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BOUNDS ON DEGREE SUM ENERGY OF GRAPH

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ABSTRACT

The sum of the absolute values of all Degree Sum eigenvalues $E_{DS}(G)$ of a graph G is called as the Degree sum energy of G. One upper and lower constraints on the degree Sum energy are obtained in this study.

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1. INTRODUCTION

Suppose that G is a simple graph, and that $V(G) = \{v_1, v_2, \ldots, v_n\}$ is its vertex set. When the vertices v_i and v_j are adjacent, the adjacency matrix $A(G)$ of the graph G is a square matrix of rank n with The $(i:j) - \text{entry}$ equal to unity, otherwise, it is equal to zero. The eigenvalues of the graph G are are $\boldsymbol{\delta_1}, \boldsymbol{\delta_2}, \ldots, \boldsymbol{\delta_n}$, of $A(G)$, which are considered to be non-increasing in order.

I. Gutman [6] originally defined the energy of G in 1978 as the total of its eigenvalues absolute values: $\boldsymbol{E(G)} = \sum_{k=1}^{n} |\boldsymbol{\delta_k}|$. There has been a steady flow of articles on this subject since I. Gutman first established the graph energy $E(G)$ of a simple graph G. For basic mathematical properties of the theory of graph energy including its upper and lower bounds one can see [4, 11]. Erich

Huckle[8], employed the energy of graphs technique in the early 1930s to develop approximations solutions for a family of organic molecules known as conjugated hydro carbons.

Numerous matrix types, including Incidence [10], Distance [9], Lapalcian [7], Maximum Degree Matrix [1] and others are established and researched for graphs, with inspiration drawn from the adjacency matrix(AM) of a graph. In their publication [12], Ramane et al. introduced and investigated sum-degree energy of G, defined as follows:

Let G be a simple graph with connections. The matrix $DSM(G) = [d_{kj}]$ needs to be defined as,

$$d_{kj} = \begin{cases} d_k + d_j, & \text{when } v_k \text{ and } v_j \text{ are adjacent} \\ 0 & \text{othewise,} \end{cases}$$

This is referred to as G's degree sum matrix.

The degree sum energy DSE of G is then written as $E_{DS}(G) = \sum_{k=1}^{n} |\xi_k|$, where, ξ_k are the eigenvalues of DSM(G), Furthermore, these eigenvalues are real numbers and are sorted in ascending order.

Note that DSM(G) has $trace = 0$, and $\sum_{k=1}^{n} {\xi_k}^2 = 2\mathfrak{C}, where\ \mathfrak{C} = \sum_{1 \le k < j \le n} (d_k + d_j)^2$.

2. BOUNDS FOR DEGREE SUM ENERGY

Throughout this section G denotes a simple graph. This section is aimed to discuss upper and lower bounds for Degree sum Energy (DSE) of G.

Theorem 3.1 Let G be a connected graph with n vertices and m edges and $2\mathfrak{C} \ge n$ then

$\boldsymbol{E_{DS}(G)} \leq \frac{2\mathfrak{E}}{\boldsymbol{n}} + \frac{\boldsymbol{1}}{\boldsymbol{n}}\sqrt{2\mathfrak{E}(n-1)(\boldsymbol{n^2}-2\mathfrak{E})}.$

Proof: Cauchy-Schwarz inequality states that if $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n)$ are $n-$ $vectors$ then:

$$\left(\sum_{k=1}^{n} a_k b_k\right)^2 \leq \left(\sum_{k=1}^{n} a_k^2\right)\left(\sum_{k=1}^{n} b_k^2\right).$$

For $a_k = 1$, $b_k = |\xi_k|$ and $2 \leq k \leq n$, in the above inequality, we obtain

$$\left(\sum_{k=1}^{n} |\xi_k|\right)^2 \leq \left(\sum_{k=1}^{n} 1^2\right)\left(\sum_{k=1}^{n} |\xi_k|^2\right).$$

Therefore,

$$(\boldsymbol{E_{DS}(G)} - \xi_1)^2 \leq (\boldsymbol{n}-\boldsymbol{1})\sum_{k=1}^{n} \xi_k{}^2 = (n-1)(2\mathfrak{E} - \xi_1{}^2),$$

$$\boldsymbol{E_{DS}(G)} = \xi_1 + \sqrt{(n-1)(2\mathfrak{E} - \xi_1{}^2)}.$$

Now consider the function,

$$\boldsymbol{f(x)} = \boldsymbol{x} + \sqrt{(n-1)(2\mathfrak{E} - \boldsymbol{x^2})}$$

Note that f is decreasing for $x \geq \sqrt{\frac{2\mathfrak{E}}{n}}$, for

$$f'(x) = 1 - \frac{(n-1)x}{\sqrt{(n-1)(2\mathfrak{E}-x^2)}} \leq 0,$$

If and only if, $x \geq \sqrt{\frac{2\mathfrak{E}}{n}}$.

Since, $1 \leq \sqrt{\frac{2\mathfrak{E}}{n}} \leq \frac{2\mathfrak{E}}{n} \leq \xi_1$, we have,

$$f(\xi_1) \leq \boldsymbol{f}\left(\frac{2\mathfrak{E}}{n}\right).$$

Therefore,

$$\boldsymbol{E_{DS}(G)} \leq f(\xi_1) \leq \boldsymbol{f}\left(\frac{2\mathfrak{E}}{n}\right).$$

Hence,

$$\boldsymbol{E_{DS}(G)} \leq \frac{2\mathfrak{E}}{\boldsymbol{n}} + \sqrt{(\boldsymbol{n}-\boldsymbol{1})\left(2\mathfrak{E} - \left(\frac{2\mathfrak{E}}{n}\right)^2\right)}$$

or equivalently,

$$\boldsymbol{E_{DS}(G)} \leq \frac{2\mathfrak{E}}{\boldsymbol{n}} + \frac{\boldsymbol{1}}{\boldsymbol{n}}\sqrt{2\mathfrak{E}(n-1)(\boldsymbol{n^2}-2\mathfrak{E})}.$$

Theorem 2.2. Let G be simple graph connected having order n and size m, then

$$E_{DS}(G) \leq \frac{4\mathfrak{E}}{(\xi_1-\xi_n)}. \qquad 2.1$$

Proof: Considering, $x = x_k$ *and* $y = y_k$, $1 \leq k \leq n$ as real sequence such that $\sum_{k=1}^{n}|x_k| = 1$ *and* $\sum_{k=1}^{n}|x_k| = 0$, the inequality stated below has been proved in[11]:

$$\left|\sum_{k=1}^{n} x_k y_k \right| \leq \frac{1}{2}\left(\max_{1\leq k\leq n}(y_k) - \min_{1\leq k\leq n}(y_k) \right) \qquad 2.2$$

Since, $\sum_{k=1}^{n}|\xi_k| = 0$, for $y_k = \xi_k$ and $x_k = \frac{\xi_k}{\sum_{k=1}^{n}|\xi_k|}$, for each $k \in \{1,2,\dots,3\}$ we have,

$$\sum_{k=1}^{n} x_k = \frac{\sum_{k=1}^{n}\xi_k}{\sum_{k=1}^{n}|\xi_k|} = 0$$

and

$$\sum_{k=1}^{n} |x_k| = \frac{\sum_{k=1}^{n}|\xi_k|}{\sum_{k=1}^{n}|\xi_k|} = 1$$

Thus, the inequality (2.2) holds.

Since, $\sum_{k=1}^{n} {\xi_k}^2 = 2\mathfrak{E}$, we have

$$\left|\sum_{k=1}^{n} x_k y_k \right| = \left|\sum_{k=1}^{n}|\xi_k| \cdot \frac{\xi_k}{\sum_{k=1}^{n}|\xi_k|}\right| = \left|\frac{\sum_{k=1}^{n}(\xi_k)^2}{\sum_{k=1}^{n}|\xi_k|}\right| = \frac{2\mathfrak{E}}{E_{DS}(G)}.$$

Applying this in (2.2), we get,

$$\frac{2\mathfrak{E}}{E_{DS}(G)} \leq \frac{1}{2}\left(max(\xi_k) - min(\xi_k)\right),$$

From which , we have

$$\frac{2\mathfrak{E}}{E_{DS}(G)} \leq \frac{1}{2}(\xi_1 - \xi_n).$$

If $G \cong K_n$, then we see that,

$$\xi_k = (n-1)^2, \xi_2 = -(n-1), \dots, \xi_n = -(n-1)$$

and,

$$\xi_1 - \xi_n = n(n-1).$$

So the equality holds in (2.1).

REFERENCES

[1] C. Adiga and Smith M ,On Maximum degree energy of a Graph,Int. J. Contemp. Math. Sciences, 4(34), (2012).

[2] D. Babi_c and I. Gutman, More lower bounds for the total π −electron energy of alternant hydrocarbons, MACTH Commun. Comput., 32 (1995), 7-17.

[3] E.H□ukel, Quantentheoretische Beitr□age zum Benzolproblem I. Die Elektronenkon guration des Benzols und verwandter Vebindungen.Z:phys.70 (1931) 204-286.

[4] B.J. McClelland, Properties of the latent roots of a matrix: The estimation of π −electron energy,J. Chem.Phys., 41, No. 1 (2007).

[5] S. S. Dragomir, A survey on cauchy-Bunyakovsky-Schwarz type discreate inequalities, J.Inequal. Pure Appl.Math. 4(3) (2003), 1-142.

[6] I. Gutman, The energy of a graph, Ber. Math. Stat. Sekt. Forschungsz.Graz, 103 (1978), 1-22.

[7] I. Gutman and B. Jhou, Laplacian energy of a Graph, Lin. Algebra Appl, 414 (2006), 29-37.

[8] G. Indulal, I. Gutman, A. Vijaykumar, On distance energy of Graphs, MATCH Commun. Math. Comput. Chem. 60(2008), 355-372.

[10] M. R. Jooyandeh, D. Kiani, M. Mirzakhah, Incidence energy of Graph, MATCH Commun. Math. Comput. Chem. bf60 (2008), 561-572.

[11] D. S. Mitrinovi_c and P. M. Vasi_c, Analytic inequalities, Springer, Berlin, (1970).

[12] H.S.Ramane, D.S.Revankar, and J.B.Patil. Bounds for the degree sum eigenvalues and degree sum energy of a graph. International Journal of Pure and Applied Mathematical Sciences, 6(2),(2013) 161-167.