Research article

Determining damage effects on asphalt concrete composites

Mohammad I. Hossain¹, Rafigul A. Tarefder²

1- Ph.D. Candidate and Graduate Research Assistant, Department of Civil Engineering, University of New Mexico, Albuquerque, New Mexico, USA.

2- Associate Professor and Regents' Lecturer, Department of Civil Engineering, University

of New Mexico, Albuquerque, New Mexico, USA.

mhossain@unm.edu

ABSTRACT

Asphalt concrete (AC) is a geological composite material, which is made with different shapes and sizes of coarse and fine aggregates bonded with asphalt binder. Cohesive and adhesive damages are observed into AC. Cohesive damage initiates into the matrix material and adhesive damage initiates at the matrix-aggregate interface. Finite element method (FEM) model is developed using ABAQUS with a coarse aggregate coated with matrix material. Damage models parameters are measured in the laboratory under dry and wet conditions to simulate FEM models. It is observed that cohesive damage occurred due to increase in shear stress carrying capacity and reached to the predefined shear strength limit. Shear stress carrying capacity caused cohesive damage due to matrix material coated on circular shape aggregate. Higher cohesive and adhesive damages are observed in cohesive damaged matrix materials and magnitude of this strain is significantly higher under wet condition than dry condition. Lower contact normal and shear stresses are observed at matrix-aggregate interface under wet condition than dry condition than dry condition. It is believed that lower contact stresses are the driving factors of higher adhesive damage at the matrix-aggregate interface.

Keyword: Asphalt concrete, Damage, Cohesive, Adhesive, Finite element method.

1. Introduction

Asphalt concrete (AC) can be defined as asphalt coated coarse aggregate particles surrounded by matrix materials. Matrix is a mixture of asphalt binder with fine aggregates passing through a #4 (4.75 mm.) sieve and retained on a #200 (0.075 mm.) sieve (Caro et al., 2010, Kim and Little, 2004). Furthermore, matrix can be divided into coarse matrix and fine matrix; coarse matrix is defined as fine aggregate passing through a #4 sieve and retained on a #10 sieve (2.0 mm.) and fine matrix defined as passing through a #10 sieve and retained on a #200 sieve (Tarefder et al., 2010).

Damages in AC can be attributed to two primary mechanisms, namely, the loss of adhesion, and the loss of cohesion due to applied load and/or environmental conditions. Loss of adhesion is caused by breaking of the adhesive bonds between the aggregate surface and the matrix. Loss of cohesion is caused by the softening or breaking of cohesive bonds within the matrix. Figure 1 shows a schematic diagram of adhesive and cohesive damage in AC (Hossain and Tarefder, 2013). In figure 1(a), coarse aggregates are coated by matrix material and without any damage, but in figure 1(b), part of the matrix coatings over aggregate are worn out and inside the matrix material are torn down. The worn out phenomenon in defined as adhesive damage and torn down phenomenon is defined as cohesive damage.



Figure 1(a): Schematic of no loss of bonding in AC.

Figure 1(b): Schematic of loss of bonding in AC.

Moisture gets into AC by diffusion mechanism and accelerates adhesive and cohesive damage in AC. Initially moisture diffuses into matrix materials and then infiltrate through matrix-aggregate interface and reaches to aggregates. It is believed that damage inside the aggregate is very minimum comparing to cohesive and adhesive damages in AC. In this study, the adhesive and cohesive damages under dry and wet conditions are determined using finite element method (FEM) modeling. The model parameters were determined by laboratory testing under dry and wet conditions (Hossain and Tarefder, 2013).

2. Problem statements, objectives and methodology

Damage in AC has been measured by laboratory investigations using indirect tensile tests on cylindrical shape AC samples (AASHTO, 2007). Cylindrical shape AC sample is loaded diametrically up to failure of the sample. The ratio of wet strength to dry strength are determined and reported for moisture sensitivity of AC mix. Laboratory tests provide comparisons between dry and wet AC samples but do not separate and provide no idea regarding basic damage phenomena, which are adhesive and cohesive damage. In addition, damage initiates in small scale such as matrix materials coated on an aggregate particle, and damage propagates through matrix materials and exposed and visible in large scale such as surface of AC pavement. For this reason, damage computation on small scale AC sample is necessary. Moreover, laboratory tests on small scale AC samples are expensive and time consuming. Numerical computation method such as FEM modeling can overcome these limitations.

The objectives of this study are to evaluate coupled cohesive and adhesive damages in AC under dry and wet conditions. Cohesive damage in matrix materials are compared by observing damage contour plots under dry and wet conditions. In addition, damage progression phenomena are determined by plotting stress-strain relationship of damaged dry and wet matrix materials. Adhesive damage at matrix-aggregate interface is measured by computing contact normal stress and contact shear stress under dry and wet conditions.

FEM modeling technique is used to evaluate cohesive and adhesive damages in AC. ABAQUS is used as FEM tool. FEM model consists of an aggregate coated with matrix materials. Maximum stress criteria is used as cohesive damage model for matrix materials; surface based traction-separation criteria is used as adhesive damage model for matrix-aggregate interface. Both damage models are available in ABAQUS. Cohesive and adhesive damage model parameters were determined from laboratory tests under dry and wet conditions. FEM model is simulated under static deformation loading condition considering the traffic wheel pressure on the AC pavements. Results are validated by measuring stiffness and strength of undamaged matrix material under dry and wet conditions in FEM simulation models and comparing with the laboratory test results.

3. Backgrounds of damage models

3.1 Cohesive damage model

Maximum stress criteria damage model is used to define cohesive damage in matrix materials. The model is defined by a monotonically increasing stress-strain up to a critical point followed by a monotonically decreasing softening curve (Lucas et al., 2007). The elastic behavior is defined by an elastic constitutive matrix that relates to the nominal stress and nominal stain in the elements (ABAQUS, 2009). The short form of the stress-strain relationship is show in Eq. (1) and the long form is shown in Eq. (2). The nominal stress vector consists of three stress components, σ_n acting to the pure normal direction, σ_s acting toward the first shear direction and σ_t acting toward second shear direction. The stress vector σ can be express in terms of stiffness *E* and ε_s

$$\{\sigma_i\} = \begin{bmatrix} E_{ij} \end{bmatrix} \{\varepsilon_j\}$$

$$\begin{bmatrix} \sigma_n \end{bmatrix} \begin{bmatrix} E_{nn} & E_{ns} & E_{nt} \end{bmatrix} \begin{bmatrix} \varepsilon_n \end{bmatrix}$$
(1)

$$\begin{cases} \sigma_s \\ \sigma_t \end{cases} = \begin{bmatrix} E_{ns} & E_{ss} & E_{st} \\ E_{nt} & E_{st} & E_{tt} \end{bmatrix} \begin{cases} \varepsilon_s \\ \varepsilon_t \end{cases}$$
(2)

where E_{nn} is the stiffness in the pure normal mode, E_{ss} is the stiffness in the first shear direction and E_{tt} is the stiffness in the second shear direction. Damage is assumed to initiate when the maximum nominal stress ratio reaches a value of one and expressed in maximum stress criteria,

$$max\left\{\frac{\langle \sigma_n \rangle}{\sigma_n^0}, \frac{\sigma_s}{\sigma_s^0}, \frac{\sigma_t}{\sigma_t^0}\right\} = 1$$
(3)

where σ_n^0 is the nominal strength towards normal direction of matrix, σ_s^0 is the nominal shear strength toward first direction and σ_t^0 is the nominal shear strength toward the second direction measured in the laboratory. The symbol () is known as Macaulay bracket, which

signify that a pure compressive stress state does not initiate damage.

3.2 Adhesive damage model

Similar to cohesive damage model, adhesive damage model can be presented in terms of load and displacement, since adhesive damage occurs at the interface, which is contact between surfaces of matrix and aggregate in this study. Adhesive damage model is presented in terms of load-displacement relationship,

$$\begin{cases} t_i \\ t_2 \\ t_3 \end{cases} = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$
(4) (5)

where t_1 , t_2 and t_3 are the three components of force on a surface in three orthogonal directions, *K*'s are the stiffness coefficients and δ_1 , δ_2 and δ_3 are three deformation components due to the respective forces. The ratio between the interface strength and load is measured along the normal to the surface and tangential direction (i.e. shear direction) of the surface. Traction-separation criteria is shown as,

$$max\left\{\frac{\langle t_1 \rangle}{t_1^0}, \frac{t_2}{t_2^0}, \frac{t_3}{t_3^0}\right\} = 1$$
(6)

where t_1^0 , t_2^0 and t_3^0 are the interface strength for normal and shear directions. In this study, only compressive strength and shear strength to the first direction of the matrix materials and tensile strength and shear strength to the first direction for the interface were measured in the laboratory.

4. Laboratory investigations and FEM model development

The following tests on the dry and wet matrix materials and the matrix-aggregate interfaces were done by controlling deformation with a rate of 1.27 mm/min (0.5 in/min). Three dry and three wet samples were prepared for each test. For wet conditioning, before testing, the samples were soaked for 48-hours under water at room temperature and subjected to a vacuum pressure of 30 mm. Hg for half an hour. Stiffness E- and K-values were determined by measuring the secant modulus, which is the slope of line connecting zero state to 50% of maximum stress and maximum load respectively (Santi et al., 2000).

4.1 Test on matrix materials

Asphalt mix was collected from a local plant. The loose mix was separated by sieving. Loose mix passing through #16 sieve (1.19 mm.) and retained on #200 sieve was collected as matrix material. Cylindrical samples of height 69.85 mm (2.75 in) and 35.31 mm (1.39 in) diameter were compacted to a target void ratio of $4.0 \pm 0.5\%$. Average of three samples' results from compression and shear tests are summarized in Table 1.

	Test type	Ultimate strength	E-value
Dry	Compression	2.61 Mpa (379 psi)	192.72 Mpa (27,952 psi)
	Shear	0.81 Mpa (118 psi)	147.64 Mpa (21,413 psi)
Wet	Compression	2.02 Mpa (293 psi)	129.44 Mpa (18,773 psi)
	Shear	0.56 Mpa (81 psi)	139.10 Mpa (20,174 psi)

Table 1: Test results of matrix materials under dry and wet conditions*.

* (Hossain and Tarefder, 2013)

4.2 Test on matrix-aggregate interface

Laboratory aggregate pull-off tests under both dry and wet conditions were done to measure the stiffness of matrix-aggregate interfaces. For tensile pull-off test, a coated aggregate was cut into half and the flat face was exposed and the other end was embedded into matrix up to the half of the aggregate. The wet and dry matrix samples were compacted to a target void ratio of $4 \pm 0.5\%$ for both tension and shear tests. The flat end is fixed with the loading frame with glue and the bottom of the matrix material container was also fixed with the base. Shear pull-off tests were done with the shear box by emerging aggregate into matrix materials and pull the aggregate horizontally by shear force. Average of three samples' results from tension and shear tests are summarized in Table 2.

	Test type	Ultimate strength	K-value
Dry	Tension	391.44 N (88 lbf)	3,706.42 N/mm (21,163 lbf/in)
	Shear	280.24 N (63 lbf)	3,150.00 N/mm (17,987 lbf/in)
Wet	Tension	244.65 N (55 lbf)	2,858.24 N/mm (16,321 lbf/in)
	Shear	124.55 N (28 lbf)	1,912.39 N/mm (10,920 lbf/in)

Table 2: Test results of matrix-aggregate interfaces under dry and wet conditions*.

* (Hossain and Tarefder, 2013)

4.3 FEM model development

The FEM model is developed using ABAQUS/CAE 6.9-EF1, commercially available software. A two-dimensional idealization of a circular aggregate with radius of 19.05 mm (0.75 in) and coated with matrix materials with thickness of 0.508 mm (0.02 in) is considered as shown in figure 2 (Hossain and Tarefder, 2013). Though AC is considered to be visco-elastic-plastic material, matrix is assumed to behave elastically since the matrix thickness in this study is very small compared to the diameter of coarse aggregate. Also, static load is applied on the FEM model; visco-elastic model is preferable if cyclic load is applied on the model. In addition, AC behaves elastically under low temperature and visco-elastically under high temperature (Zhu et al., 2010). No temperature variations are considered for simulating the model.

The loading and the shape of the FEM model are symmetrical to the vertical axis. Hinge boundary condition (BC) is used at the bottom and roller BC is used at the left side of the model. Four noded linear quadrilateral cohesive elements are used to define the matrix materials. Linear elements are used since quadratic elements are not available in ABAQUS for assigning cohesive elements. Three and four noded linear quadrilateral plane stress elements are used to define the aggregate. Combinations of both three and four noded elements are required due to the circular shape of the aggregate.

In the FEM model, instead of applying a load, a specified deformation is applied. Deformation magnitudes 1.45 mm (0.057 in) is applied on the FEM model. The magnitude of the deformation is calculated based on a standard duel tandem wheel on a pavement. It has been observed that a dual tandem wheel of total 889.64 KN (200,000 lb) load produces a 1.45 mm (0.057 in) deformation in a 203.02 mm (8.0 in) thick AC pavement (Huang, 2004). Therefore 1.45 mm (0.057 in.) deformation is considered.

The 1.45 mm (0.057 in) deformation is applied statically with a time step of 0.01 sec. the maximum time step is considered as 1.0E-5. It is observed that lower time step provide good results. The deformation load is applied in ABAQUS on 10.16 mm (0.4 in) length of matrix. Usually, indirect tensile strength of AC is determined by subjecting diametrically though a 20.32 mm-25.4 mm (0.8-1.0 in) loading strip. Since the model is symmetric, deformation load is applied over 10.16 mm (0.4 in) length.

Determining damage effects on asphalt concrete composites Mohammad I. Hossain, Rafiqul A. Tarefder



Figure 2 (a): Schematic of aggregate coated with matrix materials.



5. Results, discussions and conclusions

Damage inside the matrix materials is considered as cohesive damage and damage at the matrix-aggregate interface is considered as adhesive damage in the following sections.

5.1 Cohesive damage in matrix materials

Cohesive damage is expressed in maximum stress criteria (MAXSCRT) value and plotted in figure 3 under dry and wet conditions. A zoomed in section is presented for each figure to visualize the damage contour more clearly. Damage mostly observed under the deformation zone and for this reason the zoomed in section is only presented for the loaded region. Cohesive damage occurred in matrix materials when MAXSCRT value is 1.0. MAXSCRT value is a unit less value since it represents ratio of two stresses. MAXSCRT contour plot ranges from blue (gray in grayscale) to red (black in grayscale). It should be noted that the blue color shows zero value but it is a very small value and showing zero because the results are rounded to two decimal digits. Figure 3(a) shows the MAXSCRT values under dry condition and figure 3(b) shows the value under wet condition.

According to figures 3(a) and 3(b), cohesive damage is higher at the end of the loading zone and near 10.16 mm (0.40 in) measuring from top left corner on the matrix perimeter. Wet matrix shows higher cohesive damage than dry matrix, because both normal and shear stress carrying capacity are lower for wet matrix than dry matrix. Damaged matrix at the top surface cannot carry any stresses and the stress carrying capacity of the underlying matrix materials is increases. Due to circular shape of the model and vertical applied deformation on the certain part of the matrix, the matrix near top of the model has higher normal stresses and lower shear stresses and the matrix near higher curvature has lower normal stresses and higher shear stresses. Cohesive damage occurred when either normal or shear stress carrying capacity increases and crossed the predefined value as shown in Table 1. It is observed that, for this particular model and loading criteria, cohesive damage occurred at the location where the curvature is high since shear stress carrying capacity exceeds under both dry and wet conditions.

The matrix section, which expose to cohesive damage under the deformation zone, is divided into three regions under dry and wet conditions and numbered as 1, 2 and 3 as shown in figures 3(a) and (b). Region 1 shows matrix materials with no cohesive damage; region 2 shows cohesive damage under wet condition but no cohesive damage under dry condition;

region 3 shows cohesive damage under both dry and wet condition. More details of these regions are presented in the following sections.







Figure 3(b): Maximum stress criteria (MAXSCRT) for matrix materials under wet conditions.

5.2 Stress-strain relationships of undamaged matrix materials

Undamaged and cohesive damaged locations are separated in three regions and shown in figures 3(a) and 3(b). The stress-strain relationship for a selected element at the undamaged location (i.e. region 1 as per figure 3) is shown in figure 4. Figure 4(a) is plotted for the normal stress and normal strain and figure 4(b) is plotted for the shear stress and shear strain for a selected element. All diagram shows linear relationship as expected. Though, wet condition shows lower stiffness value than dry condition. According to figure 4(a), the maximum normal stress under dry condition is 1.7013 MPa and the corresponding normal strain is 8.83E-3 mm/mm; the maximum normal stress under wet condition is 1.4488 MPa and the corresponding normal strain is 1.12E-2 mm/mm. The calculated E-values from these plotted stress-strain relationship is 192.67 MPa and 129.36 MPa under dry and wet conditions respectively. Similarly, according to figure 4(b), the maximum shear stress under dry condition is 0.06079 MPa and the corresponding shear strain is 4.12E-4 mm/mm; the maximum shear stress under wet condition is 0.06143 MPa and the corresponding shear strain is 4.42E-4 mm/mm. The calculated E-values from these plotted stress-strain relationship is 147.55 MPa and 138.98 MPa under dry and wet conditions respectively. The calculated E-values closely matched with the E-values measured in the laboratory and given in Table 1.

5.3 Stress-strain relationship of cohesive damaged matrix materials

Figure 5 shows both undamaged and damaged stress-strain relationship of matrix materials. Region 2 is the undamaged under dry condition but damaged under wet condition and damage is observed for shear only. For this reason only shear stress-strain relationship is presented in figure 5. Figure 5(a) is an element located near the upper portion of region 2 and closer to region 1; figure 5(b) is an element located near around middle of region 2 and figure 5(c) is an element located near lower portion of region 2 and closer to region 3. The stress-strain relationship is linear under dry condition since no damage is occurred under dry condition at region 2. The maximum stresses under dry condition are 0.7540 MPa in figure 5(a), 0.7748 MPa in figure 5(b), and 0.7956 in figure 5(c); all the maximum stress values are smaller than the maximum shear strength of 0.81 MPa under dry condition. For this reason no damage is observed under dry condition for region 2.



Figure 4(a): Normal stress-strain relationship of undamaged matrix materials.



On the other hand, wet condition shows damage after shear stress reaches to 0.5574 MPa in figure 5(a), 0.5570 MPa in figure 5(b), and 0.5578 MPa in figure 5(c). According to Table 2, 0.56 MPa is the maximum shear strength of matrix under wet condition. Softening part of the stress strain returns to zero as damage progresses after the peak stress under wet condition. For figure 5(c), at the end of softening curve there is a remaining shear strain, which represents permanent shear strain in the material. This shear strain magnitude is small for figure 5(b) and not present for figure 5(a). The significance of this remaining shear strain is, lower portion of region 2 has higher damages than upper portion, also it can be said that permanent shear strain increases when distance on the perimeter increases from the left BC. Also at the end of the applied deformation zone the permanent shear strain for damaged matrix materials and the amount of permanent shear strain is higher under wet condition than that of dry condition. The stress-strain relationship for damaged dry matrix is explained in the following section.

Figure 6 presents the stress-strain relationship of cohesive damaged section under both dry and wet conditions. Shear stress and strain are presented since damage is occurred for shear stress only. Figure 6(a) is plotted for an element located near the upper region of 3 and near region 2 and figure 6(b) is plotted for an element located near the lower region of 3. Both dry and wet condition shows damages in region 3 but wet condition shows higher damage since permanent shear strain is higher under wet condition than dry condition. Maximum shear stresses under dry condition are 0.8099 MPa in figure 6(a), 0.8130 MPa in figure 6(b). The maximum shear strength under dry condition is 0.81 MPa as mentioned in the laboratory and given in Table 1. It should be noted that the tail of softening curve under wet condition in figures 6(a) and 6(b) is extending comparing to 5(b) and 5(c), means more permanent shear strain in this region under wet condition.



Figure 5(a): Stress-strain relationship at near upper region of 2 of undamaged and cohesive damaged matrix materials.



Figure 5(c): Stress-strain relationship at near lower region of 2 of undamaged and cohesive damaged matrix materials.



Figure 6(a): Stress-strain relationship at near upper region of 3 of damaged matrix materials.







Figure 6(b): Stress-strain relationship at near lower region of 3 of damaged matrix materials.

5.4 Adhesive damage in matrix-aggregate interface

Adhesive damage is expressed in cohesive surface maximum traction criteria (CSMAXSCR) and presented in figure 7 under dry and wet conditions. Adhesive damage occurred in matrix-aggregate interface when CSMAXSCR value is 1.0. CSMAXSCR contour plot ranges from blue (gray in grayscale) to red (black in grayscale). Figure 7(a) shows the FEM model with

the CSMAXSCR values under dry condition with a zoomed in section of the damaged location. Figure 7(b) shows the zoomed in section of the damaged region under wet condition.

The top of the interface is red for both dry and wet matrix, means adhesive damage occurred under deformation. But adhesive damage is higher under wet condition than dry condition. Interface cannot carry any load when it damaged and the load carrying capacity increases along the interface perimeter. Wet interface shows higher adhesive damage than dry interface, since both normal and shear traction force carrying capacities are lower for wet interface than dry interface.

The interface status can also be identified by determining contact normal stress and contact shear stress. Figure 8 shows the maximum contact stresses for the model under dry and wet conditions. The contact stresses are measured from the contact force over the contact area. ABAQUS provide total contact force for the whole domain. It is time consuming but doable to extract contact forces for any particular contact area and evaluate the results and compare or validate the results under dry and wet conditions. For this reason, the validation is not performed for this adhesive damage scenario.





Figure 7(a): Cohesive surface maximum traction criteria (CSMAXSCR) for matrix-aggregate interface under dry conditions.

Figure 7(b): Cohesive surface maximum traction criteria (CSMAXSCR) for matrix-aggregate interface under wet conditions.

According to figure 8, dry condition has higher contact stresses than wet condition. Lower contact stresses causes higher adhesive damage under wet condition than dry condition. The contact normal stress under dry condition is 3.923 MPa and contact shear stress is 0.374 MPa; on the other hand, the contact normal stress under dry condition is 2.621 MPa and contact shear stress is 0.163 MPa.

Determining damage effects on asphalt concrete composites Mohammad I. Hossain, Rafiqul A. Tarefder



Figure 8: Matrix-aggregate interface contact status under dry and wet conditions.

5.6 Conclusions

FEM model is developed considering one circular shape aggregate coated with matrix material. Cohesive damage in the dry and wet matrix materials and adhesive damage at the dry and wet matrix-aggregate interfaces are evaluated in this study. The FEM model results are summarized below,

- 1. Cohesive damage is higher under wet condition than dry condition, since both stiffness and strength of matrix materials are lower under wet condition than that of dry condition.
- 2. For this FEM model, matrix materials coated on circular shape aggregate, cohesive damage initiates and progresses due to increase in shear stress capacity and reached to the predefined shear strength under both dry and wet conditions.
- 3. Irrecoverable shear strain grows in the damaged matrix materials and the magnitude of this strain is higher under wet condition than dry condition.
- 4. Adhesive damage is higher under wet condition than dry condition since interface stiffness and strength are lower under wet condition than dry condition.
- 5. Contact normal stress and contact shear stress are significantly lower under wet condition than dry condition. Presence of moisture causes lower contact stresses in AC.

Acknowledgement

The project was funded by the National Science Foundation (NSF) through the prestigious CAREER award (NSF Grant No. 0644047).

6. References

- 1. AASHTO Standard T283, (2007), Resistance of Compacted Hot Mix Asphalt (HMA) to moisture induced damage. Standard Specifications for Transportation Materials and Methods of Sampling and Testing, 27th Edition, Capitol Street, WA, USA.
- 2. ABAQUS/CAE User's Manual, (2009), Dassault Systems, Version 6.9-EF1.

- 3. Caro, S., Masad, E., Bhasin, A., and Little, D., (2010), Micromechanical Modeling of the Influence of Material Properties on Moisture-Induced Damage in Asphalt Mixtures, Construction and Building Materials, 24, pp 1184–1192.
- 4. Hossain, M. I., and Tarefder, R. A., (2013), Effects of Moisture in Asphalt Concrete, Basic Research Journal of Engineering Innovation, 1(1), pp 16-25.
- 5. Huang, Y. H., (2004), Pavement Analysis and Design, 2nd Ed., Prentice Hall, 45-93.
- Lucas, L. J., Black, T. M., and Jones, D. P., (2007), Use of Cohesive Elements in Fatigue Analysis, ASME Pressure Vessels and Piping Division Conference, San Antonio, Texas, pp 13–25.
- Santi, P. M., Holschen, J. E., and Stephenson, R. W., (2000), Improving Elastic Modulus Measurements for Rock Based on Geology, Environmental and Engineering Geoscience, 6(4), pp 333–346.
- 8. Tarefder, R. A., Yousefi, S. S., and Kias, E., (2010), Factors that Affect the Rheological Properties of Asphalt Matrix, Pavement and Materials: Testing and Modeling in Multiple Length Scales, ASCE, Special Pu(1), pp 121–133.
- 9. Kim, Y., and Little, D. N., (2004), Linear Viscoelastic Analysis of Asphalt Mastics, Journal of Materials in Civil Engineering, ASCE, 16(4), pp 122-132.
- Zhu, X., Huang, Z., Yuan Z., and Chen, W., (2010), Micromechanics-based Analysis for Predicting Asphalt Concrete Mudulus, Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering), 11(6), pp 415-424.