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A NOVEL STUDY ON LEHMER – 3 MEAN LABELING OF SOME NEW GRAPHS

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ABSTRACT
Let $f: V(G) \to \{1,2,\dots,p+q\}$ be an injective function. For a vertex labeling f the induced edge labeling $f(e=uv)$ is defined by $f(e) = \left\lceil \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rceil$ or $\left\lfloor \frac{f(u)^3+f(v)^3}{f(u)^2+f(v)^2} \right\rfloor$, then f is called Super Lehmer -3 mean labeling, if $\{f\,(V(G))\} \cup \{f(e)/e \in E(G)\} = \{1,2,3,\dots,p+q\}$. A graph which admits Super Lehmer – 3 Mean Labeling is called Super Lehmer – 3 Mean graphs. In this paper discuss about the existence of Lehmer - 3 Mean Labeling of Some New Graphs such as $C_m \cup P_r$, $TC_m \cup P_r$, $C_m \cup TP_r$, $TC_m \cup HP_r$, $(C_m \odot K_1) \cup (P_r \odot K_1)$.

Keywords: Graph, Super lehmer – 3 mean labeling, Super lehmer - 3 mean graph, Cyvle crown

AMS subject classification: 05C78.

I. INTRODUCTION

"Graph Theory" is an important branch of Mathematics, **Euler (1707-1782)** is known as the father of Graph Theory as well as Topology. Graph Theory came into existence during the first half of the 18^{th} century. Graph Theory did not start to develop into an organized branch of Mathematics until the second half of the 19^{th} century and , there was not even a book on the subject until the first half of the 20^{th} century. Graph Theory has experienced a tremendous growth, one of the main reason for this phenomena is the applicability of Graph Theory in other disciplines such as Physics, Chemistry, Biology, Psychology, Sociology and theoretical Computer science.

A **graph labeling** is the assignment of labels, usually represented by integers, to the edges or vertices of a graph. Most graph labeling methods trace their origin to the one introduced by Rosa in 1967. Over the past three decades more than 600 papers have been published in **graph labeling.** Formally, given a graph G, a vertex labeling is a function mapping vertices of G to a set of labels. A graph with such a function defined is called a **vertex-labeled graph.** Likewise, an edge labeling is a function mapping edges of G to a set of labels. In this case, G is called an **edge labeled graph.** When used without qualification, the term labeled graph generally refers to a **vertex-labeled graph** with all labels are distinct. Any graph labeling to be considered has three important characteristics:

(i) A set of numbers from which the labels are chosen

(ii) A rule that assigns an induced value to each edge or vertex

(iii) A condition that these values must satisfy

The concept of **mean labeling** has been introduced by S.Somasundaram and R.Ponraj [7]. S. Somasundaram, S.S. Sandhya and T.S. Pavithra introduced the concept of **Lehmer - 3 Mean labeling of graphs** in [11] and studied their behavior in [8], [9], [10], [12], [13], [14] and [15]. Motivated by the above works we investigate the existence of lehmer – 3 mean labeling for some classes of graphs.

II. BASIC DEFINITIONS

Definition: 1.1

A **graph** G is an ordered triple $(V(G), E(G), \Psi_G)$ consisting of a nonempty set $V(G)$ of vertices, a set $E(G)$, disjoint from $V(G)$, of edges, and an incidence function Ψ_G that associates with each edges of G an unordered pair of vertices of G. If e is an edge and u and v are vertices such that $\Psi_G(e) = uv$, then e is said to join u and v; the vertices u and v are called the **ends** of e.

Definition: 1.2

A vertex is simply drawn as a **node** or a **dot.** The vertex set of G is usually denoted by $V(G)$or V.

Definition: 1.3

The **order** of a graph is the number of its vertices, i.e. $|\boldsymbol{V(G)}|$.

Definition: 1.4

Two vertices are said to be **adjacent** if they are the end vertices of same edge. If a vertex v is an end vertex of an edge e we say that the vertex v is **incident** on the edge e and also the edge e is incident on vertex v.

Definition: 1.5

An edge of a graph that joins a node to itself is called **loop** or **self-loop.**

Definition: 1.6

In a Multigraph, no loops are allowed but more than one edge can join two vertices. These edges are called **Multiple** or **Parallel edges.**

Definition: 1.7

A graph which has neither self-loops nor parallel edges is called **simple graph.**

Definition: 1.8

The number of edges incident on a vertex v_i with self-loops counted twice is called **degree** of a vertex v_i and is denoted by $\deg v_i$or $d(v_i)$.

Definition: 1.9

The maximum of the degrees of all the vertices is called the **maximum degree** of the graph and it is denoted by $\Delta(G)$or Δ.

The minimum of the degrees of all the vertices is called the **minimum degree** of the graph and it is denoted by $\delta(G)$or δ.

Definition: 1.10

A vertex having no incident edge is called a**isolated vertex.**

Definition: 1.11

Any vertex of degree one is called a **pendent vertex.**

Definition: 1.12

A graph without any edge is called a **null graph.**

Definition: 1.13

A simple graph in which there exists an edge between every pair of vertices is called a **complete graph.**

Definition: 1.14

A graph G in which all vertices are of equal degree is called a **regular graph.**

Definition: 1.15
A graph H is called a **subgraph**of a graph G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ and ψ_H is the restriction of ψ_G.

Definition: 1.16
A **walk** is defined as a finite alternating sequence of vertices and edges which begins and ends with vertices such that no edge appears more than once in a sequence, such a sequence is called a **walk** or **Trail** in G.

Definition: 1.17
A walk that begins and ends at the same vertex is called a **closed walk.**

A walk that is not closed is called an **open walk.**

Definition: 1.18
A closed walk with at least one edge in which no vertex except the terminal vertices appears more than once is called a **cycle** or **circuit.**

III. Lehmer – 3 Mean Labeling of Some New Graphs

Theorem 3.1.1
$C_m \cup P_r$is a Lehmer -3 Mean graph.

Proof:
Let C_m be a cycle of distance of interval $m. (1 \leq i \leq m)$

Let the points of $C_m$of $a_1, a_2, \dots, a_m (1 \leq i \leq m)$.

Let P_r be a path of length of interval r and let the points of $P_r$of $b_1, b_2, \dots, b_r (1 \leq i \leq r)$.

Let $= C_m \cup P_r$.

Define a function $f: V(G) \rightarrow \{1,2, \dots, m + r\}$ is given by

$f(a_i) = i + \frac{100}{2}for(1 \leq i \leq m)$

$f(b_i) = m + i + \frac{100}{2}for(1 \leq i \leq r)$

Then the line labels are distinct.

Hereafter f be there a Lehmer – 3 $(M_l b)$of G.

Example 3.1.2
The Lehmer – 3 $(M_l b)$ of $C_4 \cup P_4$ is specified under :

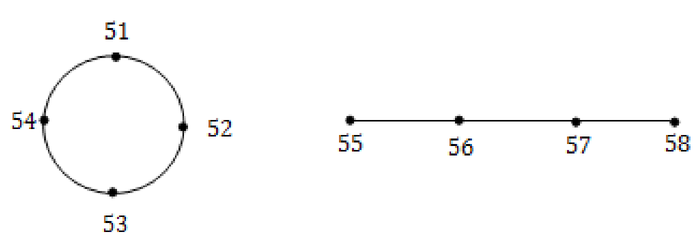

Figure 1

Theorem 3.1.3

$TC_m \cup P_r$is a Lehmer -3 Mean graph.

Proof:

Let TC_m be a cycle of distance of interval $m. (1 \leq i \leq m)$

Let the points of $TC_m$of $a_1, a_2, \dots, a_m$ $(1 \leq i \leq m)$.

Let P_r be a path of length of interval r and let the points of $P_r$of $b_1, b_2, \dots, b_r (1 \leq i \leq r)$.

Let $= TC_m \cup P_r$.

Define a function $f: V(G) \rightarrow \{1,2, \dots, 2m + r\}$ is given by

$f(a_i) = i + \frac{100}{2}for(1 \leq i \leq m)$

$f(b_i) = 2m + i + \frac{100}{2}for(1 \leq i \leq r)$

Then the line labels are distinct.

Hereafter f be there a Lehmer – 3 $(M_l b)$of G.

Example 3.1.4

The Lehmer – 3 $(M_l b)$ of $2C_4 \cup P_4$ is specified under:

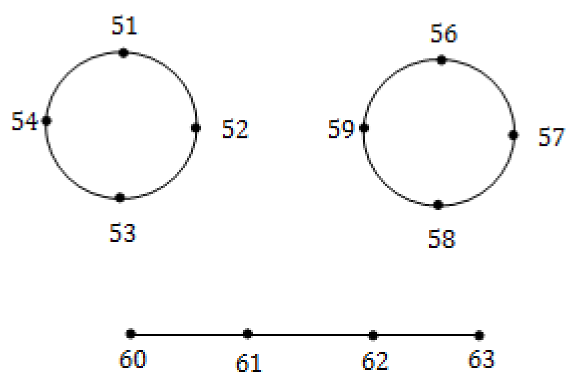

Figure 2

Theorem 3.1.5

$C_m \cup TP_r$is a Lehmer -3 Mean graph.

Proof:

Let C_m be a cycle of distance of interval $m. (1 \leq i \leq m)$

Let the points of $C_m$of $a_1, a_2, \dots, a_m$ $(1 \leq i \leq m)$.

Let TP_r be a path of length of interval r and let the points of $P_r$of $b_1, b_2, \dots, b_r (1 \leq i \leq r)$.

Let $= C_m \cup TP_r$.

Define a function $f: V(G) \rightarrow \{1,2, \dots, m + r\}$ is given by

$f(a_i) = i + \frac{100}{2}for(1 \leq i \leq m)$

$f(b_i) = m + i + \frac{100}{2}for(1 \leq i \leq r)$

Then the line labels are distinct.

Hereafter f be there a Lehmer – 3 $(M_l b)$of G.

Example 3.1.6

The Lehmer – 3 $(M_l b)$ of $C_5 \cup 2P_4$ is specified under :

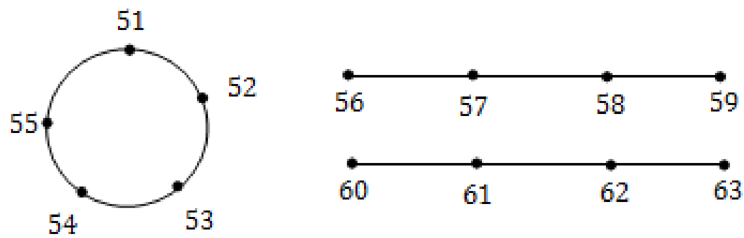

Figure 3

Theorem 3.1.7

$TC_m \cup HP_r$is a Lehmer -3 Mean graph.

Proof:

Let TC_m be a cycle of distance of interval $m. (1 \le i \le m)$

Let the points of $C_m$of $a_1, a_2, \dots, a_m (1 \le i \le m)$.

Let HP_r be a path of length of interval r and let the points of $P_r$of $b_1, b_2, \dots, b_r (1 \le i \le r)$.

Let $= TC_m \cup HP_r$.

Define a function $f: V(G) \to \{1,2, \dots ,2m + r\}$ is given by

$f(a_i) = i + \frac{100}{2}for(1 \le i \le m)$

$f(b_i) = 2m + i + \frac{100}{2}for(1 \le i \le r)$

Then the line labels are distinct.

Hereafter f be there a Lehmer – 3 $(M_l b)$of G.

Example 3.1.8

The Lehmer – 3 $(M_l b)$ of $2C_5 \cup 2P_4$ is specified under :

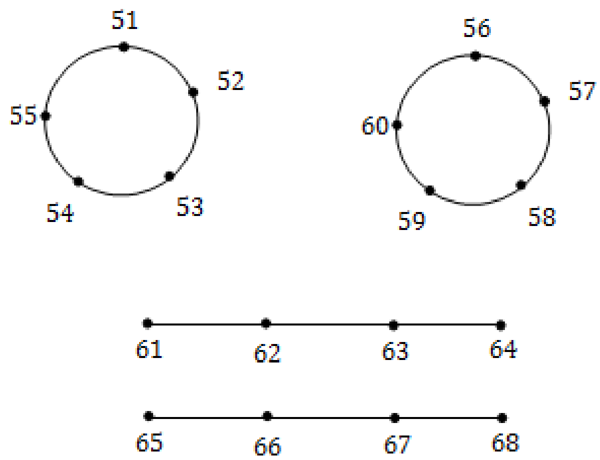

Figure 4

Theorem 3.1.9
$(C_m \odot K_1) \cup (P_r \odot K_1)$is a Lehmer -3 Mean graph.

Proof:
Let C_m be a cycle of distance of interval $m. (1 \leq i \leq m)$

Let the points of $C_m$of $a_1, a_2, \dots, a_m\,(1 \leq i \leq m)$.

Let C_m be a cycle to join a pendent points to$(a_1', a_2', \dots, a_m')$ separately.

Let P_r be a path of length of interval r and let the points of $P_r$of $b_1, b_2, \dots, b_r\,(1 \leq i \leq r)$.

Let P_r be a path to join a pendent points to$(b_1', b_2', \dots, b_m')$ separately.

Let $= (C_m \odot K_1) \cup (P_r \odot K_1)$.

Define a function $f: V(G) \rightarrow \{1,2, \dots ,2m + 2r\}$ is given by

$f(a_i) = 2i + \frac{100}{2} - 1for(1 \leq i \leq m)$

$f(a_i') = 2i + \frac{100}{2}for(1 \leq i \leq m)$

$f(b_i) = 2m + 2i + \frac{100}{2} - 1$ for $(1 \leq i \leq r)$

$f(b_i') = 2m + 2i + \frac{100}{2}for(1 \leq i \leq r)$

Then the line labels are distinct.

Hereafter f be there a Lehmer – 3 $(M_l b)$of G.

Example 3.1.10
The Lehmer – 3 $(M_l b)$ of $(C_5 \odot K_1) \cup (P_4 \odot K_1)$ is specified under :

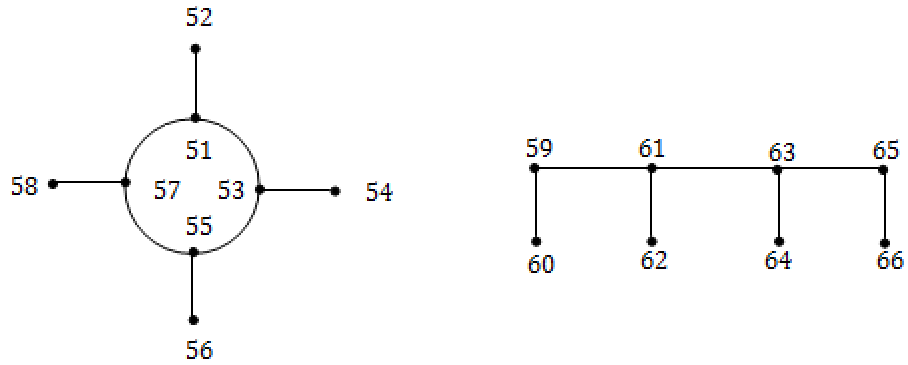

Figure 5

IV. CONCLUSION

In this thesis we discuss about Lehmer - 3 mean labelings. As all the graphs are not Lehmer - 3 mean graphs, it is very interesting to investigate graphs or graph families which admit Lehmer - 3 Mean labeling. We discuss about the existence of Lehmer - 3 Mean labeling of several families of graphs such as Path, Cycle, Crown, Comb, Dragon $(C_r@P_m) \cup P_s, (C_r@P_m) \cup H(P_s)$, $(C_r@P_m) \cup (C_r' \odot K_1)$, $T(C_r@P_m) \cup H(C_r' \odot K_1)$, $(C_r@P_m) \cup (C_r' \odot K_2)$, $T(C_r@P_m) \cup H(C_r' \odot K_2)$, $(C_r@P_m) \cup (P_s \odot K_1)$, $T(C_r@P_m) \cup (P_s \odot K_2)$. The existence of Lehmer - 3 Mean labeling for other graph families is an open area of research.

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