ISSN: 2633-4828 Vol. 4 No.3, December, 2022

International Journal of Applied Engineering & Technology

ENHANCING MATHEMATICS EDUCATION THROUGH DIVERSE MODELING STRATEGIES: AN EXPLORATORY STUDY

Nancy Thomas[1] and Dr. Annapurna Ramakrishna Sinde[2]

[1]Ph. D. Scholar and [2]Research Supervisor, Department of Mathematics, Dr. A. P. J. Abdul Kalam University, Indore, MP, India

ABSTRACT

Mathematical modeling and algebraic reasoning are two important components of mathematics education. In this study, i taught a mathematical modeling lesson to high school algebra i students. My goal was to understand how mathematical modeling and algebraic reasoning are related. To analyze students' modeling and reasoning, i adapted a coding scheme for identifying observable actions in mathematical modeling and created a coding scheme for identifying observable actions in algebraic reasoning. Using these coding templates, i analyzed three groups. I found that two groups followed iterative, non-linear modeling routes and used more algebraic reasoning, while one group followed a highly linear modeling route and did not use as much algebraic reasoning. In addition, i found that the later steps in the modeling cycle led to more algebraic reasoning than the early steps. The findings suggest that mathematical modeling does encourage algebraic reasoning, but not in all circumstances. In addition, the findings provide insight into tensions in teaching mathematical modeling and suggestions for the design of modeling lessons. To further understand how students learn algebra through mathematical modeling, i recommend further study in developing the coding template for identifying algebraic reasoning, studying the modeling behavior of more groups of students to understand other possible student modeling routes, and studying how students' modeling and reasoning changes over time.

Keywords: Mathematical Modeling , Analyze , Algebraic , Findings , Non-Linear Modeling Routes.

I. INTRODUCTION

Effective mathematics education is increasingly recognized as essential in developing critical thinking, problem-solving, and analytical skills vital in today's world, where technology and data-driven decision-making are prevalent. As traditional methods of teaching mathematics often fall short of engaging students and fostering a deep understanding, there is a growing emphasis on innovative teaching approaches, particularly modeling strategies that integrate real-world applications with mathematical concepts. Mathematical modeling, a process that uses mathematical language, symbols, and structures to represent and analyze situations from the real world, allows students to connect abstract mathematical ideas with tangible phenomena. It encourages students to apply their knowledge in various contexts, such as economics, engineering, biology, and social sciences, making learning more relevant and meaningful. Through modeling, students not only learn to manipulate numbers and equations but also to interpret and analyze the results in the context of the problem at hand, fostering a holistic understanding that goes beyond rote memorization. The use of diverse modeling strategies, such as contextual modeling, computational modeling, and data-driven modeling, caters to different learning styles and preferences, making mathematics more accessible and engaging to a broader range of students. Contextual modeling involves embedding mathematical problems within familiar or relatable contexts, helping students visualize abstract concepts and see the relevance of mathematics in their everyday lives.

For example, using real-life scenarios such as calculating interest rates, optimizing routes, or understanding statistical data in news articles makes the mathematics curriculum more engaging and applicable. Computational modeling, which integrates computer programming and mathematics, allows students to explore complex systems and scenarios that are difficult to solve using traditional methods. It also introduces them to coding and algorithmic thinking, skills increasingly valued in many professions today. Data-driven modeling, on the other hand, emphasizes the use of real-world data to solve problems, providing opportunities for students to learn about data collection, analysis, and interpretation, which are critical skills in an era dominated by big data. These various modeling strategies not only make mathematics education more effective by fostering critical thinking, creativity, and collaboration but also help in developing resilience and a growth mindset, as students learn to approach problems from multiple perspectives, recognize that making mistakes is part of the learning process, and appreciate the iterative nature of problem-solving. Furthermore, modeling activities provide an ideal platform for interdisciplinary learning, enabling students to see how mathematics interacts with other subjects, such as science, technology, engineering, and even the arts. This interconnectedness highlights the versatility and utility of mathematical knowledge in diverse fields, thus preparing students for future careers in a range of industries. Additionally, integrating modeling into mathematics education promotes equity and inclusion, as it offers multiple entry points for students with varying abilities, backgrounds, and interests. When students see that their personal experiences, ideas, and ways of thinking can be used as legitimate tools in solving mathematical problems, they are more likely to develop a positive attitude towards mathematics and perceive it as a subject they can excel in. Teacher training is crucial in this regard, as educators need to be equipped with the necessary skills and knowledge to implement modeling strategies effectively. Professional development programs that focus on modeling techniques, the use of technology in mathematics instruction, and the creation of an inclusive classroom environment are essential for supporting teachers in this transition. Moreover, there is a need for research and collaboration among educators, policymakers, and researchers to develop and refine best practices for implementing modeling in mathematics education.

Embracing various modeling strategies, mathematics education can be transformed from a traditionally abstract and often intimidating subject into a dynamic, engaging, and accessible field of study that equips students with the skills they need to navigate an increasingly complex and interconnected world. This approach not only enhances mathematical understanding but also fosters a love for learning, encouraging students to see themselves as capable mathematicians and problem-solvers ready to tackle real-world challenges. Many classroom mathematical activities are designed to develop the necessary mathematical skills and ways of thinking that students need to solve problems that may confront them in their lives. These mathematical tasks can often be sorted into two broad categories, as either purely mathematical problems or as applied mathematical problems. Purely mathematical problems refer to events found in the mathematical world involving mathematical properties, operations, theorems, and symbols. Applied problems connect mathematics to the real world and include contextualized problems and mathematical models.

Unlike most contextualized, textbook-style mathematics problems, which have elaborate narratives that contain all the information needed to determine a solution, mathematical modeling problems are considered messy real-world situations, because the tasks usually lack suggested variables or mathematical operations and lack mathematical statements that quickly lead to a solution. Mathematical modeling of real-world systems is a complex process and more like a cycle than a linear progression of steps leading to a solution. When the National Governors Association Center for Best Practices and Council of Chief State School Officers (NGA & CCSSO, 2010) released the Common Core State Standards for Mathematics (CCSSM), they included modeling with mathematics as one of the eight Standards for Mathematical Practice as well as, at the high school level, a conceptual category emphasizing the importance of mathematics in solving problems found in daily life and across disciplines.

The CCSSM describes the modeling cycle in basic steps that begin with the student interpreting the real-life problem and identifying variables; formulating a model logically, algebraically, geometrically, statistically, or graphically; computing a solution; interpreting the solution in the real-world setting; validating the solution and the model; and, reporting findings. Near the end, in the validation stage, the modeler decides whether the solution and the model are ready to be reported to the community, in that, they reasonably describe, explain, or predict the phenomena being studied. If not, the model needs to be revised by re-examining decisions made throughout the modeling process.

II. LITERATURE REVIEW

Mathematical modeling in education has emerged as a vital strategy to enhance students' understanding of mathematical concepts, promote critical thinking, and bridge the gap between abstract mathematics and real-world applications. The literature on mathematical modeling highlights various perspectives, strategies, and empirical studies that provide insights into how modeling can be effectively integrated into mathematics education. Barbosa (2006) introduces a socio-critical and discursive perspective on mathematical modeling in the classroom, emphasizing the importance of creating a learning environment where students critically engage with real-world issues through mathematical reasoning. This perspective aligns with the notion that mathematical modeling should not only be about solving mathematical problems but also about fostering social awareness and critical thinking skills among students.

Blomhøj and Jensen (2007) discuss the competencies required for effective mathematical modeling, arguing that beyond mathematical knowledge, students need to develop competencies such as problem formulation, interpretation, and validation. They suggest that these competencies are crucial for understanding and applying mathematical concepts in diverse contexts. Similarly, Blum and Niss (1991) provide a comprehensive overview of the role of applied mathematical problem-solving and modeling in linking mathematics to other subjects, highlighting its potential to make mathematics more relevant and engaging for students. Blum (2011) explores whether modeling can be taught and learned effectively, synthesizing empirical research findings that suggest that while students can learn to model through practice and guided instruction, challenges remain in integrating modeling seamlessly into the curriculum. Blum and Leiss (2007) further investigate how both students and teachers deal with modeling problems, emphasizing the need for targeted pedagogical strategies that support students in navigating the complexities of modeling tasks.

Borromeo Ferri (2006, 2007) contributes to the understanding of the cognitive processes involved in mathematical modeling, differentiating between various phases in the modeling process, such as understanding, structuring, mathematizing, working mathematically, interpreting, and validating. Her research highlights the cognitive demands of modeling and suggests that instructional support should be tailored to address these different phases to enhance students' modeling competencies.

Brousseau (1997) provides a theoretical foundation for didactical situations in mathematics, offering insights into how modeling tasks can be structured to promote deep mathematical understanding. His theory underscores the importance of designing learning environments that challenge students to explore, conjecture, and validate mathematical ideas actively. The empirical study by Boaler and Staples (2008) illustrates the impact of equitable teaching approaches in fostering mathematical modeling skills. Their research at Railside School demonstrates how a focus on collaborative learning and open-ended problem-solving tasks can create mathematical futures for all students, particularly those from underrepresented groups. This study aligns with the broader argument that modeling can democratize access to mathematics education by providing multiple entry points for diverse learners. Busse (2011) examines how upper secondary students handle real-world contexts in mathematical modeling, revealing that students often struggle with interpreting and mathematizing real-world situations. This finding suggests that effective modeling instruction should include opportunities for students to engage with authentic data and scenarios, developing their ability to transition between the real world and mathematical representations.

The importance of multiple representations in learning mathematics is emphasized by Brenner et al. (1997), who argue that understanding in algebra and other mathematical domains is enhanced when students use various representations, such as graphs, equations, and verbal descriptions. This view supports the notion that modeling should involve diverse forms of representation to cater to different learning styles and promote a more comprehensive understanding of mathematical concepts. Chazan (1996, 2000) and Chazan and Yerushalmy (2003) focus on the complexities of teaching algebra and the cognitive challenges associated with learning algebraic concepts. Their research suggests that modeling approaches, which integrate algebraic thinking with real-world applications, can make algebra more accessible and meaningful to students. They argue for curricular changes that prioritize conceptual understanding and the development of modeling skills over rote learning and procedural fluency. Cohen, Raudenbush, and Ball (2003) discuss the interplay between resources, instruction, and research in mathematics education, highlighting the need for robust instructional materials and professional development programs that support teachers in implementing modeling strategies effectively. They emphasize that for modeling to be successful, educators must be equipped with both the content knowledge and the pedagogical skills necessary to guide students through the complexities of the modeling process. The mathematical modeling in education provides a multifaceted view of the benefits, challenges, and strategies associated with integrating modeling into the mathematics curriculum. While there is consensus on the potential of modeling to enhance mathematical understanding and engagement, the effective implementation of modeling strategies requires careful consideration of cognitive processes, pedagogical practices, and equitable teaching approaches. Future research should continue to explore how diverse modeling strategies can be optimized to meet the needs of all learners and support the development of critical mathematical competencies.

III. DATA AND RESULT ANALYSIS

3.1 Data Analysis: Transcription Conventions: With the help of the audio and video recordings of the students' work, we prepared transcripts for each group. Throughout the transcripts, I labelled each of the student's comments as "turns." As soon as the speaker changed, a new turn was recorded. Each segment's first turn was labelled "turn 1" and numbered sequentially until the segment ended. I used ellipses when a student paused in the middle of a turn. Students who used non-verbal communication were marked with brackets. I used an em dash when a student switched topics mid-sentence. The results were reported using tables for each portion of the transcript that I inserted. Students' comments that were unclear were clarified by adding words in brackets to the end of the sentence.

Unit of Analysis: A lesson analysis was performed to determine what students needed to do to complete each problem, and each problem was then divided into sub-problems based on the analysis. Table 1 summarises the problems and sub-problems identified in the analysis.

Table 1: Problems and sub-problems in the modeling lesson

Problem number	Description of the problem	Sub-problem letter	Description of the sub-problem
1	Internet Music Business analysis	a	Choose businesses to analyze.
		b	Identify and report on the details of the businesses.
2	Mission Statement	a	Read and discuss the mission statements provided.
		b	Create a mission statement for your company.
3	Give a name and logo for your business.	a	Give a name and logo for your business.
4	Explain the principles on which the company's business plan is founded, and why the company will be a successful music business.	a	Explain the principles on which the company's business plan is founded, and why the company will be a successful music business.
5	Provide a mathematical representation of their artist payment plan where the artist can determine how much money they would make for different amounts of downloads / plays or sales.	a	Decide on values used to pay artists.
		b	Choose representations to represent the payment plan.
		c	Create an equation for the payment plan.
		d	Create a table for the payment plan.
		e	Create a graph for the payment plan.
6	Demonstrate the profit an artist would expect to make as a small independent artist, a popular genre artist (like a popular rap artist), and a top 10 overall artist.	a	Demonstrate the profit an artist would expect to make as a small independent artist, a popular genre artist (like a popular rap artist), and a top 10 overall artist.
7	Explain why these figures match with the principles of the company.	a	Explain why these figures match with the principles of the company.
8	Make the poster or brochure.	a	Make the poster or brochure.
9	Presentation	a	Present your business model to the class with a poster or brochure.

It took me a while to figure out how to segment my lesson, but I finally got it down. I watched a video of students working on the problem and listened to the transcript. For the purpose of examining how the students worked on the lesson, I coded whether he was introducing the class to or summarising it. If they were in the group, I coded whether or not they were present in the transcript for each turn. As a result, the final unit of analysis was defined by the problem that the group was working on at each shift, as well as whether or not the teacher assisted the group. Each day of the lesson, I randomly selected a ten-minute segment from each group, similar to Mesa and use of five-minute segments, in order to test the reliability of the segments. A 10-minute segment was used because each class period is a half-hour long, so I was able to check about 20 percent of the data. Using the template I created, a colleague coded the unit of analysis, and the initial code had 70% reliability. As a result of our discussion, we agreed on a second coding of the segments.

Identifying Modeling: As a starting point, I looked at .'s description of observable actions in students' written work that would demonstrate modelling. I modified the observable actions so that they could be identified in students' conversation or actions while completing the Internet music business lesson because I am studying students' evidence of modelling in groups rather than in their individual written work. A student is considered to be constructing if he or she is observed to "understand and recognise a mathematically manageable problem," according to Sol and colleagues. In order for students to demonstrate their understanding of the mathematical problem of what they needed to create for their Internet music business, I asked them to describe possible structures they could use to create their model. For each of the remaining observable actions, I created a list of codes in the same manner.

Then, after creating a list of codes, I went through the video to see if there were any unexpected demonstrations of modelling. This was followed by a refinement of my list of codes based on what I saw in the first viewing. My next step was to go back and look at it a second time, using the refined codes, and see what evidence of modelling was present in each transcript section. There was a member of the research team who verified the validity of six ten-minute segments of the data. The initial check resulted in an overall reliability of 71 percent and a segment-by-segment reliability of 46 percent. I rewrote the results after meeting with a colleague and discussing differences in coding. On a new data set, a second reliability check was performed by the same member of the research team who had performed the first one. On average, 79 percent of segments with modelling were found to be reliable, while only 62 percent were found to be reliable. We discussed differences in coding once more, and I rewrote the results one last time before submitting them. On page 6, you'll find a list of the final modelling cycle steps and the observable modelling actions that correspond to each step.

Table 2: Students to demonstrate their understanding

Turn no.	Speaker	Comment
2	Kalvin	Okay, let's be for real. 50 cents for a song and a dollar for a whole album.
3	Allie	What? No, they not gonna get they money worth.
4	Kalvin	Yes they is, 50 cents for a song.
5	Allie	No, but no, the artist is not gonna get their money worth. No, why am I giving you. I'm gonna put this album, working hard.
6	Kalvin	Look, look, this would be a miracle to them because look, the person gotta play the song…
7	Allie	We not doing Spotify!

Table 3: Adapted list of observable modeling actions in students' collaborative work

Constructing	Understand and recognize a mathematically manageable problem.	1. Describe possible structures for creating a model.
		2. Discuss details of what is required for a complete solution.
Simplifying / structuring	Simplify and structure. Recognize restrictions and specifications. Make decisions about a statement.	3. Specify details about what is important in the model to be created, not including mathematical values.
Mathematizing	Identify objects and relevant relationships.	4. Create the numerical values that will define the payment model.
	Choose relevant variables, distinguishing from others	5. Discuss what variables in the situation will be used to create a mathematical representation.
	State assumptions. Recognize the mathematical background that is needed.	6. Discuss how to represent the mathematical parameters.
	Explain relationships between real objects and mathematical knowledge.	7. Explain what the components of the mathematical representation represent in the real situation.
	Check the coherence in the set of assumptions and mathematical relationships according to the real situation.	8. Check the coherence in the set of assumptions and mathematical relationships according to the real situation.
Working mathematically	State the relationship among variables using mathematical language.	9. Describe the relationship between parameters in the mathematical model and components of the mathematical representation.
	Formulate hypotheses mathematically.	10. Make a prediction about the results of a mathematical process.
	Formulate problems and/or sub-problems in a mathematical way.	11. Describe what mathematical work needs to be done to find a solution.
	Problem-solving processes involved in finding the solution.	12. Use mathematical parameters to build an equation, table, or graph representing the model.
Working mathematically		13. Carry out mathematical procedures to find values for the solution.
		14. Carry out mathematical procedures to find parameters for an equation.
Interpreting	Find and interpret solutions mathematically in the model used.	15. Find and interpret solutions mathematically in the model used.
Validating	Recognise the meaning and extent of the solutions and conclusions in the real situation. Pupils can also state the model.	16. Explain the meaning of the representation created in terms of the payment plan.
	Validate the model itself. Change the model if necessary.	17. Verify that the model is accurate.
		18. Modify the model to try to more closely align to the desired results.
	Promote reflection about results.	19. Comment on whether or not the values found fit with the desired results.
Exposing	Communicate the process and results when the model is valid.	20. Communicate the process and results when the model is valid.
No modeling		21. No modeling

3.2 Results

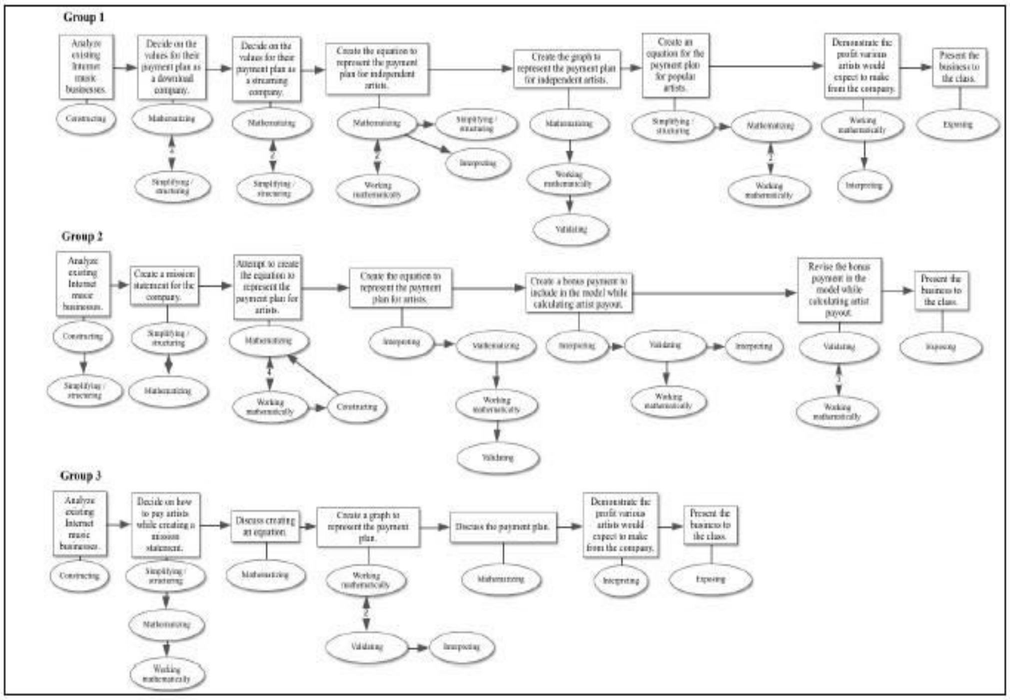
Firstly, I compiled all modelling actions demonstrated by the group into tables that show the route taken by each group to answer the first research question, "How do students engage in mathematical modelling in an Algebra I class?" To make it easier for you to follow along, I broke up the lesson into segments and included the problem the group was working on during each segment. Due to the teacher's involvement at various points during the lesson, which could have affected the students' modelling route, I listed whether or not the teacher interacted with the group during that step of the modelling cycle. Afterwards, I listed which steps in the modelling cycle the group had tackled in the time allotted to them. I also created a table that shows the modelling route in the task as it was originally designed.

It's in Table 4. It was not intended that certain parts of the problems in the lesson would lead to specific steps in the modelling process. However, these steps, such as naming the business and creating a poster, were important to immerse students in the context, as stated in the lesson design plan. In this section, I'll compare the modelling routes of each group to the task-designed modelling routes. This is followed by an analysis of how each group's modelling cycle was demonstrated in relation to the lesson's content. In Figure 2, you can see how each group went about modelling.

Table 4: Modeling route of the task as designed

Modeling route of the task as designed

Problem #	Description of the problem	Anticipated step in the modeling cycle
1a	Choose businesses to analyze.	No modeling
1b	Identify and report on the details of the businesses.	Constructing
2a	Read and discuss the mission statements provided.	Constructing
2b	Create a mission statement for your company.	Simplifying / structuring
3a	Give a name and logo for your business.	No modeling
4a	Explain the principles on which the company's business plan is founded, and why the company will be a successful music business.	Simplifying / structuring
5a	Decide on values used to pay artists.	Mathematizing
5b	Choose representations to represent the payment plan.	Mathematizing
5c	Create an equation for the payment plan.	Working mathematically
5d	Create a table for the payment plan.	Working mathematically
5e	Create a graph for the payment plan.	Working mathematically
6a	Demonstrate the profit artists would expect to make from the company given sales amounts.	Interpreting
7a	Explain why these figures match with the principles of the company.	Validating
8a	Make the poster or brochure.	No modeling
9a	Present your business model to the class with a poster or brochure.	Exposing



Group modeling routes : Note: For each group, the rectangles describe the main focus of the group's work during the lesson. The arrows between rectangles describe the order students worked through the lesson. The ovals represent the steps in the modeling cycle the group used while working on each problem. The arrows between ovals describe the order students modeled during that part of the lesson. A number between arrows represents how many times the group alternated between those steps in the modeling cycle at that time.

Group 1's Modeling Route : Table 5 shows the modelling path taken by Group 1. Figure 2 shows a diagram of Group 1's modelling path. Group 1's work on the problem will be the first thing I discuss. Then, I'll describe some of the differences between their modelling route and the task's actual modelling route.

Table 5: Group 1's modelling route.

Group 1's modeling route

Segment number	Problem	Was the teacher present?	Step in the modeling cycle
1	1.a Choose businesses to analyze.	No	No modeling
2	3.a Give a name and logo for your business.	No	No modeling
3	1.b Identify and report on the details of the businesses.	Yes	No modeling
4	1.b Identify and report on the details of the businesses.	No	Constructing
5	1.b Identify and report on the details of the businesses.	Yes	Constructing
6	1.b Identify and report on the details of the businesses.	No	Constructing
7	1.b Identify and report on the details of the businesses.	Yes	No modeling
8	1.b Identify and report on the details of the businesses.	No	No modeling
9	Whole-class summary of problem 1.b	Yes	No modeling
10	Whole-class launch of problem 2	Yes	No modeling
11	2.a Read and discuss the mission statements provided.	No	No modeling
12	2.a Read and discuss the mission statements provided.	Yes	No modeling
13	Whole-class summary of problem 2b	Yes	No modeling
14	Whole-class launch of problems 3 - 9	Yes	No modeling
15	2.b Create a mission statement for your company.	Yes	No modeling
16	2.b Create a mission statement for your company.	No	No modeling
17	2.b Create a mission statement for your company.	Yes	Constructing

Segment number	Problem	Was the teacher present?	Step in the modeling cycle
18	4.a Explain the principles on which the company's business plan is founded, and why the company will be a successful music business.	No	No modeling
19	5.a Decide on values used to pay artists.	No	Mathematizing
19	5.a Decide on values used to pay artists.	No	Simplifying / structuring
20	5.a Decide on values used to pay artists.	Yes	Simplifying / structuring
20	5.a Decide on values used to pay artists.	Yes	Mathematizing
21	5.a Decide on values used to pay artists.	No	Mathematizing
22	8.a Make the poster or brochure.	No	No modeling
23	5.a Decide on values used to pay artists.	No	Mathematizing
24	8.a Make the poster or brochure.	No	No modeling
25	5.a Decide on values used to pay artists.	No	Simplifying / structuring
26	8.a Make the poster or brochure.	No	No modeling
27	5.a Decide on values used to pay artists.	No	Mathematizing
28	2.b Create a mission statement for your company.	No	No modeling
29	5.b Choose representations to represent the payment plan.	No	No modeling
30	6.a Demonstrate the profit artists would expect to make from the company given sales amounts.	No	No modeling
31	Whole-class summary of Day 1.	Yes	No modeling
32	Whole-class launch of Day 2.	Yes	No modeling
33	5.a Decide on values used to pay artists.	Yes	Mathematizing
33	5.a Decide on values used to pay artists.	Yes	Simplifying / structuring
34	5.b Choose representations to represent the payment plan.	Yes	Mathematizing
35	5.a Decide on values used to pay artists.	No	Mathematizing
36	8.a Make the poster or brochure.	No	No modeling
37	5.c Create an equation for the payment plan.	No	Working mathematically
37	5.c Create an equation for the payment plan.	No	Mathematizing
38	5.a Decide on values used to pay artists.	No	Interpreting

Segment number	Problem	Was the teacher present?	Step in the modeling cycle
39	5.c Create an equation for the payment plan.	No	Working mathematically
39	5.c Create an equation for the payment plan.	No	Interpreting
40	8.a Make the poster or brochure.	No	No modeling
41	5.e Create a graph for the payment plan.	No	Mathematizing
42	5.b Choose representations to represent the payment plan.	No	Mathematizing
43	5.e Create a graph for the payment plan.	No	Working mathematically
44	8.a Make the poster or brochure.	No	No modeling
45	5.e Create a graph for the payment plan.	No	Working mathematically
45	5.e Create a graph for the payment plan.	No	Validating
46	5.a Decide on values used to pay artists.	No	Simplifying / structuring
46	5.a Decide on values used to pay artists.	No	Mathematizing
47	5.c Create an equation for the payment plan.	No	Working mathematically
47	5.c Create an equation for the payment plan.	No	Mathematizing
48	8.a Make the poster or brochure.	Yes	No modeling
49	5.c Create an equation for the payment plan.	No	Working mathematically
50	6.a Demonstrate the profit artists would expect to make from the company given sales amounts.	Yes	Working mathematically
51	6.a Demonstrate the profit artists would expect to make from the company given sales amounts.	No	Interpreting
52	8.a Make the poster or brochure.	No	No modeling
53	Whole-class launch of problem 9.	Yes	No modeling
54	9.a Present your business to the class.	Yes	Exposing

Modeling wasn't evident in Group 1's first three segments of the lesson. They began working on problem 1a by selecting which businesses to analyse before deciding on the name of their company. Then, Ms. Gargi showed up and asked the group which companies they liked best. They began working on problem 1b because of her question, identifying and reporting on the details of their businesses. Students did not receive any assistance from the teacher regarding their modelling work in this segment; the purpose of the discussion was to clarify instructions.

Segment 4 was the first time Group 1 showed signs of modelling. As they worked on problem 1b, they created this segment of work. As the group described the payment models of existing music businesses, they speculated on what mathematical payment models might look like if they were developed. Akanksha, for example, explained the following when describing Google Play:

Okay. Basically, you get 70 cents for each download. This means that if you sell your album, it's a good thing for you. Put your album on Google Play and say you have 23 tracks. You get–like It's 70 cents multiplied by 23. In other words, that's how much you'll get back.

After completing segments 5 and 6, the group continued to work on problem 1b. Mrs. Matthews assisted the group in segment 5 by providing some information about payment models, but did not direct the group's course of action.

There was no evidence of modelling in segments 7 through 16 for Group 1. Participants continued working on problem 1b in segments 7 and 8, but the conversation centred on how to solve it. Then, in segments 9 and 10, Mr. Deal facilitated a whole class discussion summarising problem 1 and launching problem 2, where the students created a mission statement for their company. As of the 15th and 16th segments, Group 1 began working on their mission statement. After Mr. Deal clarified the group's direction in segment 15, the group discussed what they needed to do for a mission statement in segment 16. Neither segment included any progress on the group's model.

While creating their mission statement in segment 17, Group 1 constructed again. Mrs. Matthews asked the group how much they thought the artist should be paid per song when she asked what their mission was. According to Google Play's pricing model, a song costs $1.29 and an album costs $9. That's because they gave an example of a payment model for the music business. That's an example of constructing

Having finished building in segment 17, the group discussed what they needed to do for problem 4. These segments 19 through 27 were devoted to deciding the values they would use to pay artists for problem 5a, rather than working on that problem. After five rounds of mathematizing and simplifying/structuring, the group switched back and forth five times. There, the group engaged in mathematization by proposing or discussing how much to pay artists. Kamlesh, for example, proposes in turn 2 of segment 19: "Okay, let's face it. $50 for a song and $1 for the entire album." An internal discussion ensued as to the structure of the group's company, which led to the group simplifying and clarifying the details of the type of company they were planning to create. According to Akanksha, on turn 9 of segment 19, "We're not streaming any content. You are purchasing the song. We're not a premium subscriber, either." This led to the group simplifying/structuring by clarifying what the equation they were writing would represent and mathematizing by defining new mathematical values they would use to pay artists in segment 20 when Mr. Deal arrived.

3.3 Implications for Teaching Algebra through Modeling

As a result of the study, students can learn algebra by utilising algebraic reasoning and mathematical models. As a result of research, tensions have been identified in teaching. There are three tensions in the Internet music business lesson that I want to examine here. A few of the tensions that were present in the lesson were time allocation, student assistance, and how to support students' exposition.

To encourage students to use algebraic reasoning, it's important to consider the pace of a lesson on modelling. In particular, the steps in the modelling cycle that require working in the real world are affected by this. It also means that teachers should think about how fLakshmible they will be with students as they work their way through a modelling problem. The results showed that allowing students to choose their own path had both advantages and disadvantages. A third implication is that the amount of algebraic reasoning shown in this step could be affected by how teachers facilitate exposing the model.

Assisting with the pacing of a modelling task When it comes to teaching mathematical modelling, one of the biggest challenges is determining the right pace. However, I wanted to make sure that they had enough time to construct and simplify/structure their ideas while also encouraging them to work at a pace that allowed them to complete the lesson. The results of this study indicate that it is important to ensure that students have enough time to complete the final steps of the modelling cycle. When interpreting, validating, and exposing, the last three steps of modelling, had the highest percentage of algebraic reasoning. Students used algebraic reasoning less frequently than they should have due to a relative lack of use of these steps in the modelling cycle. Some of this could be attributed to the short amount of time students were given to complete all of the final steps of the modelling cycle. This is because both groups 1 and 3 didn't finish problem 6 until the end of the lesson.

They didn't work on problem 7, in which they were supposed to demonstrate the effectiveness of their model in achieving the business' goals. Maybe if we had had a little more time to complete these problems in class, we would have had to do more interpreting and validating. Additionally, because of time constraints at the end of the lesson, I was only able to give three presentations. As a result, it's possible that I could have asked more probing questions while Group 3 presented their model, giving them the chance to show off their mathematician's skills.

You can achieve this goal by reducing the overall amount of construction and simplifying/structuring time. There was a 50-minute break in the lesson for students to work on problems that were designed to elicit these steps in the modelling cycle. In spite of this, as previously discussed, researchers have found that students struggle to complete these steps, despite the fact that they are crucial for modelling success.

It was important to me that these steps were emphasised during a modelling lesson because of their importance. Another way teachers can help is by considering how much time students need to construct, simplify, and structure a model before they begin the lesson. Due to less time spent on construction and simplification, students will have more time for algebraic reasoning in the final stages of the modelling cycle.

Assisting students with a modelling task. Teachers are also faced with the dilemma of how much guidance to give their students when they are modelling. As a result of this, students are more likely to take their own path, which increases their learning opportunities. Students may not grasp the lesson's purpose if they are not guided in their work. Different groups' approaches to this lesson's open-ended modelling problem shed light on the tension between the two.

When it comes to learning how to model, researchers have looked at what students do. Student models are non-linear, according to these studies. Students follow different modelling routes depending on how they approach the mathematical problem. For this lesson, I wanted students to follow a route more closely aligned. However, two out of the three groups' modelling paths followed a non-linear path similar to previous research. More algebraic thinking was elicited than I expected due to the non-linear, iterative nature of these modelling routes. A teacher's strategy that allows students to choose their own path through problems while giving them only minimal guidance has been shown to increase the use of algebraic reasoning.

Making modelling problems open-ended can also reduce the amount of algebraic reasoning required from students. There is a risk of a student becoming engrossed in the context of an issue that prevents them from understanding its mathematical purpose. These findings are echoed in this lesson's work by Group 3. Individual students' responses to a modelling problem's context vary, indicating that while some groups may be overwhelmed by a particular context, others will use the context to shape their ideas of mathematics in the desired way. It is important to pay close attention to how groups approach a modelling problem's mathematical aspect.

By encouraging students to discuss the problem, teachers can help them stay focused on the mathematics. There were fewer discussions of the problem in this lesson by the group that took a more linear modelling approach and focused on context over mathematic. More discussion of the problem could have led to a more mathematical understanding of the situation. In segment 20 of Group 1's work, there was an example of how to encourage discussion. For their model, the group was weighing the pros and cons of two possible values: affordable music and support for the artist. We'll have a debate about which of these values is going to work best for making music affordable and giving money to the artist in turn 16. To keep them focused on the mathematical aspect of the problem, I encouraged the group to debate. Overall, it was difficult for me to know when to guide the groups and when to let them work on their own when I was teaching the modelling lesson.

Exposing is made easier with this tool. I also struggled with how to facilitate the exposing step of the modelling cycle, which was a third source of tension for me when I was teaching mathematical modelling. When it came to algebraic reasoning, my role was necessary to encourage algebraic reasoning. When it came to exposing, two groups used algebraic reasoning, while the third did not use it whatsoever. As a result of my questions, Group 2 used algebraic reasoning without prompting, while Group 1 only used algebraic reasoning in response to my questions. I didn't ask similar questions to Group 3 during their presentation, and they didn't demonstrate any algebraic reasoning during their presentation either. There was a need to strike a balance between letting the students' presentations be immersed in context and asking specific probing questions in order to elicit algebraic reasoning during exposure.

3.4 Implications for the Modeling Lesson

When I designed the modelling lesson in this study, I was looking for specific aspects of the modelling process and algebraic reasoning to be evoked. If you want to learn more about algebraic reasoning and modelling, this is the section for you. My recommendations for future design of modelling problems will also be included. I'll talk about how the lesson's design encouraged constructing and simplifying / structuring and mathematizing, but discouraged algebraic reasoning in some ways.

To avoid having to teach students explicitly about the modelling cycle, I incorporated the steps of that cycle into the lesson itself. This choice, I believe, led to more modelling behaviour on the part of the group. If you're interested in learning more about how I designed my Internet music business analysis and mission statement activities (Problems 1 and 2) you can check out my blog post here. However, students often skip simplifying and structuring. During this lesson, all of the groups constructed and simplified / structured, with the exception of Group 3. There is a possibility that Group 3 misinterpreted the mission statement activity and created their own payment model as a result. However, I believe the results of the study show that by designing specific activities to encourage these steps of the modelling cycle, students are more likely to carry them out in greater detail.

As a way to encourage mathematizing, I designed the lesson so that students had to come up with their own values, rather than relying on the problem's data. While doing mathematical modelling, the students in this study mathematized more than any other step. These findings are in line with those, who found that when students are given fewer numbers in modelling problems, they can better distinguish between the real model and mathematical model. A greater emphasis on mathematizing was possible as a result of students' need to collect data. A useful strategy for teachers who are looking for ways of getting their students' attention on mathematization is using problems with created data.

Lesson components that encouraged construction, simplification, and mathematization did not directly lead to more algebraic reasoning, but I believe they helped prepare students for success with algebraic reasoning in other steps of the modelling cycle. It has been suggested that explicitly teaching the modelling cycle could help students avoid becoming too focused on context of the problem eg., Group 3's approach to the problem. Due to limited time constraints, teaching the modelling cycle and doing mathematical modelling in an algebra classroom isn't always an option. To encourage mathematical modelling in this situation, I believe the study's findings support incorporating a modelling cycle into the problem.

This lesson would be taught the same way if I were to teach it again, but I would incorporate the modelling cycle into the problem rather than teaching it directly. For the mission statement activity, I would be more explicit in the directions to ensure that students simplified and structured as I intended. En outre, I would explicitly require students to use a mathematical process in order to determine the values in their payment model based on a current company's payment plan. My prediction was that students who were mathematicizing were more likely to discuss variables and the relationship between a model and its context. Two out of three groups used algebraic reasoning throughout the modelling cycle.

IV. CONCLUSION

When students in an Algebra I class model mathematically, they use algebraic reasoning. As part of my Algebra I course, I developed and taught a modelling lesson, and I recorded students working on the lesson. For algebraic reasoning, I created a set of codes for identifying the observable steps in mathematical modelling, and adapted another set of codes for mathematical modelling. My analysis of the data from three groups was based on these codes and I was able to determine when they displayed modelling and algebraic reasoning. Overall, iterative modelling groups used more algebraic reasoning than those who did not. There were a lot of algebraic reasoning going on in the last steps of the modelling cycle, but they were also used less frequently overall. I discussed the implications for teaching modelling and designing modelling lessons based on the results of the study on the learning of students. To answer the question of how students learn algebra I through mathematical modelling, I will first discuss the implications of the results on a broader scale. In the next section, I'll discuss the study's limitations. On the basis of these limitations and implications, I will make recommendations for future research.

REFERENCES

[1] Barbosa, J. C. (2006). Mathematical modelling in classroom: a socio-critical and discursive perspective. ZDM, 38(3), 293-301.

[2] Blomhøj, M., & Jensen, T. H. (2007). What's all the Fuss about Competencies?. In W. Blum,

[3] P.L. Galbraith, H.-W. Henn, & M. Niss (Eds.), Modelling and applications in mathematics education: The 14th ICMI study (pp. 45-56). New York: Springer.

[4] Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research.

[5] In W. Blum, R.B. Ferri & G. Stillman (Eds.), Trends in teaching and learning of mathematical modelling (pp. 15-30). Netherlands: Springer.

[6] Blum, W., & Leiss, D. (2007). How do students and teachers deal with modelling problems. In Haines et al. (Eds.) Mathematical modelling (ICTMA12): Education, engineering and economics, (pp. 222-231). Chichester: Horwood.

[7] Blum, W., & Niss, M. (1991). Applied mathematical problem solving, modelling, applications, and links to other subjects—State, trends and issues in mathematics instruction. Educational studies in mathematics, 22(1), 37-68.

[8] Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable teaching approach: The case of Railside School. The Teachers College Record, 110(3), 608-645.

[9] Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. ZDM, 38(2), 86-95.

[10] Borromeo Ferri, R. (2007). Modelling problems from a cognitive perspective. In Haines et al. (Eds.) Mathematical modelling (ICTMA12): Education, engineering and economics, (pp. 260-270). Chichester: Horwood.

[11] Brenner, M. E., Mayer, R. E., Moseley, B., Brar, T., Duran, R., Reed, B. S., & Webb, D. (1997). Learning by understanding: The role of multiple representations in learning algebra. American Educational Research Journal, 34(4), 663-689.

[12] Brousseau,G. (1997). Theory of didactical situations in mathematics. Dordrecht: The Netherlands: Kluwer.

[13] Busse, A. (2011). Upper secondary students' handling of real-world contexts. In W. Blum, R.B.

[14] Ferri & G. Stillman (Eds.), Trends in teaching and learning of mathematical modelling (pp. 15-30). Netherlands: Springer.

[15] Chazan, D. (1996). Algebra for all students?: The algebra policy debate. The Journal of Mathematical Behavior, 15(4), 455-477.

[16] Chazan, D. (2000). Beyond formulas in mathematics and teaching: Dynamics of the high school algebra classroom. New York, NY: Teachers College, Columbia University.

[17] Chazan, D., & Yerushalmy, M. (2003). On appreciating the cognitive compLakshmity of school algebra: Research on algebra learning and directions of curricular change. A research companion to principles and standards for school mathematics. Reston, VA: NCTM.

[18] Cohen, D. K., Raudenbush, S. W., & Ball, D. L. (2003). Resources, instruction, and research.