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STRONGEST INTUITIONISTIC ANTI FUZZY BG-IDEALS IN BG-ALGEBRA

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ABSTRACT

In this paper, we investigate some properties of strongest intuitionistic anti fuzzy $\alpha -$ relation on BG-ideals and intuitionistic anti fuzzy BG-bi-ideals in BG-algebra.

Keywords: BG-algebra, intuitionistic fuzzy BG-ideal, intuitionistic anti fuzzy BG-ideals, strongest intuitionistic anti fuzzy $\alpha -$ relation on BG-ideals and intuitionistic anti fuzzy BG-bi-ideals.

1. INTRODUCTION

Iseki and Tanaka [3] introduced two classes of abstract algebra BCK-algebra and BCI-algebra.The concept of fuzzy set was introduced by zadeh [8] in 1965. Since then these ideas have been applied to other algebraic structure such as groups, rings, modules, vector spaces and topological. Kim and Kim[4] introduced the notion of BG-algebra, which is a generalization of B-algebra. Muthuraj et al.[5] investigated properties of fuzzy BG-ideals in BG-algebra. In 2005, zarandi and saeid [7] introduced the new concept called intuitionistic fuzzy ideals of BG-algebra. Biwas introduced the concept of anti fuzzy subgroups of groups. Priya and Ramachandran[6] introduced the new algebraic structure PS-algebra, which is an another generalization of BCK/BCI algebras and investigated its properties. And also they discussed the notion of anti fuzzy PS-ideals of PS-algebra. Especially, we introduce the notion of strongest intuitionistic anti fuzzy relation on BG-ideals in BG-algebra and obtain some of its results.

2.Preliminaries:

Definition 2.1

A BG-algebra is a non empty set X with a constant 0 and a binary operation "$*$" satisfying the following axioms:

(i) $x * x = 0$

(ii) $x * 0 = x$

(iii) $(x * y) * (0 * y) = x \; \forall x, y \in X.$

For brevity we also call X BG-algebra. A binary relation '$\leq$' on X can be defined by $x \leq y$ if and only if $x * y = 0$.

A non-empty set S of a BG-algebra X is called a BG-subalgebra of X if $x * y \epsilon S \; \forall \; x, y \epsilon S.$

Definition 2.2

An intuitionistic fuzzy set (μ_A, γ_A) in X is called an intuitionistic fuzzy BG-ideal of X if it satisfies the following inequalities:

(i) $\mu_A(0) \geq \mu_A(x)$

(ii) $\mu_A(x) \geq min\{\mu_A(x * y), \mu_A(y)\}$

(iii) $\mu_A(x * y) \geq min\{\mu_A(x), \mu_A(y)\}$

(iv) $\gamma_A(0) \leq \gamma_A(x)$

(v) $\gamma_A(x) \leq max\{\gamma_A(x*y), \gamma_A(y)\}$

(vi) $\gamma_A(x*y) \leq max\{\gamma_A(x), \gamma_A(y)\} \forall\, x, y \epsilon X$

Definition 2.3

An intuitionistic fuzzy set (μ_A, γ_A) in X is called an intuitionistic anti fuzzy BG-ideal of X if it satisfies the following inequalities:

(i) $\mu_A(0) \leq \mu_A(x)$

(ii) $\mu_A(x) \leq max\{\mu_A(x*y), \mu_A(y)\}$

(iii) $\mu_A(x*y) \leq max\{\mu_A(x), \mu_A(y)\}$

(iv) $\gamma_A(0) \geq \gamma_A(x)$

(v) $\gamma_A(x) \geq min\{\gamma_A(x*y), \gamma_A(y)\}$

(vi) $\gamma_A(x*y) \geq min\{\gamma_A(x), \gamma_A(y)\} \forall\, x, y \epsilon X$

Definition 2.4

An intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in X is said to be a intuitionistic fuzzy BG-bi-ideal if it satisfies the condition,

$\mu_A(x*w*y) \geq min\{\mu_A(x), \mu_A(y)\}$

$\gamma_A(x*w*y) \leq max\{\gamma_A(x), \gamma_A(y)\} \forall x, y, w \epsilon X.$

3.Strongest Intuitionistic Anti Fuzzy α −Relation on BG-Ideals

Definition 3.1

Let α be a intuitionistic fuzzy subset of X. The strongest intuitionistic anti fuzzy α −relation on BG-algebra X is the intuitionistic fuzzy subset μ_α of $X \times X$ is given by

$\mu_\alpha(x, y) = max\{\alpha(x), \alpha(y)\}$

$\gamma_\alpha(x, y) = min\{\alpha(x), \alpha(y)\} \forall x, y \epsilon X$

Theorem 3.2

Let μ_α be the strongest intuitionistic anti fuzzy α −relation on BG-algebra X.If α is an intuitionistic anti fuzzy BG-ideal of a BG-algebra X, the μ_α is an intuitionistic anti fuzzy BG-ideal of $X \times X$.

Proof:

Let $(\mu_\alpha, \gamma_\alpha)$be a intuitionistic anti fuzzy BG-ideal of a BG-algebra X.

Let $(x, y) \epsilon X \times X$

(i) $\mu_\alpha(0,0) = max\{\alpha(0), \alpha(0)\}$

$= \mu_\alpha(x,y) \ \forall x, y \epsilon X \times X$

(ii) $\mu_\alpha(x_1, x_2) = max\{\mu_\alpha(x_1), \mu_\alpha(x_2)\}$

$\leq max\{max\{\mu_\alpha(x_1*y_1), \mu_\alpha(y_1)\}, max\{\mu_\alpha(x_2*y_2), \mu_\alpha(y_2)\}\}$

$= max\{\mu_\alpha(x_1*y_1), (x_2*y_2), \mu_\alpha(y_1, y_2)\} \forall\, x_1, x_2, y_1, y_2 \epsilon X \times X$

(iii) $\mu_\alpha(x_1,x_2) * (y_1, y_2) = max\{\mu_\alpha(x_1,x_2), \mu_\alpha(y_1, y_2)\}$

$= max\{\mu_\alpha(x_1,x_2), \mu_\alpha(y_1, y_2)\} \forall\ x_1, x_2, y_1, y_2 \epsilon X \times X$

(iv) $\gamma_\alpha(0,0) = min\{\alpha(0), \alpha(0)\}$

$= \gamma_\alpha(x,y)\ \ \forall x, y \epsilon X \times X$

(v) $\gamma_\alpha(x_1,x_2) = min\{\gamma_\alpha(x_1), \gamma_\alpha(x_2)\}$

$\geq min\{min\{\gamma_\alpha(x_1 * y_1), \gamma_\alpha(y_1)\}, max\{\gamma_\alpha(x_2 * y_2), \gamma_\alpha(y_2)\}\}$

$= min\{\gamma_\alpha(x_1 * y_1), (x_2 * y_2), \gamma_\alpha(y_1, y_2)\} \forall\ x_1, x_2, y_1, y_2 \epsilon X \times X$

(vi) $\gamma_\alpha((x_1,x_2) * (y_1, y_2)) = min\{\gamma_\alpha(x_1,x_2), \gamma_\alpha(y_1, y_2)\}$

$= min\{\gamma_\alpha(x_1,x_2), \gamma_\alpha(y_1, y_2)\} \forall\ x_1, x_2, y_1, y_2 \epsilon X \times X$

Therefore, ($\mu_\alpha, \gamma_\alpha$) is an intuitionistic anti fuzzy BG-ideal of $X \times X$.

Theorem 3.3

Let ($\mu_\alpha, \gamma_\alpha$) be the strongest intuitionistic anti fuzzy $\alpha -$ relation on BG-algebra X, where α is a intuitionistic fuzzy set of a BG-algebra X . If α is an intuitionistic anti fuzzy BG-subalgebra of a BG-algebra X, then $(\mu_\alpha, \gamma_\alpha)$ is an intuitionistic anti fuzzy BG-algebra of $X \times X$.

Proof:

Let α be an intuitionistic anti fuzzy BG-subalgebra of a BG-algebra X,

Then $x_1, x_2, y_1, y_2 \epsilon X$

Then $\mu_\alpha((x_1,y_1) * (x_2,y_2)) = \mu_\alpha(x_1 * x_2, y_1 * y_2)$

$= max\{\alpha(x_1 * x_2), \alpha(y_1, y_2)\}$

$\leq max\{max\{\alpha(x_1), \alpha(x_2)\}, max\{\alpha(y_1), \alpha(y_2)\}\}$

$= max\{\mu_\alpha(x_1,y_1), \mu_\alpha(x_2,y_2)\} \forall\ x_1, x_2, y_1, y_2 \epsilon X \times X.$

Similarly

$\gamma_\alpha((x_1,y_1) * (x_2,y_2)) = \gamma_\alpha(x_1 * x_2, y_1 * y_2)$

$= min\{\alpha(x_1 * x_2), \alpha(y_1, y_2)\}$

$\geq min\{min\{\alpha(x_1), \alpha(x_2)\}, min\{\alpha(y_1), \alpha(y_2)\}\}$

$= min\{\gamma_\alpha(x_1,y_1), \gamma_\alpha(x_2,y_2)\} \forall\ x_1, x_2, y_1, y_2 \epsilon X \times X.$

Therefore ($\mu_\alpha, \gamma_\alpha$) is an intuitionistic anti fuzzy BG-algebra of $X \times X$.

Theorem 3.4

Let ($\mu_\alpha, \gamma_\alpha$) be the strongest intuitionistic anti fuzzy $\alpha -$ relation on BG-algebra X, where α is a intuitionistic fuzzy set of a BG-algebra X . If μ_α is an intuitionistic anti fuzzy BG-subalgebra of $X \times X$, then α is an intuitionistic anti fuzzy BG-subalgebra of a BG-algebra X.

Proof:

Let $x, y \epsilon X$

Now, $\alpha(x, y) = max\{\alpha(x * y), \alpha(x * y)\}$

$=max\{max\{\alpha(x), \alpha(y)\}, max\{\alpha(x), \alpha(y)\}\}$

$= max\{\alpha(x), \alpha(y)\} \forall x, y \epsilon X.$

Similary

$\alpha(x, y) = min\{\alpha(x * y), \alpha(x * y)\}$

$=min\{min\{\alpha(x), \alpha(y)\}, min\{\alpha(x), \alpha(y)\}\}$

$= min\{\alpha(x), \alpha(y)\} \ \forall x, y \epsilon X.$

Hence α is an intuitionistic anti fuzzy BG-subalgebra of X.

4. Intuitionistic Anti Fuzzy BG-bi-Ideal

Definition 4.1

An intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in X is said to be a intuitionistic anti fuzzy BG-bi-ideal if it satisfies the condition,

$\mu_A(x * w * y) \leq max\{\mu_A(x), \mu_A(y)\}$

$\gamma_A(x * w * y) \geq min\{\gamma_A(x), \gamma_A(y)\} \ \forall x, y, w \epsilon X.$

Theorem 4.2

Every intuitionistic anti fuzzy BG-ideal is a intuitionistic fuzzy anti BG-bi-ideal.

Proof :

Let $x * w * y = x * y$

Now, $\mu_A(x * w * y) = \mu_A(x * y)$

$\leq max\{\mu_A(x), \mu_A(y)\}$

Clearly, $\gamma_A(x * w * y) = \gamma_A(x * y)$

$\geq min\{\gamma_A(x), \gamma_A(y)\}$

Hence every intuitionistic anti fuzzy BG-ideal is a intuitionistic anti fuzzy BG-bi-ideal.

Conversely,

We define a intuitionistic fuzzy set $\mu_A: X \to [0,1]$ and $\gamma_A: X \to [0,1]$

Let us assume that $x \leq y$ that is $x * y = 0$

Then $\mu_A(x * w * y) = \mu_A(x * y)$

$= \mu_A(0)$

$\gamma_A(x * w * y) = \gamma_A(x * y)$

$= \gamma_A(0)$

Which is a contradiction

Hence (μ_A, γ_A) is not a intuitionistic anti fuzzy BG-bi-ideal of X.

Example 4.3

We define a intuitionistic fuzzy set $\mu_A: X \to [0,1]$ by

$\mu_A(0) = 0.7, \mu_A(x) = 0.2\ \forall x \neq 0$

Also $\gamma_A: X \to [0,1]$ by

$\gamma_A(0) = 0.2$

$\gamma_A(x) = 0.7\ \forall x \neq 0$

Clearly (μ_A, γ_A) is a intuitionistic anti fuzzy BG-ideal of X.

But (μ_A, γ_A) is not a intuitionistic anti fuzzy BG-bi-ideal of X.

Let $x = 0, w = 1, y = 0$

Then $\mu_A(x * w * y) = \mu_A(0 * 1 * 0)$

$= \mu_A(0 * 1)$

$= \mu_A(1)$

=0.2

$max\{\mu_A(x), \mu_A(y)\} = max\{\mu_A(0), \mu_A(0)\}$

$= \mu_A(0)$

=0.7

Which is a contradiction

Hence $\mu_A(x * w * y) \leq max\{\mu_A(x), \mu_A(y)\}$

Clearly, $\gamma_A(x * w * y) = \gamma_A(0 * 1 * 0)$

$= \gamma_A(0 * 1)$

$= \gamma_A(1)$

=0.7

$min\{\gamma_A(x), \gamma_A(y)\} = min\{\gamma_A(0), \gamma_A(0)\}$

$= \gamma_A(0)$

=0.2

Which is a contradiction

$\gamma_A(x * w * y) \geq min\{\gamma_A(x), \gamma_A(y)\}\ \forall x, y, w \epsilon X.$

Hence (μ_A, γ_A) is not a intuitionistic anti fuzzy BG-bi-ideal of X.

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