

Research article

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## Strengthening of plain and reinforced concrete plates using fiber reinforced polymer- computational approach

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### ABSTRACT

Formation of cracks in concrete is becoming a major threat in reducing the service life of a structure. Apart from the design standards for limit state of collapse it is also essential to analyze a structure for crack initiation and propagation. Modulus of Elasticity and Poisson's ratio of a material also play a crucial role in the release of energy during crack propagation. So, it is essential to analyze the structure considering the fracture behavior during crack propagation. The present work deals with the strengthening of plain and reinforced concrete plate subjected to fracture using Fiber Reinforced Polymer. A three-dimensional finite element analysis is carried out using ANSYS for obtaining the strain energy release rates for different types of concrete based on their Modulus of Elasticity and Poisson's ratio. Results obtained were in good agreement in arresting the crack growth when fibers are oriented at an angle of  $45^{\circ}$  in the plate.

**Keywords:** Laminated plates, Strain energy release rate, Finite element analysis, Orthotropic

### 1. Introduction

Fracture mechanics is a branch of solid mechanics which deals with the behavior of the material and conditions in the vicinity of a crack and at the crack tip. While the concept of linear elastic fracture mechanics has been well developed for more than past 40 years and successfully applied to metallic structures, several civil engineering materials such as cementitious materials, rocks, and fiber-reinforced composites commonly known as *quasibrittle* need a different fracture mechanics approach to model the fracture process. Cementitious materials can be modeled at various scales like the nano-, micro-, meso-, and macro levels. At mesolevel, they can be considered as a two-phase particulate composite, i.e., the matrix and the reinforcement. In the cement pastes, mortar, and concrete, the matrices can be considered as the hydrated cement gels, cement paste, and mortar, respectively, whereas the reinforcements in the corresponding materials can be taken as un-hydrated cement particles, fine aggregates, and coarse aggregates. Concrete is made up of many ingredients such as cement, fine aggregates, coarse aggregates, water, and admixtures in complex arrangements

Pagano (1969) studied about the limitations of classical laminated plate theory comparing with the solutions of several specific boundary value problems to the corresponding theory of elasticity solutions. He concluded that conventional plate theory leads to a very poor description of laminate response at low span to depth ratio but converges to the exact solution as this ratio increases. This is also valid in the study of sandwich plate under cylindrical bending for unidirectional laminate. (Pagano, 1970) extended his study of cylindrical bending to a bidirectional laminate i.e. cross ply laminate (0, 90), which further extended to the behavior of a rectangular plate pinned on all four edges. He (Pagano, 1970) further refined

his study on cylindrical bending of plates considering the influence of shear coupling. He formulated three-dimensional elasticity solution for the static bending of composite laminates in which the axes of elastic symmetry of the various layers are parallel to the plate axes. He also considered that the state of plane strain cannot exist under the general conditions of material symmetry which was already discussed by (Lehknitskii, 1963). (Kerr, 1968) proposed Extended Kantorovich Method (EKM) for 2D elasticity problems which further extended to 3D elasticity problem for a transversely loaded angle-ply laminate in cylindrical bending. (Lo *et al*, 1977), studied the deformation of homogeneous and laminated plates using higher order theory of plates. (Reddy J N, 1984) simplified the complexity of the study of plates by (Lo *et al*, 1977).

Kant and Swaminathan (2001), in their paper concluded that the higher order refined theory gives better results over solutions obtained by first order and higher order theories considered.

Xiao-ping Shu and Kostas Soldatos (2000) worked on Cylindrical bending of laminated plates subjected to different sets of edge boundary conditions and evaluated detailed stress distribution.

Ren (1987) using Fourier series analyzed the deformation of simply supported anti-symmetrical angle-ply laminated plate under transverse loading and results are compared with Classical laminated plate theory and Mindlin theory.

Brett Arnold Pauer (1993) in his MS thesis developed a finite element program for the analysis of laminated composite plates using first order shear deformation theory and studied the elastic behavior of plate and small deflections due to the application of the loads.

Kam Chee Zhou (2011) from University of Malaysia submitted his thesis on Finite element formulation for composite laminates. This study investigated the effect of perfect and imperfect bonding on mechanical behavior of two layered cross-ply composite laminate.

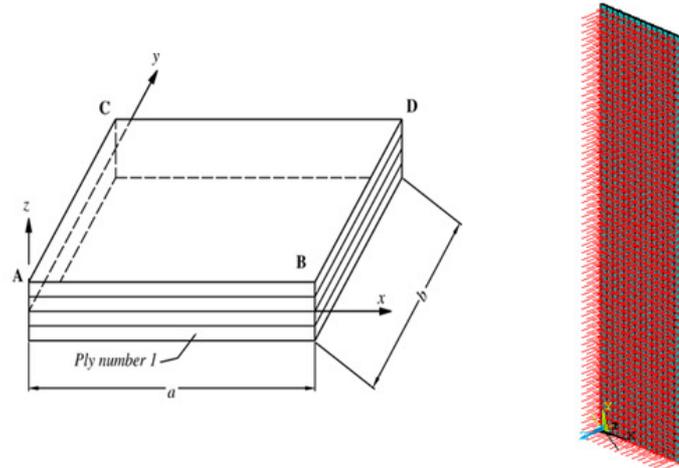
Very recently, using EKM (Santosh Kapuria and Poonam Kumari, 2011) obtained a solution of laminated composite plate in cylindrical bending. In 2012 (J.S.Kalyana Rama et.al) studied the behavior of orthotropic laminated plate under cylindrical bending, where in an infinitely long plate was modeled and made finite.

From the study of literature on cylindrical bending of FRP laminates, a three-dimensional plate has been modeled for fracture to compare Strain Energy Release Rate (SERR) for plain and reinforced concrete with and without strengthening the plate using fiber reinforced polymer. The present investigation is an extension to (J.S.Kalyana Rama et.al, 2012) work.

## **2. Problem modeling**

### **2.1 Modeling a plate for fracture**

a) A simply supported plate is modeled using SOLID 95 element for the following dimensions  $X = 50\text{mm}$ ,  $Y = 300\text{mm}$  and  $Z = 5\text{mm}$  subjected to uniform pressure of 1MPa considering five different types of properties of concrete. Strain Energy Release Rate is calculated and compared for the input data.

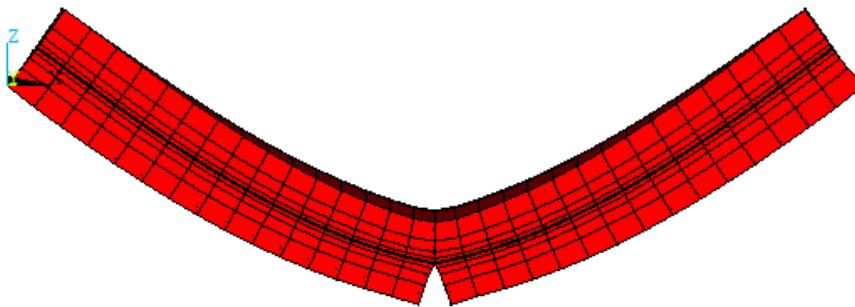


**Figure 1(a):** Four-layered orthotropic laminate and 1(b) finite element model with uniform pressure

**Table 1:** Material Properties (Balakrishna Murthy V, 2005)

Material	E (GPa)	$\nu$	G (GPa)
Graphite-epoxy-I	$E_1 = 141.6764$	$\nu_{12} = 0.257$	$G_{12} = 4.03$
	$E_2 = 12.386$	$\nu_{13} = 0.4206$	$G_{13} = 4.3592$
	$E_3 = 12.386$	$\nu_{23} = 0.257$	$G_{23} = 4.03$

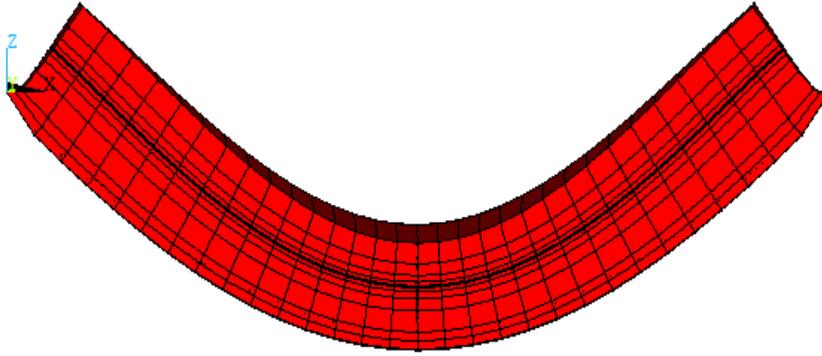
b) Behavior of five different types of plates is studied by strengthening the plate using FRP with fibers oriented with an inclination  $\theta$ . Figures 2 to 4 shows the contour plots of a plate when virtual crack and actual crack is opened and closed.



**Figure 2:** Contour plot of a plate with virtual crack and actual crack opened



**Figure 3:** Contour plot of a plate with virtual crack closed and actual crack opened



**Figure 4:** Contour plot of plate with virtual crack and actual closed

### **Properties of various plates used and SERR obtained**

1. Plain cement concrete (IS 456:2000)  
Young's Modulus = 22360, Poison ratio = 0.17  
Virtual Crack = 0.11mm  
Actual Crack = 2.5mm
2. Reinforced cement concrete (S. Elavenil et.al, 2006)  
Young's Modulus = 25000, Poison ratio = 0.2  
Virtual Crack = 0.11mm  
Actual Crack = 2.5mm
3. Reinforced cement concrete (Online material properties resource)  
Young's Modulus = 26000, Poison ratio = 0.18  
Virtual Crack = 0.11mm  
Actual Crack = 2.5mm
4. Steel Fiber Reinforced Concrete (S. Elavenil et.al, 2006)  
Young's Modulus = 28000, Poison ratio = 0.23  
Virtual Crack = 0.11mm  
Actual Crack = 2.5mm
5. High Strength Concrete (Wikipedia)  
Young's Modulus = 30000, Poison ratio = 0.18  
Virtual Crack = 0.11mm  
Actual Crack = 2.5mm

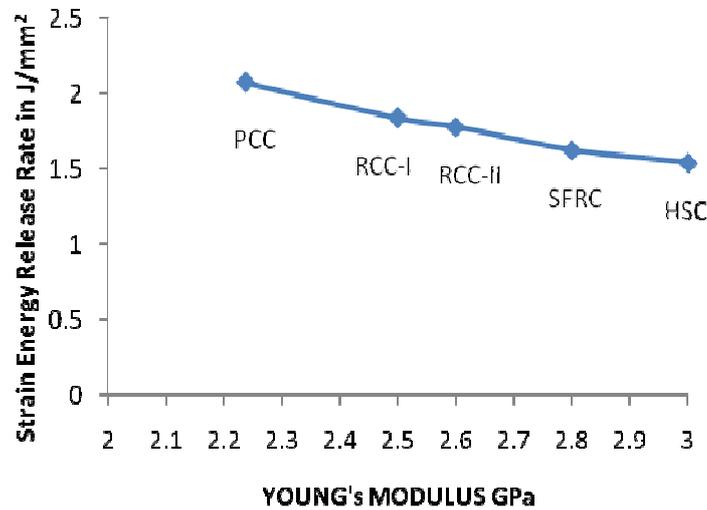
### **3. Results and discussion**

By modeling the plate using aforementioned properties strain energy release rate From the figure 5 it is observed that, with the increase in Young's modulus, the strain energy release rate decreases.

In order to arrest the crack from growing further, a 1mm thick FRP layer with fibers oriented at an angle of  $0^0$  is introduced at the position where crack began to extend. Strain energy release rate (SERR) is calculated and results are presented in Table 3 and figure 6 for  $0^0$  fiber orientation.

**Table 2:** Strain energy release rate for different types of concrete

Material	Energy –Virtual Crack closed	Energy –Virtual Crack opened	Strain Energy Release Rate(SERR)
PCC	2659.65	2796.57	2.074545
RCC-I	2355.1	2476.5	1.839394
RCC-II	2280.14	2397.59	1.779545
SFRC	2077.81	2185.04	1.624697
HSC	1976.14	2077.92	1.542121



**Figure 5:** Young's Modulus Vs strain energy release rate for different types of concrete

**Table 3:** Strain energy release rate for different types of concrete strengthened with FRP

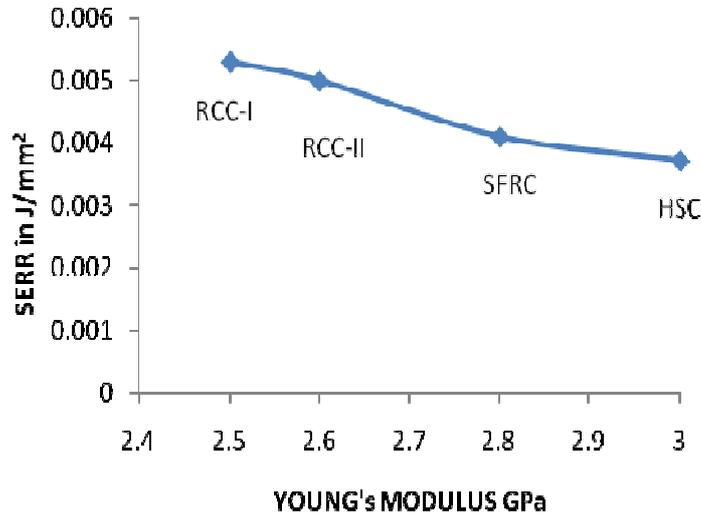
Material	Energy –Virtual Crack closed	Energy –Virtual Crack opened	Strain Energy Release Rate(SERR)
RCC-I	522.35	522.699	0.005288
RCC-II	511.186	511.515	0.004985
SFRC	484.717	484.987	0.004091
HSC	468.43	468.675	0.003712

Now, the same FRP layer is used as a strengthening technique but, this time the fibers are oriented in an inclined position i.e.  $\theta$  varying from  $0$  to  $90^0$ . The following properties are used for modeling Reinforced cement concrete plate

Young's Modulus = 26000, Poison ratio = 0.18

Virtual Crack = 0.11

Actual Crack = 2.5

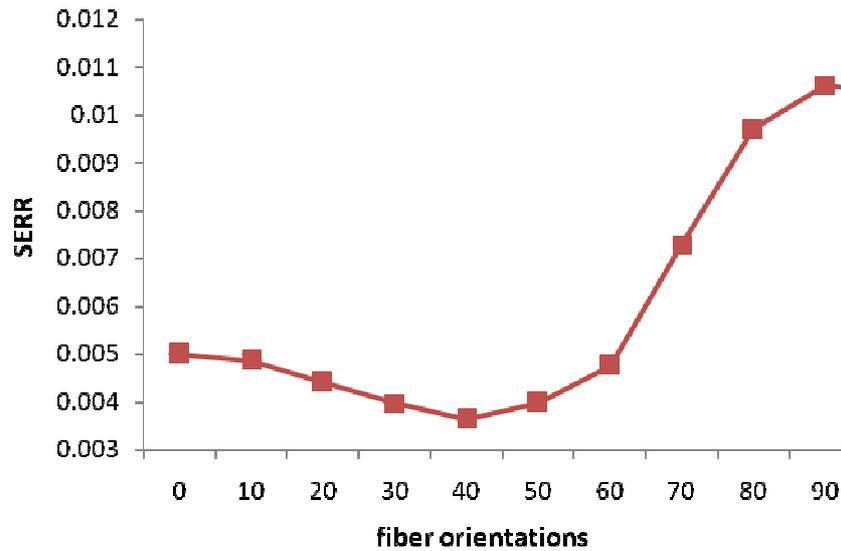


**Figure 6:** Young's modulus Vs strain energy release rate for different types of concrete strengthened with FRP

**Table 4:** Strain energy release rate for different orientations of FRP fibers in RCC plate

Fiber orientation	Strain energy release rate
0	0.004985
10	0.004848
20	0.004394
30	0.003955
40	0.003621
45	0.00397
50	0.004742
60	0.007273
70	0.009697
80	0.010606
90	0.010455

From figure 7 it is observed that the strain energy release rate decreases till 40° and then increases. When the fibers are oriented at 40° then, the obtained SERR value is minimum for a reinforced concrete plate.



**Figure 7:** Variation of strain energy release rate with respect to fiber orientation for a RCC plate

#### 4. Conclusions

Strain energy release rate decreases with the increase in Young's Modulus. Compared to plain cement concrete the strain energy release rate for RCC, SFRC, and HSC are low and hence the crack growth will be reduced. Strengthening of a RCC plate with Fiber Reinforced Polymer reduces the crack growth, For RCC plate the least value of strain energy release is obtained when fibers are oriented at an angle of  $40^{\circ}$ .

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