Chinese Journal of Mathematical Sciences
Vol. 1 No. 1 (January-June, 2021)

η-YAMABE SOLITON ON 3-DIMENSIONAL α-PARA KENMOTSU MANIFOLD

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Abstract. The aim of the current paper is to concentrate a few properties of 3-dimensional α-Para Kenmotsu manifold whose metric is η-Yamabe solitons. We have concentrated here some specific curvature conditions of 3-dimensional α-Para Kenmotsu manifold admitting η-Yamabe solitons.

1. Introduction

In 1972, Kenmotsu [4] presented Kenmotsu manifolds and the geometry of almost Kenmotsu manifolds have been explored in numerous perspectives [1]-[3]. A large portion of the outcomes contained in [1]-[2] can be well established to the class of almost α-Kenmotsu manifolds, where α is a non-zero real number [3]. The curvature identities for different classes of almost paracontact metric manifolds were obtained in [6]. A class of α-para Kenmotsu manifolds and is noted as $(\alpha - pkm)$, were studied by K. Srivastava and S. K. Srivastava [5].

Hamilton introduced the notion of Yamabe flow [8], in which the metric on a Riemannian manifold is deformed by evolving according to

$$\frac{\partial}{\partial t} g(t) = -rg(t), g(0) = g_0, \tag{1.1}$$

where r is the scalar curvature of the manifold M.

In 2-dimension, the Yamabe flow (Yf) is identical to the Ricci flow (characterized by $\frac{\partial}{\partial t} g(t) = -2Sg(t)$, where S signifies the Ricci tensor). A Yamabe soliton is signified as (YS) [7] and is compare to self-comparative arrangement of the (Yf), is characterized on a Riemannian or pseudo-Riemannian manifold (M, g) by a vector field fulfilling the condition,

$$\frac{1}{2} L_V g = (r - \lambda) g, \tag{1.2}$$

where $L_V g$ indicates the Lie subordinate of the metric g along the vector field V, r is the scalar curvature and λ is a constant. In addition a (YS) is supposed to steady, shrinking and expanding if $\lambda = 0$, $\lambda < 0$ and $\lambda > 0$ respectively. (YS) on

2010 *Mathematics Subject Classification.* Primary 53C25, 53C20, 53C21; Secondary 53C65.
Key words and phrases. η-Einstein manifold α-Para Kenmotsu manifold, Yamabe solitons, η-Yamabe solitons.
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a three-dimensional Sasakian manifold was concentrated by Sharma [11]. Presently, we characterize the idea of η-Yamabe soliton ($\eta-(YS)$) as:

$$\frac{1}{2}L_V g = (r-\lambda)g - \mu\eta\otimes\eta, \tag{1.3}$$

where $L_V g$ is the Lie derivative of the vector field with metric g and λ, μ are constants. Additionally if $\mu = 0$, the above condition lessens to (1.2) thus the η-(YS) becomes (YS).

2. Preliminaries

A differentiable manifold M of dimension $(2n+1)$ is said to have an almost paracontact (ϕ,ξ,η)-structure if it admits an (1,1) tensor field ϕ, a unique vector field ξ, 1-form η such that:

$$\phi^2 = I - \eta\otimes\xi, \eta(\xi) = 1, \phi\xi = 0, \eta\circ\phi = 0 \tag{2.1}$$

for any vector fields X, Y on M^{2n+1}. The manifold M^{2n+1} equipped with an almost paracontact structure (ϕ,ξ,η) is called almost paracontact manifold. In addition, if an almost paracontact manifold admits a pseudo-Riemannian metric satisfying:

$$g(\phi U_1,\phi U_2) = -g(U_1,U_2) + \eta(U_1)\eta(U_2), \tag{2.2}$$

$$-g(\phi U_1,U_2) = g(U_1,\phi U_2), \tag{2.3}$$

$$\eta(U_1) = g(U_1,\xi), \tag{2.4}$$

for any vector fields X, Y on M^{2n+1}, then (ϕ,ξ,η,g), is called an almost paracontact metric structure and the manifold M^{2n+1} equipped with an almost paracontact metric structure is called an almost paracontact metric manifold. Further in addition, if the structure (ϕ,ξ,η,g), satisfies

$$d\eta(X,Y) = g(X,\phi Y)$$

for any vector fields X, Y on M^{2n+1}. Then the manifold is called paracontact metric manifold and the corresponding structure (ϕ,ξ,η,g) is called a paracontact structure with the associated metric g. In a $(\alpha-pkm)_3$, the accompanying outcomes hold [5]:

$$(\nabla_{U_1}\eta)U_2 = \alpha\{g(U_1,U_2) - \eta(U_1)\eta(U_2)\}, \tag{2.5}$$

$$(\nabla_{U_1}\phi)U_2 = \alpha\{g(\phi U_1,U_2)\xi - \eta(U_2)\phi U_1\}, \tag{2.6}$$

$$\nabla_{U_1}\xi = \alpha\{U_1 - \eta(U_1)\xi\}, \tag{2.7}$$

$$\begin{aligned} R(U_1,U_2)U_3 &= \left(\frac{r}{2}+2\alpha^2\right)[g(U_2,U_3)U_1 - g(U_1,U_3)U_2] \\ &- \left(\frac{r}{2}+3\alpha^2\right)[g(U_2,U_3)\eta(U_1) - g(U_1,U_3)\eta(U_2)]\xi \\ &+ \left(\frac{r}{2}+3\alpha^2\right)[\eta(U_1)U_2 - \eta(U_2)U_1]\eta(U_3), \end{aligned} \tag{2.8}$$

$$S(U_1,U_2) = \left(\frac{r}{2}+\alpha^2\right)g(U_1,U_2) - \left(\frac{r}{2}+3\alpha^2\right)\eta(U_1)\eta(U_2). \tag{2.9}$$

$$L_\xi g(U_1,U_2) = 2\alpha g(U_1,U_2) - 2\alpha\eta(U_1)\eta(U_2), \tag{2.10}$$

for all vector fields U_1, U_2, U_3 and $W \in \chi(M)$, where r is the scalar curvature of the manifold and g is pseudo-metric.

3. η-(YS) on $(\alpha - pkm)_3$

Let M be a $(\alpha - pkm)_3$. Contemplate the η-(YS) on M as:

$$\frac{1}{2}(L_\xi g)(U_1, U_2) = (r - \lambda)g(U_1, U_2) - \mu\eta(U_1)\eta(U_2), \tag{3.1}$$

for all vector fields U_1, U_2 on M. Additionaly from (2.10) and (3.1), it generates

$$(r - \lambda - \alpha)g(U_1, U_2) = (\mu - \alpha)\eta(U_1)\eta(U_2). \tag{3.2}$$

Consider $U_2 = \xi$ in the followed condition with make use of (2.1),it obtains

$$(r - \lambda - \mu)\eta(U_1) = 0. \tag{3.3}$$

On account of $\eta(U_1) \neq 0$, it gives

$$r = \lambda + \mu \tag{3.4}$$

Presently, both together λ, μ are constants, hence r is also constant.
It is expressing as:

Corollary 3.1. *If a $(\alpha - pkm)_3$ M admits an η-(YS) (g, ξ), ξ being the Reeb vector field of M, then the scalar curvature is constant.*

In view of (3.4), if $\mu = 0$, it becomes $r = \lambda$ and so (3.1) obtains $L_\xi g = 0$. Therefore, ξ is a Killing vector field and we called M is a Killing $(\alpha - pkm)_3$. Then we have

Corollary 3.2. *If a $(\alpha - pkm)_3$ M admits a (YS) (g, ξ), ξ being the Reeb vector field of M, then the manifold is a Killing $(\alpha - pkm)_3$.*

Presently, from (2.9) and (3.4), we get,

$$S(U_1, U_2) = \left(\frac{\lambda + \mu}{2} + \alpha^2\right) g(U_1, U_2) - \left(\frac{\lambda + \mu}{2} + 3\alpha^2\right) \eta(U_1)\eta(U_2), \tag{3.5}$$

for all vector fields U_1, U_2 on M. Thus, it follows that

Corollary 3.3. *If a $(\alpha - pkm)_3$ M admits a η-(YS) (g, ξ), ξ being the Reeb vector field of M, then the manifold becomes η-Einstein manifold.*

It cognizant,

$$(\nabla_{U_1} S)(U_2, U_3) = \nabla_{U_1} S(U_2, U_3) - S(\nabla_{U_1} U_2, U_3) - S(Y, \nabla_{U_1} U_2). \tag{3.6}$$

for all vector fields U_1, U_2, U_3 on M and ∇ is the Levi-Civita connection associated with g. Presently, supplanting the expansion of S from (3.5), we get,

$$(\nabla_{U_1} S)(U_2, U_3) = -\left[\frac{\lambda + \mu}{2} + 3\alpha^2\right] [\eta(U_3)(\nabla_{U_1}\eta)U_2 + \eta(U_2)(\nabla_{U_1}\eta)U_3]. \tag{3.7}$$

for all vector fields U_1, U_2, U_3 on M. Presently, allow the manifold be Ricci symmetric i.e $\nabla S = 0$. Next from (3.7), we have

$$\left[\frac{\lambda + \mu}{2} + 3\alpha^2\right] [\eta(U_3)(\nabla_{U_1}\eta)U_2 + \eta(U_2)(\nabla_{U_1}\eta)U_3] = 0. \tag{3.8}$$

for all vector fields U_1, U_2, U_3 on M. Setting $Z = \xi$ in the followed condition and make use of (2.5) and (2.1), it yields

$$\left[\frac{\lambda+\mu}{2} + 3\alpha^2\right] [-\alpha g(\phi U_1, \phi U_2)] = 0, \tag{3.9}$$

for all vector fields U_1, U_2, U_3 on M. Subsequently, we have

$$\lambda + \mu = -6\alpha^2 \tag{3.10}$$

Thus, we have

Theorem 3.4. *Let a $(\alpha - pkm)_3$ M admits an η-(YS) (g, ξ), ξ being the Reeb vector field of M. If the manifold is Ricci symmetric, then $\lambda + \mu = -6\alpha^2$, where λ, μ, α are constants.*

Presently, in the event that the Ricci tensor S is η-recurrent, at that point we possess

$$\nabla S = \eta \otimes S. \tag{3.11}$$

It suggests

$$(\nabla_{U_1} S)(U_2, U_3) = \eta(U_1) S(U_2, U_3), \tag{3.12}$$

for all vector fields U_1, U_2, U_3 on M. As well employing (3.7), we compel

$$-\left[\frac{\lambda+\mu}{2} + 3\alpha^2\right] [\eta(U_3)(\nabla_{U_1}\eta)U_2 + \eta(U_2)(\nabla_{U_1}\eta)U_3] = \eta(U_1) S(U_2, U_3), \tag{3.13}$$

for all vector fields U_1, U_2, U_3 on M. Employing (2.5), then followed equation come to be

$$\begin{aligned} -\left[\frac{\lambda+\mu}{2} + 3\alpha^2\right] &\{\eta(U_3)[\alpha(g(U_1, U_2) - \eta(U_1)\eta(U_2))] \\ &+\eta(U_2)[\alpha(g(U_1, U_3) - \eta(U_1)\eta(U_3))]\} = \eta(U_1) S(U_2, U_3) \end{aligned} \tag{3.14}$$

Now taking $U_2 = \xi, U_3 = \xi$ and make use of (2.1) and (3.5), the above equation come to be, $2\alpha^2 \eta(U_1) = 0$. Since $\eta(U_1) \neq 0$, for all U_1 on M. we have,

$$\alpha = 0. \tag{3.15}$$

This leads the accompanying

Theorem 3.5. *Let a $(\alpha - pkm)_3$ M admits an η-(YS) (g, ξ), ξ being the Reeb vector field of M. If the Ricci tensor S is η-recurrent, then $\alpha = 0$.*

Presently, if the manifold is Ricci symmetric and the Ricci tensor S is η-recurrent, then employing (3.15) in (3.10) then (3.4) becomes $r = 0$, we have the following:

Proposition 3.6. *Let a $(\alpha - pkm)_3$ M admits an η-(YS) (g, ξ), ξ being the Reeb vector field of M. If the manifold is Ricci symmetric and the Ricci tensor S is η-recurrent, then the manifold becomes flat.*

Make V be pointwise collinear with ξ i.e., $V = b\xi$, where b is a function on M. Next, the equation (1.3) yields,

$$(L_{b\xi} g)(U_1, U_2) = 2(r - \lambda) g(U_1, U_2) - 2\mu \eta(U_1)\eta(U_2), \tag{3.16}$$

for all vector fields U_1, U_2, U_3 on M. Employing the property of Lie derivative and Levi-Civita connection, we have,

$$bg(\nabla_{U_1}\xi, U_2) + (U_1 b)\eta(U_2) + bg(\nabla_{U_2}\xi, U_1) + (U_2 b)\eta(U_1)$$
$$= 2(r-\lambda)g(U_1, U_2) - 2\mu\eta(U_1)\eta(U_2) \tag{3.17}$$

Make use of (2.7) and (2.3), the followed equation reduces to,

$$2b\alpha[g(U_1, U_2) - \eta(U_1)\eta(U_2)] + (U_1 b)\eta(U_2) + (U_2 b)\eta(U_1)$$
$$= 2(r-\lambda)g(U_1, U_2) - 2\mu\eta(U_1)\eta(U_2) \tag{3.18}$$

Setting $U_2 = \xi$ in the followed equation and employing (2.1) and (2.4), we obtain

$$U_1 b + (\xi b)\eta(U_1) = 2(r-\lambda)\eta(U_1) - 2\mu\eta(U_1) \tag{3.19}$$

Again setting $U_1 = \xi$, we get

$$\xi b = r - \lambda - \mu \tag{3.20}$$

Then, using (3.20), the equation (3.19) becomes,

$$U_1 b = (r - \lambda - \mu)\eta(U_1) \tag{3.21}$$

Employing exterior differentiation in (3.21), we receive,

$$(r - \lambda - \mu)d\eta = 0 \tag{3.22}$$

In view of $d\eta \neq 0$ [13] ,the followed equation generates

$$r = \lambda + \mu \tag{3.23}$$

Using (3.23), the equation (3.21) gets,

$$Xb = 0, \tag{3.24}$$

which implies that b is constant. Hence, we have the following theorem:

Theorem 3.7. *Let M be a $(\alpha - pkm)_3$ admitting an η-(YS) (g, V), V being a vector field on M. If V is pointwise collinear with ξ, then V is a constant multiple of ξ, where ξ being the Reeb vector field of M.*

Employing (3.23), the equation (1.3) yields,

$$(L_V g)(U_1, U_2) = 2\mu[g(U_1, U_2) - \eta(U_1)\eta(U_2)], \tag{3.25}$$

for all vector fields U_1, U_2, U_3 on M. We develop,

Theorem 3.8. *Let M be a $(\alpha - pkm)_3$ admitting an η-(YS) (g, V), V being a vector field on M which is pointwise collinear with ξ, where ξ being the Reeb vector field of M. Then, V is a Killing vector field iff the soliton reduces to a (YS).*

From the equation (3.5), it becomes,

$$QU_1 = \left(\frac{\lambda+\mu}{2} + \alpha^2\right)U_1 - \left(\frac{\lambda+\mu}{2} + 3\alpha^2\right)\eta(U_1)\xi, \tag{3.26}$$

for all vector fields U_1 on M and Q. We have,

$$(\nabla_\xi Q)U_1 = \nabla_\xi QU_1 - Q(\nabla_\xi U_1), \tag{3.27}$$

for all vector fields U_1 on M Next employing (3.26), the equation (3.27) becomes,

$$(\nabla_\xi Q)U_1 = -\left(\frac{\lambda+\mu}{2} + 3\alpha^2\right)((\nabla_\xi \eta)U_1)\xi \tag{3.28}$$

Using (2.5)

$$(\nabla_\xi Q)U_1 = 0, \tag{3.29}$$

for all vector fields U_1 on M. Therefore Q is parallel along ξ. Once again from (3.7),we accomplish,

$$(\nabla_\xi S)(U_1, U_2) = -\left(\frac{\lambda+\mu}{2} + 3\alpha^2\right)[\eta(U_2)(\nabla_\xi \eta)U_1 + \eta(U_1)(\nabla_\xi \eta)U_2], \tag{3.30}$$

for all vector fields U_1, U_2 on M. Using (2.5) in the followed equation, we make out,

$$(\nabla_\xi S)(U_1, U_2) = 0, \tag{3.31}$$

for all vector fields U_1, U_2 on M. since S is parallel along ξ. So, we state the following theorem:

Theorem 3.9. *Let M be a $(\alpha - pkm)_3$ admitting an η-(YS) (g, ξ), ξ being the Reeb vector field of M. Then Q and S are parallel along ξ, where Q is the Ricci operator, defined by $S(U_1, U_2) = g(QU_1, U_2)$ and S is the Ricci tensor of M.*

4. Curvature properties on $(\alpha - pkm)_3$ admitting η-(YS)

The projective curvature tensor P of type (1,3) in 3-manifolds M is defined by

$$P(U_1, U_2)U_3 = R(U_1, U_2)U_3 - \frac{1}{2}[S(U_2, U_3)U_1 - S(U_1, U_3)U_2], \tag{4.1}$$

for all vector fields U_1, U_2, U_3 on M (See [12]). Imposing $U_3 = \xi$ in the followed equation also make use of (2.8) and (2.9), we obtain

$$P(U_1, U_2)\xi = -\alpha^2[\eta(U_2)U_1 - \eta(U_1)U_2] - \alpha^2[-\eta(U_2)U_1 + \eta(U_1)U_2], \tag{4.2}$$

which implies that,

$$P(U_1, U_2)\xi = 0. \tag{4.3}$$

So, we state the following theorem:

Theorem 4.1. *If M is a $(\alpha - pkm)_3$ admitting η-(YS) (g, ξ), ξ being the Reeb vector field on M, then M is ξ-projectively flat.*

The concircular curvature tensor $\tilde{C}$ of type (1,3) in 3-manifold [9] is given by

$$\tilde{C}(U_1, U_2)U_3 = R(U_1, U_2)U_3 - \frac{r}{6}[g(U_2, U_3)U_1 - g(U_1, U_3)U_2], \tag{4.4}$$

for any vector fields U_1, U_2, U_3 on M. Setting $U_3 = \xi$ in the followed equation further make use of (2.4) and (2.8), it yields

$$\tilde{C}(U_1, U_2)U_3 = -\alpha^2[\eta(U_2)U_1 - \eta(U_1)U_2] - \frac{r}{6}[\eta(U_2)U_1 - \eta(U_1)U_2], \tag{4.5}$$

Now using (3.4), we get

$$\tilde{C}(U_1, U_2)\xi = \left[-\alpha^2 - \frac{\lambda+\mu}{6}\right][\eta(U_2)U_1 - \eta(U_1)U_2], \tag{4.6}$$

This implies that $\tilde{C}(U_1, U_2)\xi = 0$, if and only if $\lambda + \mu = -6\alpha^2$. It can be expressed as

Theorem 4.2. *A $(\alpha - pkm)_3$ admitting η-(YS) (g, ξ), ξ being the Reeb vector field on M is ξ-concircularly flat iff $\lambda + \mu = -6\alpha^2$.*

Presently, if the Ricci tensor S is η-recurrent and applying (3.15) in (4.5), we obtain,

Corollary 4.3. *Let M be a $(\alpha - pkm)_3$ admitting η-(YS) (g, ξ), ξ being the Reeb vector field on M. If the manifold is ξ-concircularly flat and the Ricci tensor is η-recurrent, then the manifold M becomes flat.*

The Weyl-conformal curvature tensor W of type (1,3) in 3-manifold M is represented by

$$\begin{aligned} W(U_1, U_2)U_3 = & R(U_1, U_2)U_3 - [S(U_2, U_3)U_1 - S(U_1, U_3)U_2 \\ & - g(U_2, U_3)QU_1 - g(U_1, U_3)QU_2] \\ & + \frac{r}{2}[g(U_2, U_3)U_1 - g(U_1, U_2)U_3], \end{aligned} \tag{4.7}$$

for any vector fields U_1, U_2, U_3 on M (See [12]). Fixing $U_3 = \xi$ in the followed equation as well as employing (2.4),(2.8),(3.5) and (3.26). we get,

$$\begin{aligned} W(U_1, U_2)\xi = & \left(\frac{\lambda + \mu}{2}\right)[\eta(U_2)U_1 + \eta(U_1)U_2] - 2\left(\frac{\lambda + \mu}{2}\right)\eta(U_1)\eta(U_2)\xi \\ & + \left(\frac{\lambda + \mu}{2}\right)[\eta(U_2)U_1 - \eta(U_1)U_2] \end{aligned} \tag{4.8}$$

Thus, we get

$$W(U_1, U_2)\xi = (\lambda + \mu)[\eta(U_2)U_1 - \eta(U_1)\eta(U_2)\xi], \tag{4.9}$$

which implies that $W(U_1, U_2) = 0$ iff $\lambda + \mu = 0$. Hence, it gives the following theorem:

Theorem 4.4. *A $(\alpha - pkm)_3$ admitting η-(YS) (g, ξ), ξ being the Reeb vector field on M is ξ-Weyl-conformally flat iff $\lambda + \mu = 0$.*

The Pseudo-projective curvature tensor $\bar{P}$ of type (1,3) in 3-manifold M is defined by [10]

$$\begin{aligned} \bar{P}(U_1, U_2)U_3 = & aR(U_1, U_2)U_3 + b[S(U_2, U_3)U_1 - S(U_1, U_3)U_2] \\ & - \frac{r}{3}(\frac{a}{2} + b)[g(U_2, U_3)U_1 - g(U_1, U_3)U_2], \end{aligned} \tag{4.10}$$

for any vector fields U_1, U_2, U_3 on M and a, b are constants. Taking $U_3 = \xi$ in the followed equation as well as playing (2.4),(2.9), (3.4), (3.5) and (3.26), the followed equation proceeds,

$$\bar{P}(U_1, U_2)\xi = \left[-a\alpha^2 - 2\alpha^2 b - \frac{\lambda + \mu}{3}\left(\frac{a}{2} + b\right)\right][\eta(U_2)U_1 - \eta(U_1)U_2] \tag{4.11}$$

This implies that $\bar{P}(U_1, U_2)\xi = 0$ if and only if $a\alpha^2 + 2\alpha^2 b + \frac{\lambda+\mu}{3}\left(\frac{a}{2} + b\right) = 0$.
At that point by explaining, we get $\bar{P}(U_1, U_2)\xi = 0$ iff $(a + 2b)\left[\alpha^2 + \frac{\lambda+\mu}{6}\right] = 0$.
i.e., either $a + 2b = 0$ or $\lambda + \mu = -6\alpha^2$. So, we can state the following:

Theorem 4.5. *A $(\alpha - pkm)_3$ admitting η-(YS) (g, ξ), ξ being the Reeb vector field on M is ξ-Pseudo-projectively flat iff $a + 2b = 0$ or $\lambda + \mu = -6\alpha^2$.*

Presently, on the off chance that the Ricci tensor S is η-recurrent at that point utilizing (3.15) in (4.11), we produce,

$$\bar{P}(U_1, U_2)\xi = \left(\frac{a+2b}{6}\right)(\lambda + \mu)[\eta(U_2)U_1 - \eta(U_1)U_2]. \tag{4.12}$$

Consequently utilizing (3.4) in (4.12), we obtain,

Corollary 4.6. *Let a $(\alpha - pkm)_3$ admitting η-(YS) (g, ξ), ξ being the Reeb vector field on M. If the manifold is ξ-Pseudo-projectively flat and the Ricci tensor is η-recurrent, then the manifold M becomes flat, provided $a + 2b \neq 0$.*

We have,

$$R(\xi, U_1) \cdot S = S(R(\xi, U_1)U_2, U_3) + S(U_2, R(\xi, U_1)U_3), \tag{4.13}$$

for any vector fields U_1, U_2, U_3 on M.
Presently, let the manifold be ξ-semi symmetric, i.e., $R(\xi, U_1) \cdot S = 0$.
Next, from (4.13), we compel

$$S(R(\xi, U_1)U_2, U_3) + S(U_2, R(\xi, U_1)U_3) = 0. \tag{4.14}$$

for any vector fields U_1, U_2, U_3 on M. Utilizing (2.8), supplanting the declaration of S from (3.5) and clarifying, we concur,

$$\alpha^2\left(\frac{\lambda+\mu}{2}\right)[g(U_1, U_2)\eta(U_3) + g(U_1, U_3)\eta(U_2) - 2\eta(U_1)\eta(U_2)\eta(U_3)] = 0 \tag{4.15}$$

Taking $U_3 = \xi$ in the followed equation and utilizing (2.1) and (2.4), we obtain

$$\alpha^2\left(\frac{\lambda+\mu}{2}\right)[g(U_1, U_2) - \eta(U_1)\eta(U_2)] = 0, \tag{4.16}$$

for any vector fields U_1, U_2 on M.
Using (2.2), the followed equation becomes.

$$\alpha^2\left(\frac{\lambda+\mu}{2}\right)[g(\phi U_1, \phi U_2)] = 0, \tag{4.17}$$

for any vector fields U_1, U_2, U_3 on M. Therefore, we have

$$\alpha^2\left(\frac{\lambda+\mu}{2}\right) = 0 \tag{4.18}$$

Then, either $\alpha^2 = 0$ or $\lambda + \mu = 0$. Hence, we have the following theorem:

Theorem 4.7. *If a $(\alpha - pkm)_3$ admitting η-(YS) (g, ξ), ξ being the Reeb vector field on M is ξ-semi symmetric, then either $\alpha = 0$ or $\lambda + \mu = 0$.*

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