Chinese Journal of Mathematical Sciences
Vol. 1 No. 1 (January-June, 2021)

REALITY OR MATHEMATICAL FORMALITY – EINSTEIN'S GENERAL RELATIVITY ON MULTI-FIELDS

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ABSTRACT. A physical law is described by differential equation. Meanwhile, Einstein's general relativity asserts the equation should be the same form in all coordinate systems, which results in a revolution for finding covariant equations on physical laws in last century. However, there are no essential difference in covariant or differential equation for the conclusion unless its form of expression of a physical law, one attends to trifles but neglect the essential in applying Einstein's general relativity. In this paper, we review the Einstein's general relativity in philosophy, not only its pursuit on mathematical form but also its enlightening for understanding things in the universe by remaind its philosophical implications, including the discussion on the essence of invariants and general relativity, the contradictory system's universal with combinatorics, new mathematical elements for understanding the reality of things and Einstein's general relativity to multi-fields with differential equations on new elements, i.e., continuity flows which are more general for understanding things in the universe.

1. Introduction

A well-known natural law is that all things are continuously changing in the universe, concluded by Heraclitus, a philosopher in ancient Greek that one can not step enter the same river twice, which implies an initial work for understanding things, i.e., holding on the change laws of things in the world. Then, *how do we describe the change event of a thing T?* Usually, we characterize the state function $F(t,x)$ of T on position x in space and the moment, i.e., the time t in a coordinate system $\mathcal{F}$ in the eyes, ears, nose, tongue, body and mind of humans, where $x=(x^1,x^2,x^3)\in\mathbb{R}^3$. Because of the continuously change, all values x and t on a thing T can only be observed in themselves difference, not the absolute one and furthermore, its state $F(t,x)$ can be determined if its initial position and speed are known before hand, which leads to characterize the change law of T by differential equation

$$\begin{cases} \mathfrak{F}\left(t,x,F_t,F_x,\cdots\right)=0 \\ F(x)|_{t=t_0}=F_0,\ F_x|_{t=t_0}=F_1 \end{cases} \tag{1.1}$$

2010 *Mathematics Subject Classification.* Primary 05C21,05C78,15A03,34B45; Secondary 37N25,92B05.

Key words and phrases. Einstein's general relativity, invariant, harmonious system, contradictory systems, continuity flow, mathematical element, Smarandache multi-space, multi-field, mathematical combinatorics.

on observed x and t, a solvable equation by its physical meaning. For example, the Maxwell equation

$$\begin{cases} \nabla \cdot \mathbf{E}(t,x) = \dfrac{1}{\varepsilon_0}\rho(x), \\ \nabla \times \mathbf{E}(t,x) = -\dfrac{\partial}{\partial t}\mathbf{B}(t,x), \\ \nabla \cdot \mathbf{B}(t,x) = 0, \\ \nabla \times \mathbf{B}(t,x) = \mu_0 \mathbf{j}(t,x') + \dfrac{1}{c^2}\dfrac{\partial}{\partial t}\mathbf{E}(t,x) \end{cases} \tag{1.2}$$

on an electromagnetic field, where $\mathbf{E}$ and $\mathbf{B}$ are respectively the electric and magnetic fields dependent on both the position x and time t with an operator action ∇, i.e.,

$$\begin{aligned} \mathbf{E} &= (E_{x^1}, E_{x^2}, E_{x^3}), \quad \mathbf{B} = (B_{x^1}, B_{x^2}, B_{x^3}), \\ \nabla &= \left(\frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right) \end{aligned}$$

and $\rho(t,x)$, $\mathbf{j}(t,x)$ are respectively the densities of electric charge and electric current.

Certainly, different coordinate systems $\{O; t, x\}$ result in differential equations on a thing T, maybe in different forms of (1.1). However, all these equations are characterizing the same thing T. Then, *how are these differential equations related in mathematics?* Einstein answered this question with a long time speculation on the gravitation with the inertial force, i.e., the general relativity in 1915 following.

Principle 1.1 (Equivalence,[3]) *These gravitational forces and inertial forces acting on a particle in a gravitational field are equivalent and indistinguishable from each other.*

Principle 1.2 (Covariance,[3]) *An equation describing the law of physics should have the same form in all coordinate systems.*

According to Principles 1.1 and 1.2, Einstein thought that a physical equation should be a tensor equation because tensors are covariant, He presented his gravitational equation $R_{ij} - \frac{1}{2}g_{ij}R = \kappa T_{ij}$ in [5] by tensors, which is equivalent to the minimum variational principle, also includes the Newtonian's law of universal gravitation as a limitation case ([3]). For testing the correctness of general relativity, Einstein predicted 3 experiments, i.e., Mercury's motion on the perihelion, curved light and gravitational redshift, which were all verified by experiments later, arrival at the rightness of general relativity.

Einstein's great achievements led to a revolution movement for finding covariant in mathematics on physical laws, i.e., tensors which promoted physics greatly. However, a few peoples consider the philosophical implication of Einstein's general relativity in revolution. Certainly, it gives unified mathematical forms of physical laws, but *what is its essence for understanding things in the universe?* The answer is not certain. Lots of researchers were absorbed in finding covariant forms of physical laws in classical, rewriting them in tensors but innovative achievements rarely. *Why did this happen?* We should think back to its reason deeply that happened, i.e., too emphasizing the form but neglecting the conclusion.

Notice that the science's role is understanding the nature and then, developing our society in coordination with the universal laws. For this objective, the first question is *whether the reality can be all characterized by solvable differential equations or not?* Notice that all things in the universe can be characterized by solvable differential equation is only one's priori hypotheses. The answer is likely uncertain because few peoples consider the non-solvable differential equations with reality. The second question is on the mathematical expression of reality, i.e., *which is more important for the expression, the form or its conclusion?* The answer is certainly the conclusion because the form of expression always serves that of the conclusion. Certainly, getting an unified mathematical form of physical law is beautiful but we can not attend to trifles, neglect the essential in applying Einstein's general relativity to the reality of things. The main purpose of this paper is to review the Einstein's general relativity in philosophy, not only its pursuit on mathematical form but also its enlightening for understanding things in the universe by remaind its philosophical implications, including the discussion on the essence of invariants and general relativity, the contradictory system's universal with combinatorics, new mathematical elements for understanding the reality of things and Einstein's general relativity to multi-fields with differential equations on new elements in mathematical combinatorics ([6]), i.e., continuity flows which are more general for understanding things in the universe.

For terminologies and notations not mentioned here, we follow the reference [1] for mechanics, [3] and [4] for general relativity, [5] for algebraic invariants, [11] for combinatorial geometry, [27] for elementary particles and [11], [28] for Smarandache systems and multispaces.

2. Invariants with Physical Equations Background

Einstein's general relativity concludes the mathematical form $\mathfrak{F}(x)$ of a physical equation $\mathfrak{F}(x) = 0$ should be invariant under all transformations T on its coordinate system $\{O; x\}$, i.e.,

$$\mathfrak{F}(x) = \mathfrak{F}(T(x)), \qquad (*)$$

which should be an invariant. We look back the invariant with finding of the Einstein's general relativity.

2.1. Algebraic Invariants A physical law should be an expression or solution of differential equation in a coordinate system $\{O; x^1, x^2, \cdots, x^n\}$. Consider invariants under linear transformation T of coordinate system following Hilbert ([5]). Let $\mathcal{C}(a_0, a_1, \cdots, a_n; x^1, x^2, \cdots, x^n)$ be a polynomial with an image $\mathcal{C}(a'_0, a'_1, \cdots, a'_n; x'^1, x'^2, \cdots, x'^n)$ under a linear transformation $T = (\alpha_{ij})_{n\times n}\, x'^t$, where $x' = (x'^1, x'^2, \cdots, x'^n)$. Then, $\mathcal{C}$ is said to be a covariant if

$$\mathcal{C}(a'_0, a'_1, \cdots, a'_n; x'^1, x'^2, \cdots, x'^n) = \delta^p \mathcal{C}(a_0, a_1, \cdots, a_n; x^1, x^2, \cdots, x^n),$$

where $\delta = \det(T)$, the determinant of T which is a constant and p is an integer.

For example,

$$(a_0a_1 - a_1^2)(x^1)^2 + (a_0a_3 - a_1a_2)x^1x^2 + (a_1a_3 - a_2^2)(x^2)^2,$$

$$\begin{aligned}&(a_0^2a_3 - 3a_0a_1a_2 + 2a_1^3)(x^)1^3 + 3(a_0a_1a_3 + a_1^2a_2 - 2a_0a_2^2)(x^1)^2x^2\\ &\quad -3(a_0a_2a_3 - 2a_1^2a_3 + a_1a_2^2)x^1(x^2)^2 - (a_0a_3^2 - 3a_1a_2a_3 + 2a_2^3)(x^2)^3\end{aligned}$$

are the only covariant of degree 2 in $p = 2$ or degree 3 in $p = 3$, respectively [5].

Particularly, if $\det(T) = 1$,

$$\mathcal{C}(a_0', a_1', \cdots, a_n'; x'^1, x'^2, \cdots, x'^n) = \mathcal{C}(a_0, a_1, \cdots, a_n; x^1, x^2, \cdots, x^n),$$

i.e., its form is invariant under the transformation T is particularly important in physics because it is in accord with Principle 1.2. For example, the Lorentz transformation T, i.e.,

$$\begin{cases} x'^1 = \dfrac{x^1 - vt}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \\ x'^2 = x^2 \\ x'^3 = x^3 \\ t' = \dfrac{t - \frac{v}{c^2}x^1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \end{cases} \quad \text{with matrix} \quad (\alpha_{ij})_{4\times 4} = \begin{pmatrix} \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} & 0 & 0 & \frac{-v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-\frac{v}{c^2}}{\sqrt{1-\left(\frac{v}{c}\right)^2}} & 0 & 0 & \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \end{pmatrix}$$

it is easily verified that

$$\det(T) = \begin{vmatrix} \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} & 0 & 0 & \frac{-v}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{-\frac{v}{c^2}}{\sqrt{1-\left(\frac{v}{c}\right)^2}} & 0 & 0 & \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \end{vmatrix} = 1.$$

Thus, a covariant $\mathcal{C}(a_0, a_1, \cdots, a_n; x^1, x^2, x^3, t)$ transferred by the Lorentz transformation holds with

$$\mathcal{C}(a_0', a_1', \cdots, a_n'; x'^1, x'^2, x'^3, t) = \mathcal{C}(a_0, a_1, \cdots, a_n; x^1, x^2, x^3, t).$$

Furthermore, it can be applied to the covariance of physical laws under special transformation, i.e., the Lorentz transformation, which is nothing else but the Einstein's special relativity.

2.2. General Relativity with Equation Notice that the transformation T of coordinate system maybe not linear but differentiable in Principle 1.2 and a physical law is usually describing by differentials. We should find the differential, not only the algebraic invariants of physical laws in general. Einstein found to solve this problem is a little easy than that of Hilbert et al. on algebraic invariants because of the chain rule of differential applicable, which concludes that if $u = g(y^1, y^2, \cdots, y^m)$ is a differential function at $b = (b_1, b_2, \cdots, b_m) \in \mathbb{E}^m$ and $f_i(x)$ differentiable at $a \in \mathbb{E}^n$ with $b_i = f_i(a)$ for $1 \leq i \leq m$, then the chain rule of differential concludes that $u = g \circ f$ is also differentiable at a with

$$\frac{\partial u}{\partial x^i} = \sum_{j=1}^{m} \frac{\partial u}{\partial y^j}\frac{\partial y^j}{\partial x^i} = \sum_{j=1}^{m} \frac{\partial g}{\partial y^j}\frac{\partial f_j}{\partial x^i}, \quad 1 \le i \le n, \tag{2.1}$$

and the complete differential

$$du = \sum_{i=1}^{n} \frac{\partial u}{\partial y^i} dy^i = \sum_{i=1}^{n} \frac{\partial u}{\partial y^i} \sum_{j=1}^{n} \frac{\partial f_i}{\partial x^j} dx^j = \sum_{i=1}^{n} \frac{\partial u}{\partial x^i} dx^i, \tag{2.2}$$

is invariant under a transformation f of coordinate system $\{O; y^1, y^2, \cdots, y^n\}$, no matter whatever f is linear or not. Generally, let $f = (f_1, f_2, \cdots, f_n) : \mathbb{E}^n \to \mathbb{E}^n$ be differentiable at $x \in \mathbb{E}^n$, then we know the differential of f at x should be the Jacobian matrix

$$J_{f;x} = \frac{\partial (f_1, f_2, \cdots, f_n)}{\partial (x^1, x^2, \cdots, x^n)} = \begin{pmatrix} \frac{\partial f_1}{\partial x^1} & \frac{\partial f_1}{\partial x^2} & \cdots & \frac{\partial f_1}{\partial x^n} \\ \frac{\partial f_2}{\partial x^1} & \frac{\partial f_2}{\partial x^2} & \cdots & \frac{\partial f_2}{\partial x^n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x^1} & \frac{\partial f_n}{\partial x^2} & \cdots & \frac{\partial f_n}{\partial x^n} \end{pmatrix}$$

with a complete differential

$$df = J_{f;x} dx^t, \tag{2.3}$$

which is invariant under transformations of coordinate system $\{O; x^1, x^2, \cdots, x^n\}$, where $dx = (dx^1, dx^2, \cdots, dx^n)$ and furthermore,

$$\det(J_{f;x}) = \det\left(\frac{\partial (f_1, f_2, \cdots, f_n)}{\partial (x^1, x^2, \cdots, x^n)}\right) \|dx\| \tag{2.4}$$

with $\|dx\| = dx^1 dx^2 \cdots dx^n$.

Let M be an manifolds of dimension n with a finite cover $\{\mathfrak{C}_i; 1 \le i < \infty\}$ such that each $\mathfrak{C}_i$ is homeomorphic to $\mathbb{R}^n$, i.e., there is a $1-1$ continuous mapping $\rho : \mathfrak{C}_i \to \mathbb{R}^n$ and an inverse ρ^{-1} for an integer $n \ge 1$. Thus, an manifold is a combination of finite local Euclidean space $\mathbb{R}^n$. Then, *what is a tensor?* A tensor is tensor product of tangent vectors X, cotangent vectors X^* generated by ([4])

$$\left\{ \frac{\partial}{\partial x^i}\bigg|_p \mid 1 \le i \le n \right\} \quad \text{and} \quad \left\{ dx_p^i \ 1 \le i \le n \right\}$$

on local chart $(\mathfrak{C}_p; p)$ of manifold M, i.e.,

$$X|_{\mathfrak{C}_p} = \sum_{i=1}^{n} X^i \frac{\partial}{\partial x^i} \quad \text{or} \quad X^*|_{\mathfrak{C}_p} = \sum_{i=1}^{n} X^{*i} dx^i,$$

where X^i, X^{*i} are smooth functions on $(\mathfrak{C}_p; p)$ or exactly, a (r,s)-tensor $\tau_{\mathfrak{C}_p}$ is

$$\tau_{\mathfrak{C}_p} = \sum_{i_1,\cdots,i_r; j_1,\cdots,j_s} \tau^{i_1,i_2,\cdots,i_r}_{j_1,j_2,\cdots,j_s} \frac{\partial}{\partial x^{i_1}} \otimes \cdots \otimes \frac{\partial}{\partial x^{i_r}} \otimes dx^{j_1} \otimes \cdots \otimes dx^{j_s}, \tag{2.5}$$

where $\tau^{i_1,i_2,\cdots,i_r}_{j_1,j_2,\cdots,j_s}$ is a smooth function on $\mathfrak{C}_p$. Notice that both of the tangent and cotangent vectors are invariant by equations (2.2), (2.3) and (2.4), which concludes that tensors are invariant under transformations T of coordinate system $\{O; x^1, x^2, \cdots, x^n\}$, consistent with the requirements of Principle 1.2.

For establishing metric space on tensors, Riemann defined the length of a curve γ on a manifold to be the integral of the length of tangent vector along γ, resulted in a metric g on manifold M such that $g\langle X, Y\rangle = g(X,Y)$, $g(X,X) \geq 0$ with equality hold only if $X = \mathbf{0}$ for $\forall X, Y \in T_p(M)$, where TM is the tangent vector space at a point $p \in M$ ([4]). Notice that g is a $(2,0)$ covariant tensor determined by

$$g = \sum_{i,j} g_{ij} dx^i \otimes dx^j, \qquad g_{i,j} = g\left(\frac{\partial}{\partial x^i}, \frac{\partial}{\partial x^j}\right) \tag{2.6}$$

and $d\gamma = \sqrt{\sum\limits_{i,j} g_{ij}(p) dx^i \otimes dx^j}$ on $\mathfrak{C}_p$ of $p \in \gamma$.

Then, *how to differentiate a tensor X with respect to Y on a Riemmanian manifold for $X, Y \in T_pM$?* Generally,

$$(D_Y X)_{\mathfrak{C}_p} = \sum_{i,j,k} Y^k \left(\frac{\partial X_i}{\partial x^k} + X^j \Gamma^i_{jk}\right) \frac{\partial}{\partial x^i} \tag{2.7}$$

in the local chart $\left(\mathfrak{C}_p; x^i\right)$, where

$$\Gamma^k_{ij} = \frac{1}{2} \sum_l g^{kl} \left(\frac{\partial g_{il}}{\partial x^j} + \frac{\partial g_{lj}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^l}\right) \tag{2.8}$$

with $(g_{ij})_{n\times n} \left(g^{kl}\right)_{n\times n} = I_{n\times n}$, and a curvature tensor $\mathcal{R}$ is defined by

$$\mathcal{R} = \sum_{i,i,k,l} R^l_{kij} dx^k \otimes \frac{\partial}{\partial x^l} \otimes dx^i \otimes dx^j \tag{2.9}$$

with

$$R^l_{kij} = \frac{\partial \Gamma^l_{kj}}{\partial x^i} - \frac{\partial \Gamma^l_{ki}}{\partial x^j} + \Gamma^h_{kj}\Gamma^l_{hi} - \Gamma^h_{ki}\Gamma^l_{hj}, \quad R_{ij} = \sum_k R^k_{ikj}, \tag{2.10}$$

which implies that

$$\nabla_s R^l_{kij} = \frac{\partial^2 \Gamma^l_{kj}}{\partial x^s \partial x^i} - \frac{\partial^2 \Gamma^l_{ki}}{\partial x^s \partial x^j} \quad \text{and} \quad \nabla_i R^l_{kjs} + \nabla_j R^l_{ksi} + \nabla_s R^l_{kij} = 0, \tag{2.11}$$

where

$$\nabla_i V_j = \frac{\partial V_j}{\partial x^i} - \sum_k \Gamma^k_{ij} V_k, \quad \nabla_s V_{ij} = \frac{\partial V_{ij}}{\partial x^s} - \sum_k \left(\Gamma^k_{si} V_{kj} + \Gamma^k_{sj} V_{ik}\right)$$

on tensors $V_i, V_{ij}, \cdots$. Applying equations (2.10) and (2.11), one gets that

$$\sum_j \nabla_j \left(R_{ij} - \frac{1}{2} g_{ij} R\right) = 0 \ \text{ where } \ R = \sum_{i,j} g^{ij} R_{ij}.$$

Notice that the conservation law of matters implies the distribution of matter T in the universe satisfies $\sum\limits_j \nabla_j T_{ij} = 0$, which concludes the gravitational equations

$$R_{ij} - \frac{1}{2} g_{ij} R = kT_{ij}$$

by Einstein. If it concludes Newtonians law of universal gravitation as a limitation, then it can be determined ([3]) that $k = 8\pi G/c^4 = 2.08 \times 10^{-48} s^2/cm \cdot g$.

Similarly, Einstein's method on gravitational field can be applied also to other classical fields of physical laws in covariant forms. For example, the equation (2.12) following is nothing else but the covariant form ([3])of Maxwell equations.

$$\begin{cases} \partial_\mu F^{\mu\nu} = \mu_0 j^\nu, \\ \partial_\kappa F_{\mu\nu} + \partial_\mu F_{\nu\kappa} + \partial_\nu F_{\kappa\mu} = 0 \end{cases} \tag{2.12}$$

where $\partial_\mu = (\partial_t, \partial_{x_1}, \partial_{x_2}, \partial_{x_3})$,

$$F_{\mu\nu} = \begin{bmatrix} 0 & E_{x_1}/c & E_{x_2}/c & E_{x_3}/c \\ -E_{x_1}/c & 0 & -B_{x_3} & B_{x_2} \\ -E_{x_2}/c & B_{x_3} & 0 & -B_{x_1} \\ -E_{x_3}/c & -B_{x_2} & B_{x_1} & 0 \end{bmatrix},$$

and

$$F^{\mu\nu} = \begin{bmatrix} 0 & -cB_{x_1} & -cB_{x_2} & -cB_{x_3} \\ cB_{x_1} & 0 & E_{x_3} & -E_{x_2} \\ cB_{x_2} & -E_{x_3} & 0 & E_{x_1} \\ cB_{x_3} & E_{x_2} & -E_{x_1} & 0 \end{bmatrix}.$$

3. Differential Equations on Multi-Fields

Let $\mathfrak{M}_r$ be the sets of things holding on by mathematics and let $\mathfrak{U}_r$ be things in the universe. Then, *is there* $\mathfrak{M}_r = \mathfrak{U}_r$ *or not?* There is popular conjecture on mathematics with the reality of things in the universe which claims that the physical universe is not merely described by mathematics but is mathematics proposed by Max Tegmark [26], which concludes $\mathfrak{M}_r = \mathfrak{U}_r$, a duplicate of the *Theory of Everything*. Clearly, if this conjecture is true, the mathematics could provides methods and tools for understanding all things in the universe. However, mathematics is homogenous without contradictions, i.e. a compatible one in logic but the contradictions exist everywhere in the eyes of humans, which implies that $\mathfrak{M}_r \subset \mathfrak{U}_r$ ([21]). Therefore, how to view a contradictory system to be mathematics is an important step for hold on the reality of things in the universe. We have to find new elements for establishing new mathematics and then, understanding things in the universe.

3.1. Contradictory Equations Notice that there is a priori assumption for describing a physical law by differential equations, i.e., the state function $F(t;x)$ of a thing in the universe is in accordance with the differential property. Even so, it can not be applied to the systems of things not synchronized such as adaptive systems in case by classical mathematics. Formally, assume the solution manifold of the ith second order differential equations

$$\begin{cases} \mathfrak{F}^i\left(t, x, F^i_t, F^i_x, \cdots\right) = 0 \\ F^i(x)|_{t=t_0} = F^i_0, \ F^i_x|_{t=t_0} = F^i_1 \end{cases} \tag{Eq_i}$$

is $S_i(t;x)$ for an integer $i,\ 1 \leq i \leq m$ but

$$\bigcap_{i=1}^{m} S_i(t;x) = \emptyset,$$

i.e., the system $(Eq_i), 1 \leq i \leq m$ is non-solvable in the classical. Then, *the non-solvable system $(Eq_i), 1 \leq i \leq m$ is valuable for understanding things in the universe or not?* The central point for answering this question is to determine whether there are no things or subspace $F^i(t;x),\ 1 \leq i \leq m$ in the universe holding with this system. The answer is No in general. For example, let H_i be a horse with active region restricted to the solution of differential equation

$$\frac{d^2x}{dt^2} + (1+2i)\frac{dx}{dt} + i(i+1)x = 0 \qquad (H_i)$$

for integer $1 \leq i \leq 6$. Clearly, the solution basis of equation (H_i) is $\left\{e^{-it}, e^{-(i+1)t}\right\}$ with

$$\bigcap_{i=1}^{6} \left\{e^{-it}, e^{-(i+1)t}\right\} = \emptyset,$$

i.e., there are no solution of the system $(H_i),\ 1 \leq i \leq 6$. *Could one concludes there are meaningless of the equation system $(H_i),\ 1 \leq i \leq 6$ in the universe?* Of course not because the horses $H_i,\ 1 \leq i \leq 6$ are actual living on the earth. Then, *what is wrong with his conclusion?* He is wrong from the beginning, i.e., concludes incorrectly that the active region of the 6 horses is the solution of equations $(H_i),\ 1 \leq i \leq 6$, i.e., the intersection of the 6 active regions.

Generally, it is certainly right that applies differential equations to describing the behavior of one horse H_i because each equation characterizes its one character and the horse should posses all characters that described by the equations. However, it can not be applied to a group of horses because the behaviors of the group of horses or an adaptive system is not the intersection

$$\bigcap_{i=1}^{m} S_i(t;x) \quad \text{but} \quad \bigcup_{i=1}^{m} S_i(t;x),$$

the union of solutions of the differential equations $(Eq_i),\ 1 \leq i \leq m$.

Then, *how to view such non-solvable differential systems for understanding things in the universe?* In classical mathematics, such a case is abandoned without attentions or discussed one by one, i.e., the variables x are assumed in different spaces but lost all of them are in one systems. That is why classical mathematics be limited only to isolated things. However, all things in the universe are connected in the universe, particularly, elements in a system. We should across this gap to characterize systems by elements, not only those of isolated elements.

For this objective, the topological graph G^L is the best candidate because each thing inherits a topological structure in the universe defined by

$$\begin{aligned} V\left(G^L\right) &= \left\{S_i(t;x) |\ 1 \leq i \leq m\right\}, \\ E\left(G^L\right) &= \left\{(S_i(t;x), S_j(t;x)) \mid S_i(t;x) \bigcap S_j(t;x) \neq \emptyset, 1 \leq i,j \leq m\right\} \end{aligned}$$

with a labeling

$$L:\ S_i(t;x)\in V(G)\to\{S_i(t;x);1\leq i\leq m\},$$
$$L:\ (S_i(t;x),S_j(t,x))\in E(G)\to\{S_i(t;x)\bigcap S_j(t;x);1\leq i,j\leq m\},$$

where

$$\mathfrak{L}=\{S_i(t;x);1\leq i\leq m\}\bigcup\{S_i(t;x)\bigcap S_j(t;x);1\leq i,j\leq m\}$$

is the set of labels. For example, the labeled graph of the non-solvable system of differential equations (H_i), $1\leq i\leq 6$ is shown in Figure 1, where, $\mathfrak{L}$ is denoted simply by the solution basis.

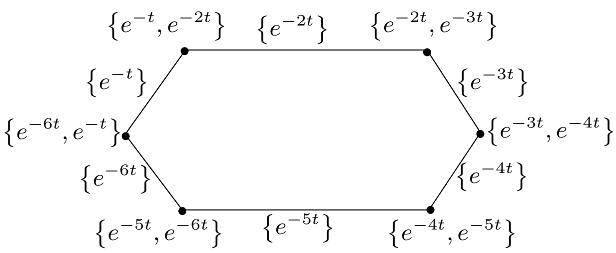

Figure 1. Graph solution

Consequently, the dynamical behavior of a system described by a non-solvable system of differential equations can be characterized by labeled graph G^L, a complex network ([2]) which can be abstracted to mathematical elements for understanding the group behavior of things in the universe. More details on non-solvable differential equations with reality can be found in [12]-[16].

3.2. Mathematical Elements A non-harmonious system is such a system $\mathfrak{S}$ consisting of elements P_i, $1\leq i\leq p, p\geq 2$ with interrelations that P_i is constrained on equation $\mathfrak{F}_i=0$ in space on time t. Certainly, there are elements for describing behavior of things in classical mathematics but none of them can be applied to such a non-harmonious system described by a non-solvable system of differential equations. However, they are indeed exist in the universe. Notice that if we view the labels on G^L as continuity flows, then it holds with conservation laws on vertices of G^L, which enables us to introduce 2 globally mathematical elements holding with conservation laws for such systems ([16]-[25]).

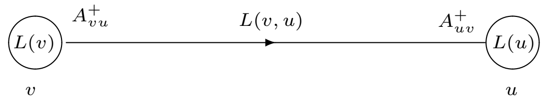

Figure 2

Element I. *Element I is called continuity flow $\overrightarrow{G}^L$, which is an oriented embedded graph $\overrightarrow{G}$ in a topological space $\mathfrak{S}$ associated with a mapping $L: v\to L(v)$, $(v,u)\to L(v,u)$, 2 end-operators $A^+_{vu}: L(v,u)\to L^{A^+_{vu}}(v,u)$ and $A^+_{uv}:$*

$L(u,v) \to L^{A^+_{uv}}(u,v)$ on a Banach space $\mathfrak{B}$ over a field $\mathfrak{F}$ such as those shown in Figure 2, with $L(v,u) = -L(u,v)$, $A^+_{vu}(-L(v,u)) = -L^{A^+_{vu}}(v,u)$ for $\forall (v,u) \in E\left(\overrightarrow{G}\right)$ and holding with continuity equation

$$\sum_{u \in N_G(v)} L^{A^+_{vu}}(v,u) = L(v) \ \ \text{for} \ \ \forall v \in V\left(\overrightarrow{G}\right).$$

Element II. *Element II is called harmonic flow $\overrightarrow{G}^L$, which is an oriented embedded graph $\overrightarrow{G}$ in a topological space $\mathfrak{S}$ associated with a mapping $L : v \to L(v) - iL(v)$ for $v \in E\left(\overrightarrow{G}\right)$ and $L : (v,u) \to L(v,u) - iL(v,u)$, 2 end-operators $A^+_{vu} : L(v,u) - iL(v,u) \to L^{A^+_{vu}}(v,u) - iL^{A^+_{vu}}(v,u)$ and $A^+_{uv} : L(v,u) - iL(v,u) \to L^{A^+_{uv}}(v,u) - iL^{A^+_{uv}}(v,u)$ on a Banach space $\mathfrak{B}$ over a field $\mathfrak{F}$ such as those shown in Figure 3,*

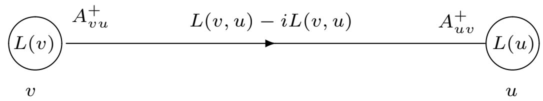

Figure 3

where $i^2 = -1$, $L(v,u) = -L(u,v)$ for $\forall (v,u) \in E\left(\overrightarrow{G}\right)$ and holding with continuity equation

$$\sum_{u \in N_G(v)} \left(L^{A^+_{vu}}(v,u) - iL^{A^+_{vu}}(v,u)\right) = L(v) - iL(v) \ \ \text{for} \ \ \forall v \in V\left(\overrightarrow{G}\right).$$

All continuity flows, i.e., Elements I are denoted by $\mathfrak{G}_{\mathfrak{B}}$, where $\mathfrak{G}$ is a graph family. Notice that if we let the Banach space to be $\mathfrak{B} \times \mathfrak{B}$ then the Element II is only a special case of Element I with complex vector $\mathbf{v} + i\mathbf{u}$, $i^2 = -1$. However, it reflects living bodies with respective real and imaginary parts $L(v,u)$, $-L(v,u)$ appearing in pair with the property that $\mathbf{v} + \mathbf{u} = \mathbf{0}$, which can be applied to characterize livings ([21]-[24]).

Are Elements I and II really mathematical elements with operations $+$, $\cdot$ and furthermore, differential and integral as the usual? The answer is Yes in case because they can be viewed as vectors underlying a topological graph. For example, we define ([25])

$$\begin{aligned}
\overrightarrow{G}^L + \overrightarrow{G'}^{L'} &= \left(\overrightarrow{G} \setminus \overrightarrow{G'}\right)^L \bigcup \left(\overrightarrow{G} \bigcap \overrightarrow{G'}\right)^{L+L'} \bigcup \left(\overrightarrow{G'} \setminus \overrightarrow{G}\right)^{L'}, \\
\overrightarrow{G}^L \cdot \overrightarrow{G'}^{L'} &= \left(\overrightarrow{G} \setminus \overrightarrow{G'}\right)^L \bigcup \left(\overrightarrow{G} \bigcap \overrightarrow{G'}\right)^{L \cdot L'} \bigcup \left(\overrightarrow{G'} \setminus \overrightarrow{G}\right)^{L'}
\end{aligned}$$

and

$$\frac{df}{dt} = \lim_{\Delta t \to 0} \frac{f\left(\overrightarrow{G'}^{L'}[t + \Delta t]\right) - f\left(\overrightarrow{G}^L[t]\right)}{\overrightarrow{G'}^{L'}[t + \Delta t] - \overrightarrow{G}^L[t]} = G^{\lim\limits_{\Delta t \to 0} \frac{f(L'[t+\Delta t]) - f(L[t])}{L'[t+\Delta t] - L[t]}},$$

where f is G-isomorphic operator which is an isomorphism of graph on $\mathfrak{G}_{\mathfrak{B}}$ hold with $L_2 = f \circ \varphi \circ L_1$ for $\forall (v,u) \in E\left(\overrightarrow{G}_1\right)$, $\overrightarrow{G}^L[t] \in \mathfrak{G}_{\mathfrak{B}}$ dependent on variable t and the image $f\left(\overrightarrow{G}'^{L'}[t+\Delta t]\right) \to f\left(\overrightarrow{G}^L[t]\right)$ as $\Delta t \to 0$.

Similarly, we define partial differentials

$$\frac{\partial f\left(\overrightarrow{G}[t]\right)}{\partial t} = \overrightarrow{G}^{\frac{\partial f(L)}{\partial t}} \quad \text{and} \quad \frac{\partial f\left(\overrightarrow{G}[t]\right)}{\partial x^i} = \overrightarrow{G}^{\frac{\partial f(L)}{\partial x^i}}$$

and also tensors $V_{ij\cdots}$ generated by $\frac{\partial}{\partial x^i}$, dx^i for integers $1 \leq i, j \cdots \leq n$. Thus, the initial problem

$$\left\{\begin{array}{l} \dfrac{\partial^2 \overrightarrow{G}^L}{\partial t^2} = \overrightarrow{G}^{\frac{\partial^2 L}{\partial t}} = \overrightarrow{G}_0^{L_0} \\ \overrightarrow{G}^L\Big|_{t=t_0} = \overrightarrow{G}_1^{L_1}, \quad \dfrac{\partial \overrightarrow{G}^L}{\partial t}\Bigg|_{t=t_0} = \overrightarrow{G}_2^{L_2} \end{array}\right. \tag{3.1}$$

on $\mathfrak{G}_{\mathfrak{B}}$ with solution $\overrightarrow{G}^L$ is valuable in mathematics.

3.3. Differential Equation on Multi-Fields A multi-field $\widetilde{S}$ is a union of distinct fields S_i constraint with solvable differential equation (Eq_i) for integers $i \geq 1$, i.e., a *Smarandache multi-space* [28] on non-harmonious systems in the universe. Then, *what is the differential equations on a multi-field $\widetilde{S}$ and are there G^L solutions?* Clearly, a multi-field $\widetilde{S}$ is characterized by a non-solvable system of second order differential equations

$$1 \leq i \leq m \quad \left\{\begin{array}{l} \mathfrak{F}^i\left(t, x, F_t^i, F_x^i, \cdots\right) = 0 \\ F^i(x)|_{t=t_0} = F_0^i, \ F_x^i|_{t=t_0} = F_1^i \end{array}\right. \tag{3.2}$$

and the solution manifold of the ith equation is $S_i(t;x)$. Similar to the subsection 3.1, we construct a continuity flow $\overrightarrow{G}^L[t]$ with bi-direction on each edge by

$$\begin{aligned} V\left(\overrightarrow{G}^L[t]\right) &= \{S_i(t;x)|\ 1 \leq i \leq m\}, \\ E\left(\overrightarrow{G}^L[t]\right) &= \left\{(S_i(t;x), S_j(t;x))\ |\ S_i(t;x) \bigcap S_j(t;x) \neq \emptyset, 1 \leq i, j \leq m\right\} \end{aligned}$$

with a labeling

$$\begin{aligned} &L: \ S_i(t;x) \in V\left(\overrightarrow{G}[t]\right) \to \{S_i(t;x); 1 \leq i \leq m\}, \\ &L: \ (S_i(t;x), S_j(t,x)) \in E\left(\overrightarrow{G}[t]\right) \to \{S_i(t;x) \bigcap S_j(t;x); 1 \leq i, j \leq m\}. \end{aligned}$$

Then, $\overrightarrow{G}^L[t]$ is the $\overrightarrow{G}^L$ solution of the system (3.1) with initial conditions

$$\overrightarrow{G}^L\Big|_{t=t_0} = \overrightarrow{G}^{L_{F_0}} \quad \text{and} \quad \frac{\partial \overrightarrow{G}^L}{\partial x^i}\Bigg|_{t=t_0} = \overrightarrow{G}^{L_{F_1}},$$

where

$$L_{F_0}: \ S_i(t;x) \in V\left(\overrightarrow{G}^L[t]\right) \to F_0^i, \quad L_{F_1}: \ S_i(t;x) \in V\left(\overrightarrow{G}^L[t]\right) \to F_1^i,$$

$$L_{F_0}: \ (S_i(t;x), S_j(t;x)) \in E\left(\overrightarrow{G}^L[t]\right) \to F_0^i \bigcap F_0^j,$$

$$L_{F_1}: \ (S_i(t;x), S_j(t;x)) \in E\left(\overrightarrow{G}^L[t]\right) \to F_0^i \bigcap F_1^j.$$

For example, it is known that the Einstein's gravitational equation $R_{ij} - \frac{1}{2}g_{ij}R = \kappa T_{ij}$ is established by an assumption that curvature tensors are sufficiently smooth. However, *if it is not true, what will happens?* In this case, we can not describe the gravitational field $\mathcal{M}$ only by one tensor equation $R_{ij} - \frac{1}{2}g_{ij}R = \kappa T_{ij}$. We should decompose $\mathcal{M}$ into m subfields $M_1, M_2, \cdots, M_m$ whose curvature tensor is sufficiently smooth ([14]). Then, the Einstein's gravitational equation can be applied on each subfield M_i, $1 \le i \le m$, i.e., the gravitational field $\mathcal{M}$ should be described by a system of tensor equations

$$\left\{\begin{array}{lcl} R^1_{i_1j_1} - \frac{1}{2}g_{i_1j_1}R^1 & = & \kappa^1 T^1_{i_1j_1} \\ R^2_{i_2j_2} - \frac{1}{2}g_{i_2j_2}R^2 & = & \kappa^2 T^1_{i_2j_2} \\ \cdots\cdots\cdots\cdots\cdots & \cdots & \cdots\cdots \\ R^m_{i_mj_m} - \frac{1}{2}g_{i_mj_m}R^m & = & \kappa^m T^m_{i_mj_m} \end{array}\right. \tag{3.3}$$

where $R^k_{i_kj_k}$, R^k, $g_{i_kj_k}$, $T^k_{i_kj_k}$ and κ^k denote respectively the tensors R_{ij}, R, g_{ij}, T_{ij} and the constant κ in the kth smooth manifold M_k, $1 \le k \le m$. Then, *what is $\mathcal{M}$ looks like in geometry?* Certainly, it should be the combination of the smooth manifolds M_k, $1 \le k \le m$, i.e., $\widetilde{M} = \bigcup\limits_{i=1}^{m} M_i$ underlying a labeled graph G^L with

$$\begin{aligned} V\left(G^L\right) &= \{M_i |\ 1 \le i \le m\}, \\ E\left(G^L\right) &= \left\{(M_i, M_j) |\ M_i \bigcap M_j \ne \emptyset, 1 \le i, j \le m\right\} \end{aligned}$$

with

$$L: M_i \in V\left(G^L\right) \to M_i, \quad L: (M_i, M_j) \in E\left(G^L\right) \to M_i \bigcap M_j,$$

which is nothing else but a combinatorial manifold with Riemannian metric

$$ds^2_{\mathcal{M}} = ds^2_{M_1} + ds^2_{M_2} + \cdots + ds^2_{M_m}, \tag{3.4}$$

where ds_{M_i} is the Riemannian metric of the smooth manifold M_i ([7]-[10]). Thus, the Einstein's gravitational field is a projection of $\mathcal{M}$ on M_k for an integer k, $1 \le k \le m$, only a local or partial holding on the gravitational field of the universe.

4. Einstein's General Relativity for Reality

Holding on the reality of things advances one naturally to speculate a philosophical questions on the reality with mathematics, i.e., what is the reality implied by a mathematical form on the behavior of things or what is an appropriate form on the reality of things, particularly, the Einstein's general relativity with the differential equations of physical laws.

4.1. Philosophy of Einstein's General Relativity Notice the relative relationship in humans with the objective things determines that there two kind of observing systems: the system S_{out} in which the observing things do not depend on human's will and the system S_{in} in which humans are included in the observing things or in other words, the human's behavior is acting on the observing things with influence on the observing things. As is known to all, human is independent of objective things in the macroscopic world, whose will can not affects or changes the objective existence. However, an observer with an observed microscopic particle form a mutual interaction system, which results in the observation can not accurately determine a microscopic particle ([26]). Certainly, this is because of the limitation of humans, namely we cannot simultaneously determine the speed and position of a microscopic particle which results in that one only describes a microscopic particle by filed, i.e., the possibility of its location in space.

A coordinate system $\{O; x^1, x^2, \cdots, x^n\}$ is a reference frame for humans to quantitatively characterize the behavior of objective things, and it is also the basis for establishing equations of physical laws. Clearly, Einstein's general relativity asserts that physical equations should have the same mathematical form in all reference frames, which concludes that humans and the objective things consist of a system S_{out}, namely, objective things are independent of humans and then, follows the law that objective things do not change on human's will and their behavior should be expressed by mathematical invariants, i.e., not change depending on the coordinates one sets up, which is indeed true in the macroscopic world by human's observation, for instance the object's motion and the gravitational field. That is why Einstein applied tensor equations for describing the gravitational law. However, *is Einstein's notion also true in the case of microscopic particles?*

Usually, one characterizes a microscopic particle P by a field, which is a space $\mathbb{R}^4$ associated with the probability of P appearing at location $(t, x^1, x^2, x^3) \in \mathbb{R}^4$. It should be noted that characterizing the microscopic particles by fields is actually to apply the same notion that used in the macroscopic cases, i.e., observing things do not depend on human's will. Whence, applying the Einstein's general relativity in the microscopic cases should be with the assumption applied in macroscopic particles because only in this way the physical laws of microscopic particles can be described by tensor equations, i.e., invariant under the transformation of the coordinate system $\{O; t, x^1, x^2, x^3\}$. But, *is the behavior of a microscopic particle really so?* We are not certain on this analogical pattern because their behavior is very different from that of macroscopic particles.

4.2. Mathematical Form with Reality Notice that the universe is not well-distributed in matter. Thus, the non-solvable differential equations (3.2) is a general form that of equation (1.1) on things in the universe, particularly, the fields. Clearly, the equation (1.1) is a projection of the system (3.2) of equations on an integer k, $1 \leq k \leq m$, maybe invariant or not under transformations of the coordinate system $\{O; x^1, x^2, \cdots, x^n\}$. Then, *what is the essential difference in the two equations?* Certainly, the equation (1.1) is a special case of equation (3.2) in $m = 1$. However, if we generalize the conception of solution of equations to continuity flows, i.e., elements I or II, they have no difference because both of them are solvable equations describing physical laws. Clearly, to apply non-solvable system (3.2) of solvable differential equations (1.1) to physical laws is a great leap for understanding things of the universe because of the universal contradiction in the eyes of humans.

Then, *what is the difference of differential equation with its tensor equations on reality of things in the universe?* For example, *what is the difference between equations* (1.2) *and* (2.12) *on electromagnetic field?* Certainly, the equation (2.12) is a tensor equation on electromagnetic fields with form invariant under the transformation of the coordinate system $\{O; t, x^1, x^2, x^3\}$ but the equation (1.2) is not. However, there are no essential difference because both of them characterize electromagnetic fields, and the equation (1.2) is more useful in practise. We have to transfer the form of the tensor equation (2.12) to (1.2) for examining or experimental in an electromagnetic field.

Certainly, the tensor equation is more concise in the expression form, more like a criterion or a philosophical notion on the form but there are no essential difference on reality of things. Thus, the expression of a physical law in tensor equations or usual differential equations is insubstantial because both of them are equally important for understanding things in the universe.

4.3. A Review on Einstein's General Relativity Einstein's general relativity is actually the mathematical realization of one philosophical words, i.e., the objective things do not depend on humans will in the universe. Indeed, he concluded that the equation describing a physical law should be invariant under all transformations of the coordinate systems in his general relativity, which brought about him to find the tensor equation on gravitational field and more researchers pursued further the mathematical form of physical laws that concluded by Einstein but neglected the physical meaning of a mathematical form, a little deviation from the understanding of things in the universe. Then, *how can we get out of this situation and avoid the limitation of humans for understanding?* We should take two steps at least for this objective.

First, we should recognize the limitations of human's understanding on things in the universe, i.e., all understandings are local or partial, an approximation on the reality of things and then, establish the equation (3.1) or (3.2) for physical laws, i.e., the understandings of humans is a projection of the reality of things.

Principle 4.1(Projection) *All known characters of thing T is the projection of that T on an assumed well-distributed universe of humans.*

Second, even if the universe is well-distributed in matter, *what should we know in Einstein's general relativity? Is it only concluded the tensor form of physical laws?* Of course not! If we understand the philosophical implication of Einstein's general relativity, we should realize that the tensor equations are only the expression form for physical laws, which concludes the unimportance of coordinate systems in understanding things of the universe. However, if we consider the expression form of physical laws in an unimportant point, we could view the Einstein's general relativity in a new way on coordinate systems. Thus, the central thing is not throw away but the equal importance on coordinate systems in Einstein's general relativity, i.e., the equal-right principle for understanding things of the universe.

Principle 4.2(Equal-Rights) *All coordinate systems are equally important for understanding things in the universe.*

By Principle 4.2, *how could we understand things in the universe by differential equations?* Certainly, we can establish differential equation describing a physical law in any coordinate system because each of them characterizes the same physical law, no matter what its mathematical form. However, we should choose such a coordinate system $\{O; t, x^1, x^2, x^3\}$ that holds with a brevity and easily verified mathematical form of the physical laws for characterizing and then, rewrite the covariant equation by tensors if we hope so. Thus, the most important thing for understanding a thing in the universe is not throw away but establish an appropriated coordinate system, a new view on the Einstein's general relativity.

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