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STUDY OF FREE VIBRATION OF TRAPEZOIDAL VISCO ELASTIC PLATE WITH THICKNESS VARIATION

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ABSTRACT

To investigate how a non-homogeneous trapezoidal plate, whose density differs linearly in one direction and whose thickness differs linearly in both the directions, is affected by a 2D linearly variable temperature. S-S-S-S is the boundary condition, and 2-term deflection is considered. The problem's solution is found using the Rayleigh-Ritz approach. Studies have also been conducted on the impact of additional plate factors, such as aspect ratios, taper constants, and no n-homogeneity constant. Accurate calculations are made, and graphical results are displayed.

Keywords: non-homogeneity, thermal gradient, trapezoidal plate, vibration, thickness.

1. INTRODUCTION

Since the 19th century, when German physicist Chladni conducted experiments to find different modes of free vibration, plate theory was used to lessen vibration as well as noise in the complex. Since then, it has grown into a vast and expanding subject that deals with progressively complex problems and offers a wide range of theoretical and academic approaches.In the fields of maritime and aviation engineering, where lightweight structural parts made of orthotropic materials are fundamentally important, the orthotropic trapezoidal plate is often used. In real, they are thin plate constructions with improved stability and stiffness properties. They are frequently subjected to both static as well as time-varying loads in their numerous applications in the various disciplines of contemporary civil, mechanical, and structural engineering. As a result, scholars have long been quite interested in the study of orthotropic trapezoidal plates under various circumstances.The dynamic properties of orthotropic trapezoidal plates have been the subject of extensive investigation since many researchers have used various techniques to analyze these designs dynamically. Electromechanical transducers for mirrors & lenses in optical systems, electronic telephones, naval constructions, nuclear reactor structures, as well as aeronautical fields all require plates of varying thickness. Vibration analysis is crucial for preventing resonance caused by internal or external forces because of the practical significance of these plates. The requirement for solutions to a variety of difficulties involving plates as well as other structures supported on the elastic medium has also increased in the last few years due to the advancement of solid propellant rocket motors, the growing usage of soft filaments in structures of aerospace, and the construction of higher-speed runways. Examples of these practical solid structure interaction difficulties include multistory buildings'floor slabs and building activities in cold climates.

Numerous researchers have been encouraged to pursue vibration analysis as a field of study, according to existing literature. Few of them are classified as beneath. Kumar and Lal [1] investigated the non-homogeneous orthotropic rectangular plates'vibrations having bilinear thickness variation laying on the Winkler foundation. The monoclinic rectangular plates'free transverse vibrations having continuously variable thicknesses and densities were investigated by Kumar and Tomar [2]. The exponential thermal impact on the non-homogeneous orthotropic rectangular plate' vibration with bi-directional linear thickness change has been studied by Johri and Johri [3]. Non-homogeneous circular plates'vibration analysis with non-linear thickness change was performed by Gupta et al. [4] utilizing the differential quadrature technique. The shooting technique for non-linear vibration as well asthe sizzling buckling of heated orthotropic circular plates was covered by Li and Zhou [5]. The non-homogeneity effect on the natural frequencies of elliptic plates'vibration was investigated by Chakraverty et al. [6].The thermal gradient on the trapezoidal plates'vibration having different thicknesses was investigated by Gupta et al. [7]. The Rayleigh-Ritz approach is repurposed for determining the fundamental frequencies. For a trapezoidal plate, the frequencies of the first two vibration modes are derived for a range of values of the aspect ratio, temperature gradient, as well as taper constant. Gupta and Sharma [8] conducted a recent study on the impact of heat on "the free vibration of an orthotropic thin trapezoidal plate with a variation of thickness that is linear.

2. METHOD OF ANALYSIS

To research transverse vibration, a thin, symmetric, non-homogeneous trapezoidal plate having a changeable thickness as well as density was taken.**Fig. 1.1** displays" the plate's geometry.

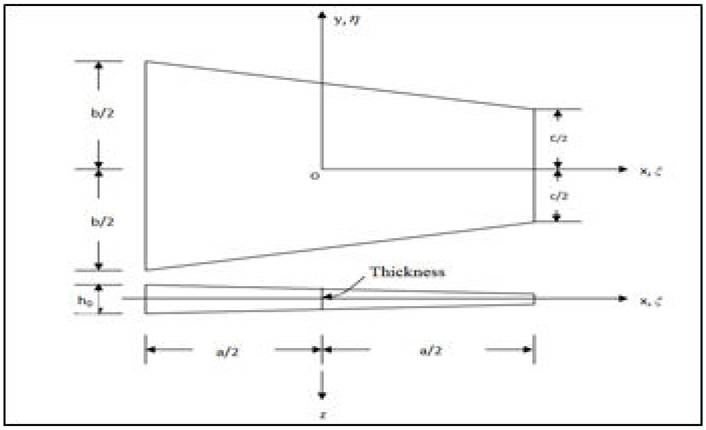


Fig. 1.1 Trapezoidal plate geometry

3. THICKNESS AND DENSITY

Thickness q of trapezoidal plate is taken into account to be bilinear namely,linear in ζ and φ direction, respectively, as:

$$q(\zeta)=q_o\left[1-(1-\beta_1)\left(\zeta+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\varphi+\frac{1}{2}\right)\right] \quad (1)$$

The linear variation in density along ζ a direction that causes non-homogeneity of the plate is as follows:

$$\psi=\psi_0\left[1-(1-\alpha_1)\left(\zeta+\frac{1}{2}\right)\right] \quad (2)$$

The plate's temperature should be viewed as two dimensions, and it is represented as $\varepsilon=\varepsilon_o\left(\frac{1}{2}-\zeta\right)\left(\frac{1}{2}-\varphi\right)$

(3)

When ε the is temperature extra greater than the "reference at some point stage at the boundary of the plate, it is signified by τ_o

The modulus of elasticity for flexible substance isexplained as:

R = $R_O(1-\gamma\varepsilon)$ (4)

Where R_O is the representative of Young's modulus at $\varepsilon = 0$ and γ is studied as the distinction of" slope R.

When the value of ε is changed from Eq. (3) to (4):

$$R = R_O\left(1-\alpha\left(\frac{1}{2}-\zeta\right)\left(\frac{1}{2}-\varphi\right)\right) \quad (5)$$

Where $\alpha = \gamma\varepsilon_O$ $(0 \leq \alpha \leq 1)$.

4. Equation of motion

The following is the guiding differential equation for strain energy K and kinetic energy P:

$$P = \frac{ab}{2} w^2 \int c(\zeta)\psi w^2 dA, \quad (6)$$

$$K = \frac{ab}{2}\int G(\zeta)\left[\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\zeta^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\varphi^2}\right)^2 - 2(1-\nu)\left[\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\zeta^2}\frac{\partial^2 w}{\partial\varphi^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\zeta\partial\varphi}\right)^2\right]\right]dA \quad (7)$$

where G, (ξ) sometimes referred to as flexural rigidity, is determined by

$$G(\xi) = G_O\left\{\left[1-(1-\beta_1)\left(\zeta+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\varphi+\frac{1}{2}\right)\right]\right\}^3 \quad (8)$$

Also

$$G_O = \frac{Rq_O{}^3}{12(1-\nu^2)} \quad (9)$$

is the plate's flexural rigidity at the oblique $\zeta = -\frac{1}{2}$, and A represents the plate area.

On utilizing Eq. (9) and (5), Eq. (8) provides the flexural rigidity value as mentioned below

$$G(\xi) = \frac{Rq_O{}^3}{12(1-\nu^2)}\left\{\left[1-(1-\beta_1)\left(\zeta+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\varphi+\frac{1}{2}\right)\right]\right\}^3 \times\left(1-\alpha\left(\frac{1}{2}-\zeta\right)\left(\frac{1}{2}-\varphi\right)\right) \quad (10)$$

Later, kinetic energy as well as strain energy become, assuming eqs. (1), (2) into eq. (6) and (10) into eq. (7).

$$P = \frac{ab}{2}\psi_o q_o w^2 \int\left[\left\{\left[1-(1-\beta_1)\left(\zeta+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\varphi+\frac{1}{2}\right)\right]\right\}\times\left[1-(1-\alpha_1)\left(\zeta+\frac{1}{2}\right)\right]\right]w^2 dA \quad (11)$$

K =

$$\frac{ab}{2}\frac{R_O q_O{}^3}{12(1-\nu^2)} \int\left\{\left[1-(1-\beta_1)\left(\zeta+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\varphi+\frac{1}{2}\right)\right]\right\}^3 \times\left(1-\alpha\left(\frac{1}{2}-\zeta\right)\left(\frac{1}{2}-\varphi\right)\right)\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\zeta^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\varphi^2}\right)^2 - 2(1-\nu)\left[\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\zeta^2}\frac{\partial^2 w}{\partial\varphi^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\zeta\partial\varphi}\right)^2\right]\right\}dA \quad (12)$$

The deflection function with two terms is regarded

$$w=\left[\zeta+\frac{1}{2}\right]\left[\varphi-\left(\frac{b-c}{2}\right)\zeta+\left(\frac{b+c}{4}\right)\right]$$

"as $\times\left[\zeta-\frac{1}{2}\right]\left[\varphi+\left(\frac{b-c}{2}\right)\zeta-\left(\frac{b+c}{4}\right)\right]$

$$\times\left[A_1+A_2\left(\zeta+\frac{1}{2}\right)\left(\varphi-\left(\frac{b-c}{2}\right)\zeta+\left(\frac{b+c}{4}\right)\right)\times\left(\zeta-\frac{1}{2}\right)\left(\varphi+\left(\frac{b-c}{2}\right)\zeta-\left(\frac{b+c}{4}\right)\right)\right], \tag{13}$$

Where A_1 and A_2 are constants.

$$\varphi_2=\frac{c}{4b}-\frac{\zeta}{2}+\frac{1}{4}+\frac{c\zeta}{2b},\ \varphi_1=-\frac{c}{4b}+\frac{\zeta}{2}-\frac{1}{4}-\frac{c\zeta}{2b}$$

$$\zeta_2=-\frac{1}{2},\ \zeta_1=\frac{1}{2} \tag{14}$$

The Rayleigh-Ritz approach, that is on the basis of the idea of energy conservation—that is, the maximum kinetic energy and the maximum strain" energy should be balanced—is used to compute the frequency in extensions. Consequently, the following equation can be explained by

$$\delta(P-K)=0. \tag{15}$$

Boundary condition (14) is applied to Equations (11) and (12) to obtain

$$P=\frac{ab}{2}\psi_o q_o w^2\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\zeta}{2}-\frac{1}{4}-\frac{c\zeta}{2b}}^{\frac{c}{4b}-\frac{\zeta}{2}+\frac{1}{4}+\frac{c\zeta}{2b}}\left[\left\{\left[1-(1-\beta_1)\left(\zeta+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\varphi+\frac{1}{2}\right)\right]\right\}\times\left[1-(1-\alpha_1)\left(\zeta+\frac{1}{2}\right)\right]\right]w^2\,d\eta\,d\xi \tag{16}$$

$$K=\frac{ab}{2}\frac{R_O q_O{}^3}{12(1-\nu^2)}\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\zeta}{2}-\frac{1}{4}-\frac{c\zeta}{2b}}^{\frac{c}{4b}-\frac{\zeta}{2}+\frac{1}{4}+\frac{c\zeta}{2b}}\left\{\left[1-(1-\beta_1)\left(\zeta+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\varphi+\frac{1}{2}\right)\right]\right\}^3$$
$$\times\left(1-\alpha\left(\frac{1}{2}-\zeta\right)\left(\frac{1}{2}-\varphi\right)\right)\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\zeta^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\varphi^2}\right)^2-2(1-v)\left[\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\zeta^2}\frac{\partial^2 w}{\partial\varphi^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\zeta\partial\varphi}\right)^2\right]\right\}d\varphi\,d\zeta \tag{17}$$

Now Equations (15) becomes:

$$\delta\left(K_1-\lambda^2 P_1\right)=0 \tag{18}$$

Where;

$$P_1 = \frac{ab}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\zeta}{2}-\frac{1}{4}-\frac{c\zeta}{2b}}^{\frac{c}{4b}-\frac{\zeta}{2}+\frac{1}{4}+\frac{c\zeta}{2b}}\left[1-(1-\beta_1)\left(\zeta+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\varphi+\frac{1}{2}\right)\right]\times\left[1-(1-\alpha_1)\left(\zeta+\frac{1}{2}\right)\right]w^2 d\varphi d\zeta \tag{19}$$

$$K_1 = \int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\zeta}{2}-\frac{1}{4}-\frac{c\zeta}{2b}}^{\frac{c}{4b}-\frac{\zeta}{2}+\frac{1}{4}+\frac{c\zeta}{2b}}\left[\left[1-(1-\beta_1)\left(\zeta+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\varphi+\frac{1}{2}\right)\right]\right]^3\times\left(1-\alpha\left(\frac{1}{2}-\zeta\right)\left(\frac{1}{2}-\varphi\right)\right)$$
$$\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\zeta^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\varphi^2}\right)^2-2(1-\nu)\left(\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\zeta^2}\frac{\partial^2 w}{\partial\varphi^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\zeta\partial\varphi}\right)^2\right)\right\}d\varphi d\zeta \tag{20}$$

$$\lambda^2 = \frac{12\psi_o a^4\omega^2(1-\nu^2)}{R_O {q_O}^2} \tag{21}$$

is known asthe frequency parameter.

Eq. (18) includes2 unknowns A_1 as well as A_2 as a result of the substitution of equation (13). It is necessary to evaluate these two unknowns using Eq. (18), in the form shown “below:

$$\frac{\partial}{\partial A_1}\left(K_1-\lambda^2 P_1\right)=0,\ \frac{\partial}{\partial A_2}\left(K_1-\lambda^2 P_1\right)=0. \tag{22}$$

On simplifying Eq. (21), we get:

$$f_{s1}A_1+f_{s2}A_2=0,\ \text{s=1,2}. \tag{23}$$

Where $f_{s1}, f_{s2}\,(s=1,2)$ involves parametric constant as well as the frequency parameter.

The determinant of the coefficient of Eq. (22) should vanish for a non-0 solution. We obtain the frequency equation” as

$$\begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix}=0 \tag{24}$$

Equation (24) shows the 2 values of λ^2 the 1st and 2nd modes of vibration, correspondingly, and can be used to generate the quadratic equation λ^2.

Result and discussion

With the aid of the MAPLE software, frequency modes for a non-homogeneous S-S-S-S trapezoidal plate must be determined for various combinations of the “thermal gradient α, non-homogeneity constant α_1, tapering parameters β_1, β_2, aspect ratios (a/b, c/b), and a fixed” Poisson ratio of 0.33. Figures are used to illustrate each result.

Fig. 1, it is clear that frequency falls for both scenarios when $\alpha_1 = 0.4$ and $\alpha_1 = 1$ as the thermal gradient rises α from 0 to 1. Additionally, it is noted that as the taper constant rises from 0 to 0.4, the frequency of both vibration modes rises in both scenarios.

Fig. 2 clearly shows that when the "tapering parameters β_1, β_2 rise from 0 to 1 for both cases $\alpha_1 = 0.4$ and $\alpha_1 = 1$, the vibrational frequency for both modes" increases. Additionally, the data indicated that in the two aforementioned examples, the frequency of both vibration modes reduces as the temperature gradient value rises from 0 to 0.4.

Fig. 3 emphasizes that when the aspect ratio c/b rises from 0.25 to 1, frequency declines; yet, as one traverses the table from left to right for the ensuing examples, frequency increases for both modes.

Case1: $\alpha = \beta_1 = \beta_2 = 0$ and $\alpha = 0, \beta_1 = \beta_2 = 0.4$

Case2: $\alpha = 0.4, \beta_1 = \beta_2 = 0$ and $\alpha = 0.4,\ \beta_1 = \beta_2 = 0.4$

Fig. 4. As "we examined the table from left to right for the situations mentioned here, it is evident that growth in non-homogeneity α_1 from 0 to 1 suggests that the vibrational frequency for both modes falls whereas, on the other hand, the frequency" grows.

Case1: $\alpha = \beta_1 = \beta_2 = 0$ and $\alpha = 0, \beta_1 = \beta_2 = 0.4$

Case2: $\alpha = 0.4, \beta_1 = \beta_2 = 0$ and $\alpha = 0.4,\ \beta_1 = \beta_2 = 0.4\ (\alpha)$

Fig.1 Thermal Gradient α v/s Frequency λ

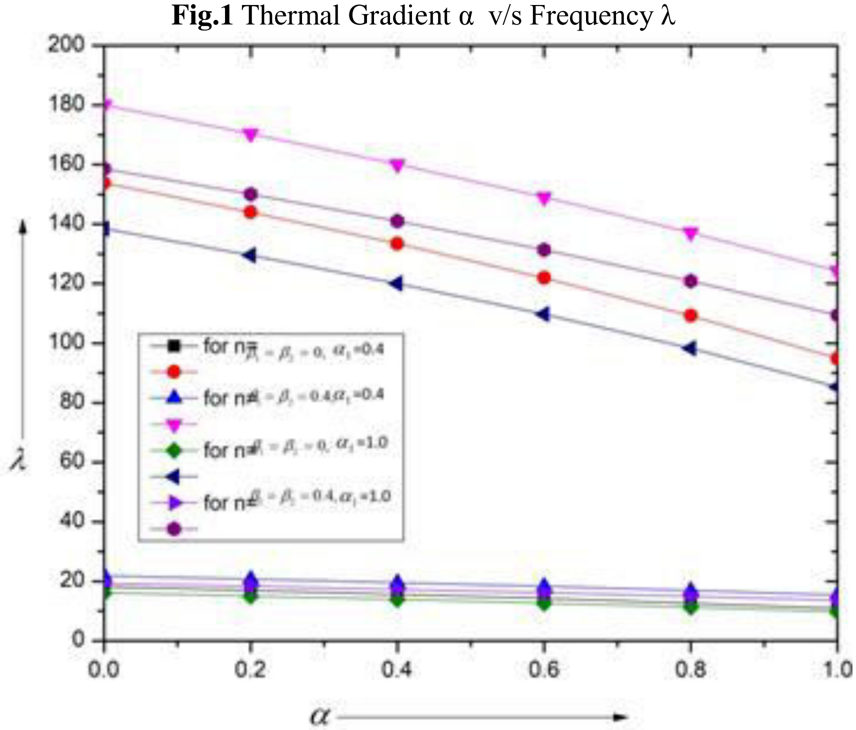


Fig. 2: Taper Constant $\beta_1 = \beta_2$ v/s Frequency λ

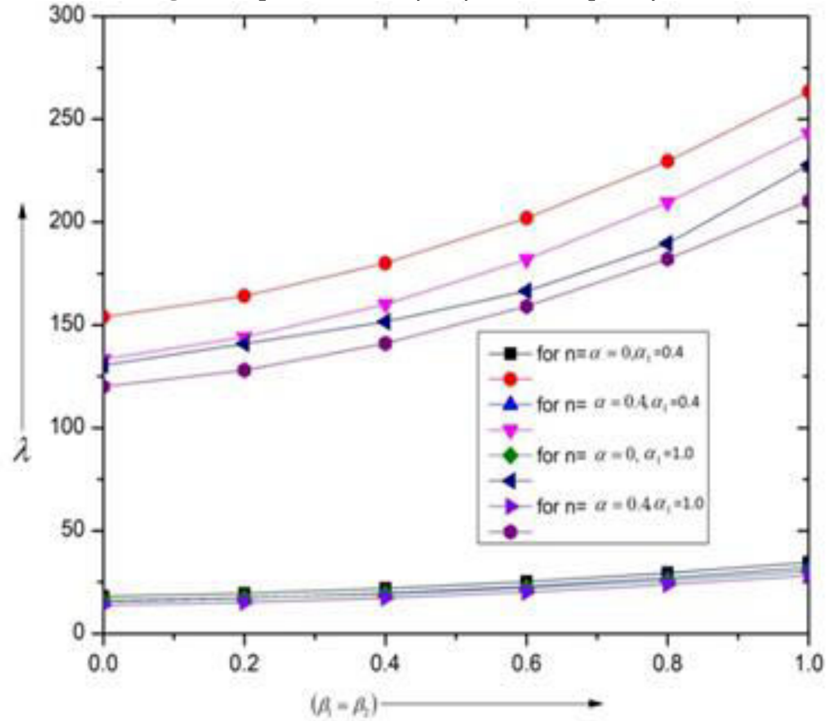


Fig. 3: Aspect Ratio (c/b) v/s Frequency λ

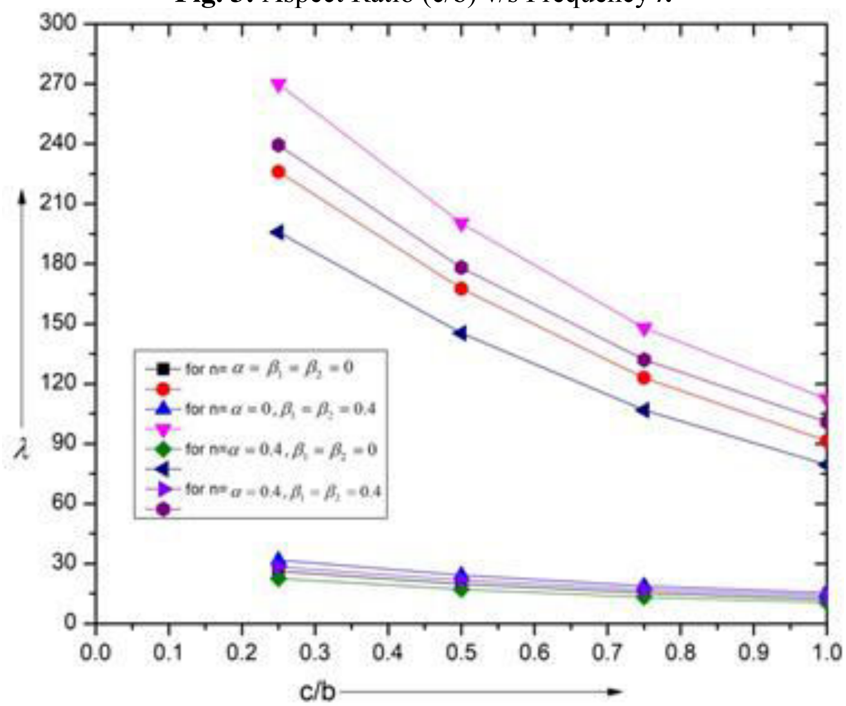
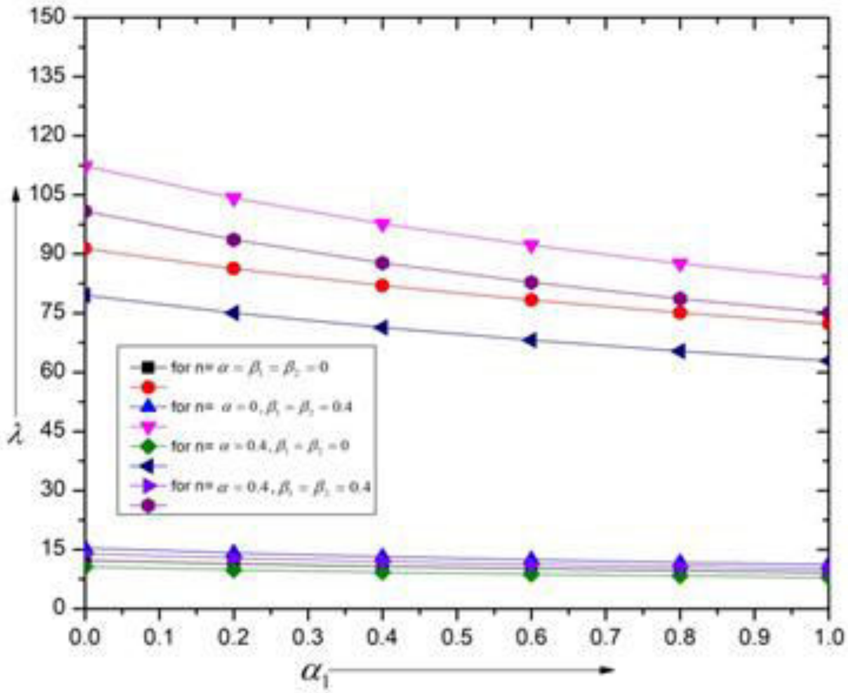


Fig. 4 Non-Homogeneity Constant α_1 v/s Frequency λ



CONCLUSION

Because plate vibration has a huge variety of engineering applications, including those in civil, mechanical, and aeronautical engineering, it is an important field of study. An excellent as well as computationally effective approach for determining the vibration properties of a trapezoidal plate's transverse vibration is provided by the Rayleigh-Ritz method. In this way, various“values of the taper constants, thermal gradient, aspect ratio, as well as non-homogeneity constant” have been able to capture the natural frequencies for a symmetric, non-homogeneous trapezoidal plate.

REFERENCES

1. **A.W. Leissa,** Vibration of the plate, NASA SP-160(1969).
2. **Gupta AK and Sharma P.** (2010) Review the thermal gradient effect on frequencies of a trapezoidal plate of linearly varying thickness. Applied Mathematics 1(5): 357–365.
3. **Gupta A. K., Sharma S.** Thermally induced vibration of an orthotropic trapezoidal plate of linearly varying thickness. Journal of Vibration and Control, Vol. 17, Issue 10, 2011, p. 1591-1598.
4. **Gupta, A.K. and Sharma, P.** (2012) Consequence of Linear Thermal Gradient on Vibrations of Trapezoidal Plates whose Thickness Varies Parabolically. Journal of Vibration and Control, 18, 395-403.
5. **Gupta, Arun Kumar, and Shanu Sharma.** "Effect of thermal gradient on oscillation of non-homogeneous orthotropic trapezoidal plate of linearly varying thickness." *Ain Shams Engineering Journal* 4, no. 3 (2013): 523-530.
6. **Khanna A, Ashish Kumar Sharma,** (2013): “Natural Frequency of Viso-Elastic Square Plate with Thickness Variation”, Elixir Mech. Engg.,56, pp. 13264-13267.
7. **Sharma A. K., Sharma S. K**. 2014. Free vibration analysis of a visco-elastic orthotropic rectangular plate with bi-parabolic thermal effect and bi-linear thickness variation. Journal of Applied Mechanics and Computational Fluid Dynamics. 1(1): 10-23.

8. **A. Sharma, A.K. Raghav, V. Kumar,** Mathematical study of vibration on non-homogeneous parallelogram plate with thermal gradient, International Journal of Mathematical Science,36(1)(2016), 1801-1809.

9. **A.K. Sharma and V. Verma** Inspection of Vibration using Rayleigh-Ritz method for Orthotropic Non-Homogeneous Plate with varying Thickness and Thermal Effect, RJAV vol XIV issue 1/2017.

10. **A.K. Sharma and Silky BENSAL** Free Vibration Analysis of Square Structure Plate with Different Boundary Conditions, RJAV Vol 20 issue 2/2023.