problem of system (1). Using the variational principle, we analytically derive the stationary conditions that must be satisfied for a performance index to be optimized. The control input at each time t is determined so as to minimize the performance index given by

$$J=\varphi(x(t+N))+\sum_{k=t}^{t+N-1}L(x(k),u(k)) \tag{2}$$

Therein, $N\in\mathbb{N}$ denotes the length of prediction horizon. $\varphi:\mathbb{R}^n\to\mathbb{R}_+$ and \$$L:\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}_+$ are so-called terminal cost function and stage cost function, respectively, and assumed to be continuously differentiable functions with $\varphi(0)=0$ and $L(0,0)=0$.

The minimization problem of (2) subject to (1) can be reduced to the minimization of the following performance index introduced using the costate $\lambda\in\mathbb{R}^n$ associated with system equation (1):

$$\bar{J}=\varphi(x(t+N))+\sum_{k=t}^{t+N-1}\Big[L(x(k),u(k))$$
$$+\lambda^T(k+1)\{\hat{F}(x(k),u(k))-E(x(k))x(k+1)\}\Big] \tag{3}$$

where $\hat{F}$ is defined by

$$\hat{F}=E(x(t))x(t)+F(x(t),u(t)).$$

Let $\hat{H}$ denote the Hamiltonian defined by

$$\hat{H}:=L(x(k),u(k))+\lambda^T(k+1)\hat{F}(x(k),u(k)). \tag{4}$$

In the above equation, note that the Hamiltonian considered here is different from the conventional Hamiltonian for standard optimal control problem which is defined using F instead of $\hat{F}$.

On the basis of the variational principle, it has been shown in [9] that we obtain the necessary conditions for a stationary value of $\bar{J}$ over the horizon $(t \leq k \leq t+N)$ as follows.

$$E(x(t))x(t+1) = E(x(t))x(t) + F(x(t),u(t)) \tag{5a}$$

$$\lambda^T(t+N)E(x(t+N-1)) = \frac{\partial\varphi(x(t+N))}{\partial x(t+N)} \tag{5b}$$

$$\lambda^T(k)E(x(k-1)) = \frac{\partial\hat{H}(x(k),\lambda(k+1),u(k))}{\partial x(k)} - \lambda^T(k+1)\frac{\partial\{E(x(k))x(k+1)\}}{\partial x(k)} \tag{5c}$$

$$\frac{\partial\hat{H}(x(k),\lambda(k+1),u(k))}{\partial u(k)} = 0 \tag{5d}$$

A well-known difficulty in solving nonlinear optimal control problems is that the obtained stationary conditions cannot be solved analytically in general. A fast numerical algorithm for solving the above stationary conditions has been proposed in [9].

STABILITY ANALYSIS

In this section, we consider the stability problem of the implicit MPC for discretized Brayton-Moser equations. It is known that the regularity of the solution is effected by the rank of $E(x(t))$. Thus, it is difficult to analyze the solution of system (1) for the case where $\mathrm{rank}E(x(t))$ varies with $x(t)$. To avoid this difficulty, we impose the following assumption.

Assumption 1: There exist nonsingular matrices $S, T \in \mathbb{R}^{n\times n}$ such that

$$E(x(t)) = S\begin{bmatrix}\hat{E}(x(t)) & 0\\ 0 & 0\end{bmatrix}T, \tag{6}$$

$$\det(\hat{E}(x(t))) \neq 0, \tag{7}$$

$$S^{-1}F(x(t),u(t)) = \begin{bmatrix}G(x(t),u(t))\\ R(x(t))\end{bmatrix}, \tag{8}$$

are satisfied for all $x(t)$, where $\hat{E}(x(t)): \mathbb{R}^n \to \mathbb{R}^{r\times r}$,

$R(x(t)): \mathbb{R}^n \to \mathbb{R}^{n-r}$.

Let $w \in \mathbb{R}^r$ and $z \in \mathbb{R}^{n-r}$ be defined by

$$\begin{bmatrix}w(t)\\ z(t)\end{bmatrix} := Tx(t). \tag{9}$$

Under Assumption 1, we can rewrite system (1) as the following form:

$$S\begin{bmatrix}\hat{E}(w(t),z(t)) & 0\\ 0 & 0\end{bmatrix}TT^{-1}\begin{bmatrix}w(t+1)\\ z(t+1)\end{bmatrix} =$$

$$S\begin{bmatrix}\hat{E}(w(t),z(t)) & 0\\ 0 & 0\end{bmatrix}TT^{-1}\begin{bmatrix}w(t)\\ z(t)\end{bmatrix} + S\begin{bmatrix}G(w(t),z(t),u(t))\\ R(w(t),z(t))\end{bmatrix}$$

Then, we obtain the following equation.

$$\begin{bmatrix}\hat{E}(w(t),z(t)) & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}w(t+1)\\ z(t+1)\end{bmatrix} =$$

$$\begin{bmatrix} \hat{E}(w(t),z(t)) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w(t) \\ z(t) \end{bmatrix} + \begin{bmatrix} G(w(t),z(t),u(t)) \\ R(w(t),z(t)) \end{bmatrix}$$

Let $\hat{G}(w(t),z(t),u(t))$ be defined by

$\hat{G}(w(t),z(t),u(t)) = \hat{E}(w(t),z(t))w(t) + G(w(t),z(t),u(t))$.

Consequently, it can be seen that we can rewrite system (1) as the following form:

$$\begin{bmatrix} \hat{E}(w(t),z(t))w(t+1) \\ 0 \end{bmatrix} = \begin{bmatrix} \hat{G}(w(t),z(t),u(t)) \\ R(w(t),z(t)) \end{bmatrix} \quad (10)$$

Remark 1: Assumption 1 implies that the system can be divided into the dynamical equation and the static constraint as shown in (10) and its structure doesn't vary with $x(t)$. Hence, there is a limitation on the variability of algebraic constraints imposed on the system. However, there many constrained systems that satisfy Assumption 1 in mechanical and electrical systems.

Next, the following assumption is imposed to guarantee the regularity of the solution.

Assumption 2: The Jacobian matrix $\partial R(w,z)/\partial z$ is nonsingular for all $w(t)$, $z(t)$.

Here, we state the following theorem.

Theorem 1: Suppose that Assumptions 1-2 are satisfied. Then, there exists the unique solution of system (1).

Proof: Note that the regularity of solution of system (1) is equivalent to the one of system (10), because T in (9) is assumed to be nonsingular. Consider x and y of $f(x,y)$ in Lemma 2 as $x = w$ and $y = z$, respectively. For given $w(t)$, it is obvious from Lemma 2 that there exists the unique solution $z(t) = g(w(t))$ satisfying $R(w(t),g(w(t))) = 0$. Consequently, $w(t+1)$ can be uniquely determined by $\hat{E}^{-1}\hat{G}(w(t),g(w(t)),u(t))$. Likewise, for given $w(t+1)$, we can uniquely determine $z(t+1)$ using Lemma 2. Repeating this procedure, we can conclude that system (10) has a unique solution. This completes the proof.

Assumption 3: $G(0,0,0) = 0$ and $z = g(0) = 0$ at $x = 0$, $u = 0$, i.e., the origin of w and z is the equilibrium point.

Let $X(t)$ be defined by

$$X(t) := \begin{bmatrix} x(t) \\ x(t+1) \\ \vdots \\ x(t+N) \end{bmatrix} \quad (11)$$

Let $U^*(t)$ denote the sequence of the optimal control input over the prediction horizon defined by

$$U^*(t) := \begin{bmatrix} u^*(t) \\ \vdots \\ u^*(t+N-1) \end{bmatrix} := \arg\min_{U(t)} J \text{ subject to (1).} \quad (12)$$

Likewise, let $X^*(t)$ denote the optimal state sequence of the closed-loop system over the prediction horizon using $U^*(t)$.

Assumption 4: There exists the unique solution $X^*(t)$ and $U^*(t)$ that satisfy generalized Euler-Lagrange equations (5).

Let a function $V(x(t))$ be defined by

$$V(x(t)) := J(X^*(t),U^*(t)). \quad (13)$$

Let $\hat{U}^*(t+1)$ be defined by

$$\hat{U}^*(t+1) := \begin{bmatrix} u^*(t+1) \\ \vdots \\ u^*(t+N-1) \\ u(t+N) \end{bmatrix}. \quad (14)$$

Therein, the final optimal control input $u^*(t+N)$ is replaced with any feasible control input $u(t+N)$. Accordingly, let $X^*(t+1)$ be the state sequence of the closed-loop system using $U^*(t+1)$.

Here, we introduce the well-known standard assumption for the stability analysis of MPC systems.

Assumption 5: There exists a function $\alpha \in K^{\infty}$ such that

$$V(x(t)) \le \alpha(\|x(t)\|) \quad (15)$$

is satisfied for all t.

Note that if there exists a positive constant ρ such that

$$\|u^*(t)\| \le \rho\|x(t)\|$$

is satisfied for all t, then Assumption 5 is satisfied. Thereby, Assumption 5 is called the weak controllability assumption [10].

Here, we provide the stability criteria for the closed-loop system using the implicit MPC.

Theorem 2: Under Assumptions 1-5, the closed-loop system using control input $U^*(t)$ is asymptotically stable at the origin if there exists $u(t)$ such that the following inequality is satisfied for all .t

$$\varphi(x(t+1)) - \varphi(x(t)) \le -L(x(t), u(t)) \quad (16)$$

Proof: Because of Assumption 2, $z(t)$ and $x(t)$ can be given by

$$z(t) = g(w(t)),$$

$$x(t) = T^{-1}\begin{bmatrix} w(t) \\ g(w(t)) \end{bmatrix} := h(w(t))$$

Here, we consider the explicit system in (10).

$$w(t+1) = D(w(t), u(t)) \quad (17)$$

where $D(w(t), u(t))$ is given by

$$D(w(t), u(t)) := \hat{E}^{-1}(w(t), g(w(t)))\hat{G}(w(t), g(w(t)), u(t)).$$

For system (17), we can apply Lemma 1 as shown below. Let $\hat{V}$, $\hat{\varphi}$, $\hat{L}$ be given by

$$\hat{V}(w(t)) = V(h(w(t))),$$

$$\hat{\varphi}(w(t)) = \varphi(h(w(t))),$$

$$\hat{L}(w(t), u(t)) = L(h(w(t)), u(t)),$$

respectively. Now, using the relation

$$J(X^*(t+1), U^*(t+1)) \le J\left(\hat{X}^*(t+1), \hat{U}^*(t+1)\right) \quad (18)$$

we have the following:

$$\hat{V}(w(t+1)) = \sum_{k=t+1}^{t+N} L(x^*(k), u^*(k)) + \varphi(x^*(t+N+1))$$

$$\leq \textstyle\sum_{k=t+1}^{t+N-1} L(x^*(k), u^*(k)) + L(x^*(t+N), u(t+N)) + \varphi(x(t+N+1)) \quad =: \hat{V}^+(w(t+1)) \quad (19)$$

Let $\hat{V}^+(w(t+1))$ be defined as above. Using the above inequality, we have the following:

$$\hat{V}(w(t+1)) - \hat{V}(w(t)) \leq \hat{V}^+(w(t+1)) - \hat{V}(w(t))$$

$$= -L(x(t), u^*(t)) + L(x^*(t+N), u(t+N))$$

$$+\varphi(x(t+N+1)) - \varphi(x^*(t+N)) \quad (20)$$

From the assumption in Theorem 2, we can see that there exists $u(t+N)$ such that the following inequality holds.

$$\varphi(x(t+N+1)) - \varphi(x^*(t+N)) \leq -L(x^*(t+N), u(t+N)) \quad (21)$$

Substituting (21) into (20) yields

$$\hat{V}(w(t+1)) - \hat{V}(w(t)) \leq -L(x(t), u^*(t)) \quad (22)$$

Here, note that there exists a positive constant ν such that the following inequalities hold.

$$\hat{V}(w(t)) \geq L(x(t), u^*(t))$$

$$\geq L(h(w(t)), 0)$$

$$\geq \nu \|w(t)\| \quad (23)$$

Therefore, it follows that

$$\hat{V}(w(t+1)) - \hat{V}(w(t)) \leq -\nu \|w(t)\| \quad (24)$$

Consequently, under Assumption 5, we can see that there exist K^∞ functions α_1, α_2 such that the following inequalities are satisfied.

$$\alpha_1(\|w(t)\|) \leq \hat{V}(w(t)) \leq \alpha_2(\|w(t)\|)$$

$$\hat{V}(w(t+1)) - \hat{V}(w(t)) \leq -\alpha_1(\|w(t)\|)$$

Hence, using Lemma 1, we can conclude that $w(t) = 0$ is asymptotically stable. Then, $z(t) = 0$ is also asymptotically stable because of Assumption 3. Consequently $x(t) = 0$ is asymptotically stable. This completes the proof.

Remark 2: The stabilization problem of nonlinear systems usually can be reduced to finding the so-called control Lyapunov function. Note that the difficulty of finding $u(t)$ satisfying inequality (16) in Theorem 2 is almost same as the difficulty of finding the control Lyapunov function φ.

CONCLUSION

In this study, the continuous-time Brayton-Moser equation is discretized as a discrete-time nonlinear implicit system. Then, we examined the model predictive control problem of discretized Brayton-Moser equations that belongs to a class of discrete-time nonlinear implicit systems with a particular structure. Using the variational principle, the stationary conditions that must be satisfied for a performance index to be optimized have been derived. Also, a fast numerical algorithm for solving the model predictive control problem of discretized Brayton-Moser equations has been established in [9] using C/GMRES algorithm. In this paper, we consider the stability problem of the implicit MPC for discretized Brayton-Moser equations. The stability criterion for the closed-loop system with implicit MPC for discretized Brayton-Moser equations has been shown in this paper. To establish the MPC method for a class of implicit systems belonging to spatiotemporal dynamic systems [12]-[13], delay systems [14]-[15], uncertain systems [16]-[17], and probabilistic constrained systems [18]-[19] is a possible future work.

ACKNOWLEDGEMENT

This work was supported in part by Japan Society for the Promotion of Science, Grant-in-Aid for Scientific Research (C), Grant Number JP18K04216.

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