ACCURATE EDGE DOMINATION IN FUZZY GRAPHS

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Abstract: *A strong edge dominating set* D' *of a fuzzy graph* $G=(\sigma,\mu)$ *is an accurate edge dominating set if* $S-D'$ *has no strong edge dominating set of cardinality* $|D'|$ *where S is the set of all strong arcs of* $G=(\sigma,\mu)$ *.This paper introduces the concept of accurate edge domination in fuzzy graphs using strong arcs. We determine the accurate edge domination number* $\gamma'_a(G)$ *of some fuzzy graphs such as fuzzy paths and fuzzy cycles.*

Keywords: *Fuzzy graph, strong arc, non strong arc, accurate edge domination number, accurate domination, fuzzy cycle, fuzzy path.*

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1. INTRODUCTION

The fuzzy graph was introduced by Rosenfeld A[7] using fuzzy relation, introduced by Zedah[1], represents the relationship between the objects by particularly indicating the level of the relationship between the objects of the given set. Also he coined many fuzzy related graph theoretic concepts like bridge, cut vertex and tree. Fuzzy graphs have many more applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. It is quite well known that graphs are simply models of relations. A graph is a suitable way of representing information involving relationship between objects. The study of fuzzy graphs further studied by Kaufmann [2]. The study of domination in graphs was developed in the late 1950s and 1960s. Claude Berge [13] wrote a book on graph theory where he introduced the coefficient of external stability, which is known as the domination number of a graph. In 1962, Oystein Ore[6] introduced the terms, dominating set and domination number in his book on graph theory. A. Somasundaram and S. Somasundaram present the concept of domination in fuzzy graphs [5]. K.R. Bhutani and A. Rosenfeld introduced the concept of strong arcs in fuzzy graphs [10]. O.T. Manjusha and Sunitha discussed some concepts in domination and total domination in fuzzy graphs using strong arcs [4]. As an anology to vertex domination, the concept of edge domination was introduced by S.Mitchell and S.T. Hedetniemi[14] . In 1987, S.R. Jayaram refered edge domination as line domination in graphs and studied some properties. Later S. Arumugam and S. Velammal,[16] developed the concept of edge domination. Since the publication of these papers, the edge domination in graphs has been studied extensively by many researchers and several papers have been published on this topic.

The accurate domination in graphs was introduced by kulli and kattimani [12]. In this paper, we present the concept of accurate edge domination in fuzzy graphs using strong arcs**.** For graph-theoretic terminology, we refer to Harary, 1969 [8].

2. PRELIMINARIES

Fuzzy graph $G=(\sigma,\mu)$ is a pair of function $\sigma:V\to[0,1]$ and $\mu:V\times V\to[0,1]$ such that $\mu(u,v)\le\sigma(u)\wedge\sigma(v)$ for all u, v in V. The fuzzy graph $H(\tau,\rho)$ is called a fuzzy sub graph of $G=(\sigma,\mu)$

if $\tau(u) \le \sigma(u)$ for all u in V and $\rho(u,v) \le \mu(u,v)$ for all u, v in V. The underlying crisp graph of a fuzzy graph $G = (\sigma, \mu)$ is denoted by $G^* = (\sigma^*, \mu^*)$, where $\sigma^* = \{u \in V / \sigma(u) > 0\}$ and $\mu^* = \{(u,v) \in V \times V / \mu(u,v) > 0\}$. A fuzzy graph $G = (\sigma, \mu)$ is said to be bipartite if the node-set V can be partitioned into two non-empty sets V_1 and V_2 such that $\mu(v_1, v_2) = 0$ if $v_1, v_2 \in V_1$ or $v_1, v_2 \in V_2$. Further if $\mu(v_1, v_2) > 0$ for all $v_1 \in V_1$ and $v_2 \in V_2$ then G is called a complete bipartite graph. A complete bipartite graph on 'n' vertices is denoted by $F(K_n)$. An arc (u, v) of a fuzzy graph $G = (\sigma, \mu)$ is called an effective edge if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ and effective edge neighborhood of $u \in V$ is $N_e(u) = \{v \in V / edge(u,v)\ is\ effective\}$. A path ρ in a fuzzy graph $G = (\sigma, \mu)$ is a sequence of distinct nodes $v_0, v_1, v_2, ...v_n$ such that $\mu(v_{i-1}, v_i) > 0$ where $1 \le i \le n$ and n is called the length of ρ. The strength of the path ρ is defined to be $\wedge_{i=1}^{n} \mu(v_{i-1}, v_i)$. A single vertex considers as a path of length zero. The strongest path joining any two nodes u, v is a path corresponding to maximum strength between u and v. The strength of the strongest path is denoted by $\mu^\infty(u,v)$ and it is defined by $\mu^\infty(u,v) = \sup\{\mu^k(u,v) / k = 1,2,3,...\infty\}$. An arc (u, v) of a fuzzy graph $G = (\sigma, \mu)$ is called a strong arc if $\mu^\infty(u,v) = \mu(u,v)$, otherwise it is called a non-strong arc. A fuzzy graph $G = (\sigma, \mu)$ is said to be a strong fuzzy graph if it contains no non-strong arcs. Let $G = (\sigma, \mu)$ be a fuzzy graph and u, v be two nodes of G. We say that u dominates v if the arc (u, v) is strong. A subset D of V is called a strong dominating set of $G = (\sigma, \mu)$ if for every $v \in V - D$, there exists $u \in D$ such that u dominates v. A strong dominating set D is called a minimal strong dominating set if no proper subset of D is a strong dominating set. The minimum cardinality taken over all minimal strong dominating set is called strong domination number, denoted by $\gamma_s(G)$ and the corresponding dominating set is called minimum strong dominating set. Let $G = (\sigma, \mu)$ be a fuzzy graph. Let e_i and e_j be two edges of G. We say that e_i dominates e_j if e_i is a strong arc and e_i is adjacent to e_j. A subset D′ of E(G) is called strong edge dominating set if for every $e_j \in E(G) - D'$ there exist $e_i \in D'$ such that e_i dominates e_j. The smallest number of edges in any strong edge dominating set of G is called its edge domination number and its denoted by $\gamma_s'(G)$. An edge dominating set D' of G such that $|D'| = \gamma'(G)$ is called minimum edge dominating set.

3. ACCURATE EDGE DOMINATION BY STRONG ARCS

Definition 3.1. Let $G = (\sigma, \mu)$ be a fuzzy graph and let S be the set of all strong arcs of $G = (\sigma, \mu)$. The subset $D' \subseteq S$ is said to be accurate edge dominating set if D' is a strong edge dominating set and $S - D'$ contains no strong edge dominating set of same cardinality as $|D'|$.

Definition 3.2. The accurate edge domination number is the minimum cardinality of an accurate edge dominating set and it is denoted by $\gamma'_a(G)$

Definition 3.3. The upper edge domination number $\Gamma'_a(G)$ of $G = (\sigma, \mu)$ is the maximum cardinality of an accurate edge dominating set

Example 3.4. In the fuzzy graph represented in figure 3.1, the set $D' = \{(a,b),(c,d)\}$ is an accurate edge dominating set since $|S - D'| = 1$.

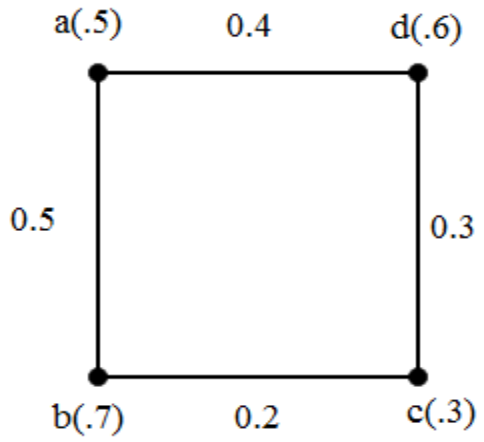


Fig. 3.1 Fuzzy Graph

Remark 3.5.

1. For any fuzzy graph $G = (\sigma, \mu)$, $\gamma'_s(G) \leq \gamma'_a(G)$.
2. A minimal strong edge dominating set of a fuzzy graph $G = (\sigma, \mu)$ may or may not be an accurate edge dominating set and vice versa.

Definition 3.6. Let u be a vertex in a fuzzy graph $G = (\sigma, \mu)$. The neighborhood of $u \in V$ is $N(u) = \{v \in V / \mu(u,v) > 0\}$. The degree of the vertex u is is defined to be d(u) = |N(u)|

Definition 3.7. Let u be a vertex in a fuzzy graph $G = (\sigma, \mu)$. The strong neighborhood of $u \in V$ is $N_s(u) = \{v \in V / arc(u,v) \text{ is strong}\}$. $N_s[u] = N_s(u) \cup \{u\}$ is the closed neighborhood of u. A vertex u in a fuzzy graph $G = (\sigma, \mu)$ is said to be isolated if $d(u) = 0$.

Theorem 3.8. For every non isolated vertex u in a fuzzy graph $G = (\sigma, \mu)$, $N_s(u) \neq \phi$.

Proof: Let u be any vertex in $G = (\sigma, \mu)$. Since u is non isolated, we have $d(u) > 0$. Let $N(u) = \{u_1, u_2, u_3, ...u_n\}$. Select the vertex u_i such that $\mu(u,u_i) \geq \mu(u,u_k)$ for all $1 \leq k \leq n$. Suppose the edge (u,u_i) is not a strong arc then there exist a $u - u_i$ path ρ in G such that $S_\rho > \mu(u,u_i)$. This path ρ must contain an edge of the form (u,u_k), $k \neq i \,\&\, 1 \leq k \leq n$. Also $\mu(u,u_k) \leq \mu(u,u_i)$. This implies $S_\rho \leq \mu(u,u_i)$. This is a contradiction and hence the edge (u,u_i) is a strong arc. Therefore $N_s(u) \neq \phi$.

Theorem 3.9. Let $G = (\sigma, \mu)$ be a fuzzy graph having no isolated vertices. The set of all strong arcs of $G = (\sigma, \mu)$ form a strong edge dominating set of G.

Proof: Let S be the set of all strong arcs of $G = (\sigma, \mu)$ and let $(u,v) \in E(G) - S$. By theorem 3.8, $N_s(u) \neq \phi$ and hence there exist a strong arc say (u, w) in G. Since it is a strong arc, $(u, w) \in S$. Therefore the edge (u, v) is dominated by the edge $(u, w) \in S$. Therefore the set of all strong arcs of $G = (\sigma, \mu)$ form a strong edge dominating set.

Corollary 3.10. Let $G=(\sigma,\mu)$ be a fuzzy graph having no isolated vertices. The set of all strong arcs of $G=(\sigma,\mu)$ is an accurate edge dominating set.

Corollary 3.11. Let $G=(\sigma,\mu)$ be a fuzzy graph having no isolated vertices. Then $\gamma'_a(G)\leq|S|$ where S is the set of all strong arcs of G.

Remark 3.12.

1. Throughout this paper S denote the set of all strong arcs of a fuzzy graph $G=(\sigma,\mu)$.
2. If a fuzzy graph $G=(\sigma,\mu)$ has no isolated vertices then $\left\lceil\frac{n}{2}\right\rceil\leq|S|\leq nC_2$, n is the number of vertices in $G=(\sigma,\mu)$

Definition 3.13. A subset $I\subseteq S$ is said to be independent edge set if any two edges of I are not adjacent. An independent edge set M is said to be maximal edge independent set if there does not exist an independent set I' such that $M\subset I'\subset S$. The minimum number of edges in a maximal edge independent set is called independence number and it is denoted by α'_0

Remark 3.14.

1. For a fuzzy graph, maximal edge independent set is not unique.
2. A maximal edge independent set of a fuzzy graph need not be an strong edge dominating set
3. A maximal edge independent set of a fuzzy graph need not be an accurate edge dominating set

Example 3.15.

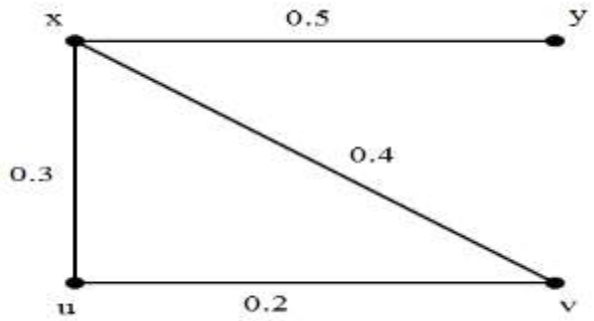


Fig. 3.2 Fuzzy Graph

In the fuzzy graph represented in figure 3.2, the maximal edge independent set $M=\{(x,y)\}$ is not a strong edge dominating set where as the maximal edge independent set $M=\{(u,x)\}$ is a strong edge dominating set.

Definition 3.16. Let D be any subset of the set of all strong arcs of a fuzzy graph $G=(\sigma,\mu)$. The base set of D is defined by $D_b=\{u\in V(G)/u$ is an end vertex of any edge in D$\}$.

Definition 3.17. Let D' be any subset of the set of all strong arcs of a fuzzy graph $G=(\sigma,\mu)$. The set of all edges dominated by D' is denoted by $D'_s=\{e_i \in E(G)-D' / e_i \text{ is adjacent to some edge in D'}\}$.

Remark 3.18.

1. For any strong edge dominating set D' we have $|D'|+|D_s'|=n$, where n is the total number of edges in $G=(\sigma,\mu)$.
2. Let D' be any subset of S, the set of all strong arcs of a fuzzy graph $G=(\sigma,\mu)$. If $|D'|+|D_s'|<n$, then D' could not be a strong edge dominating set of G.
3. The total number of edges in $P_n(\sigma,\mu)=n-1$ and the total number of edges in $C_n(\sigma,\mu)=n$.

Theorem 3.19. Let $G=(\sigma,\mu)$ be a fuzzy graph having no isolated vertices such that $|S|=\frac{n}{2}$, where n is the number of vertices in G and if n is even then $\gamma'_a(G)=\gamma'_s(G)=\frac{n}{2}$.

Proof: Let $G=(\sigma,\mu)$ be a fuzzy graph having n vertices. Let the set of all strong arcs $S=\{e_1,e_2,e_3,.....e_{\frac{n}{2}}\}$. Since for each vertex v_i, $N_s(v_i)\neq\phi$ and since $|S|=\frac{n}{2}$, we have the base set of S, $S_b=V(G)$. Consequently S is a strong edge dominating set of G. Let D' be any subset of S such that $|D'|=\frac{n}{2}-1$. It is clear that $|D'_b|=n-2$. Therefore there exist two vertices say v_i and v_j such that $v_i,v_j\notin D'_b$. Also we have $N_s(v_i)\neq\phi$ and $N_s(v_j)\neq\phi$, since $|S|=\frac{n}{2}$, there must be the strong arc (v_i,v_j) in G and this arc is not dominated by any edge in D'. Consequently D' is not a strong edge dominating set of G. Therefore the set S is the minimum strong edge dominating set and hence S is an accurate dominating set of G. Therefore $\gamma'_a(G)=\gamma'_s(G)=\frac{n}{2}$.

Corollary 3.20. Let $G=(\sigma,\mu)$ be a fuzzy graph having no isolated vertices such that $|S|=\left\lceil\frac{n}{2}\right\rceil$, where n is the number of vertices in G and if n is odd then $\gamma'_a(G)=\gamma'_s(G)=\frac{n-1}{2}$.

Theorem 3.21. Let $C_n(\sigma,\mu)$ be a strong fuzzy cycle with p vertices, then $\gamma'_a(C_n)=\frac{n}{3}+2$ when $n=3k$ and $n>3$.

Proof. Let $v_1,v_2,v_3.....v_n$ be the vertices and $e_1,e_2,e_3.....e_n$ be the edges of $C_n(\sigma,\mu)$ where $e_i=(v_i,v_{i+1})$, $i=1,2,....,n-1$. Let S be the set of all strong arcs of $C_n(\sigma,\mu)$. Since $C_n(\sigma,\mu)$ is a strong fuzzy graph we have $S=\{e_1,e_2,e_3.....e_n\}$. Let $S_1=\{e_1,e_4,e_7.....e_{n-2}\}$; $S_2=\{e_2,e_5,e_8.....e_{n-1}\}$; $S_3=\{e_3,e_6,e_9.....e_n\}$. Let D' be the strong edge dominating set of $C_n(\sigma,\mu)$. Since $\gamma'(C_n)=\frac{n}{3}$, we

have $D' = S_i$ for some i = 1, 2, 3 and $S_1 \cap S_2 \cap S_3 = \phi$. Therefore $\gamma'_a(C_n) > \frac{n}{3}$. Let D'_1 be the strong edge dominating set of $C_n(\sigma, \mu)$ such that $|D'_1| = \frac{n}{3} + 1$. Then $D'_1 = S_i \cup \{e_k\}$ where $e_k \in S - S_i$ for some i. Let S_j be the strong edge dominating set such that $j \neq i$ and $e_k \notin S_i \cup S_j$. Consider the set $D'_2 = S_j \cup \{e_j\}$, $j \neq k$. It is clear that $D'_1 \cap D'_2 = \phi$, also $D'_2 \subset S - D'_1$ and $|D'_2| = |S_j \cup \{e_j\}| = \frac{n}{3} + 1$. Since S_j itself a strong edge dominating set we conclude that D'_2 is a strong edge dominating set of $C_n(\sigma, \mu)$ and hence $\gamma'_a(C_n) > \frac{n}{3} + 1$. Consider the set $D'_3 = \{e_1, e_2, e_3, e_4, e_7 \ldots\ldots e_{n-2}\}$, $|D'_3| = \frac{n}{3} + 2$. In $C_n(\sigma, \mu)$ e_2 is adjacent only with e_1 and e_3 only. Therefore any edge in $S - D'_3$ does not form an edge dominating set of $C_n(\sigma, \mu)$. Therefore $\gamma'_a(C_n) = \frac{n}{3} + 2$

Theorem 3.22. Let $C_n(\sigma, \mu)$ be a strong fuzzy cycle with n vertices, then $\gamma'_a(C_n) = \left\lceil \frac{n}{3} \right\rceil + 1$ when $n \neq 3k$ and $n > 3$.

Proof. Let D' be the minimum strong edge dominating set of $C_n(\sigma, \mu)$. Therefore $|D'| = \left\lceil \frac{n}{3} \right\rceil$. Consider the mapping $\phi : D' \to S - D'$ such that $\phi(ei) = \begin{cases} e_{i-1} & if\ e_{i-1} \notin D' \\ e_{i+1} & Otherwise \end{cases}$

Claim: $\phi(D')$ is strong edge dominating set.

Suppose not then there exist an edge $x \in C_n$ such that x is not dominated by $\phi(D')$. Let $x = e_j$ for some j. Therefore $e_{j-1}, e_j, e_{j+1} \notin \phi(D')$. Consequently $e_{j-1}, e_j, e_{j+1} \in D'$. This is a contradiction to D' is a minimum strong edge dominating set. Therefore $\phi(D')$ is strong edge dominating set. Since ϕ is 1-1 $|\phi(D')| = \left\lceil \frac{n}{3} \right\rceil$.Therefore D' is not an accurate edge dominating set. Therefore $\gamma'_a(C_n) > \left\lceil \frac{n}{3} \right\rceil$.

Consider the strong edge dominating set $D' = \{e_1, e_2, e_5, e_8, \ldots.. e_n\}$ when $n = 3s + 2$ and $D'_1 = \{e_1, e_2, e_5, e_8, \ldots.. e_{n-2}, e_n\}$ when $n = 3s + 1$. It is clear that $|D'| = |D'_1| = \left\lceil \frac{n}{3} \right\rceil + 1$. In both the case $N_s(e_1) = \{e_n, e_2\} \subset D'$ when $n = 3s + 2$ and $N_s(e_1) \subset D'_1$ when $n = 3s + 1$. Therefore any edges in $S - D'$ could not dominate the edge e_1. Therefore any set of edges in $S - D'$ does not form an edge dominating set. Consequently $\gamma'_a(C_n) = \left\lceil \frac{n}{3} \right\rceil + 1$

Theorem 3.23. Let $P_n(\sigma,\mu)$ be a fuzzy path with n vertices, then $\gamma'_a(P_n)=\left\lceil\frac{n}{3}\right\rceil$ when $n=3k+1$ and $n>3$.

Proof. Let $S=\{e_1,e_2,e_3,.....e_{n-1}\}$ be the edges of $P_n(\sigma,\mu)$. In P_n all the edges are strong arcs. Since $\gamma'_a(P_n)=\left\lceil\frac{n}{3}\right\rceil$ we have $D'=\{e_2,e_5,e_8,.....e_{n-1}\}$ is the unique minimum strong edge dominating set. Therefore $S-D'$ contains no strong edge dominating set. In particular it contains no strong edge dominating set of same cardinality as $|D'|=\left\lceil\frac{n}{3}\right\rceil$. Consequently $\gamma'_a(P_n)=\left\lceil\frac{n}{3}\right\rceil$.

Theorem 3.24. Let $P_n(\sigma,\mu)$ be a fuzzy path with n vertices, then $\gamma'_a(P_n)=\frac{n}{3}$ when $n=3k$ and $n>3$.

Proof. Let $S=\{e_1,e_2,e_3,.....e_{n-1}\}$ be the edges of $P_n(\sigma,\mu)$, all of its edges are strong. Consider the set of strong arcs $D'=\{e_2,e_5,e_8,.....,e_{n-4},e_{n-2}\}$. It is clear that $e_1\in S-D'$ and $e_{n-1}\in S-D'$. Also since

$$d_s(e_i)=\begin{cases}2 & if\ i=2,3,\ldots..n-2\\ 1 & Otherwise\end{cases}$$

e_1 and e_{n-1} does not have strong neighbor in $S-D'$. Therefore any strong edge dominating set in $S-D'$ must contain the edge e_1 and e_{n-1}. Let D'' be any strong edge dominating set of $P_n(\sigma,\mu)$ such that $|D''|=\frac{n}{3}$. Note that e_1 and e_{n-1} each dominate exactly one edge. Except the edges e_1 and e_{n-1} remaining edges of D'' each can dominate at most 2 edges. Therefore D'' can dominate $2(\frac{n}{3}-2)+2=\frac{2n}{3}-2$ edges. Also $|D''|+|D''_s|=\frac{n}{3}+\frac{2n}{3}-2=n-2<n-1=|E(C_n)|$. Therefore D'' could not be a strong edge dominating set. Consequently $\gamma'_a(P_n)=\frac{n}{3}$.

Theorem 3.25. Let $P_n(\sigma,\mu)$ be a fuzzy path with n vertices, then $\gamma'_a(P_n)=\left\lceil\frac{n}{3}\right\rceil$ when $n=3k+2$ and $n>3$.

Proof. Let $P_n(\sigma,\mu)$ be a fuzzy path and $S=\{e_1,e_2,e_3,.....e_{n-1}\}$ be the edges of $P_n(\sigma,\mu)$. Let D' be a minimum strong edge dominating set. Since $\gamma'_a(P_n)=\left\lceil\frac{n}{3}\right\rceil$, we have $|D'|=\left\lceil\frac{n}{3}\right\rceil$.

Case i) When n is even and $n=3k+2$. Consider the minimum strong edge dominating set $D'_1=\{e_2,e_5,e_8,.....e_{\frac{n}{2}-2},e_{\frac{n}{2}},e_{\frac{n}{2}+2},e_{\frac{n}{2}+5},.....e_{n-2}\}$ It is clear that $e_1,e_{n-1}\in S-D'_1$ and $|D_1'|=\left\lceil\frac{n}{3}\right\rceil$. Since e_1 and e_n does not have the strong neighbor in $S-D'_1$ any edge dominating set in $S-D'_1$ must contain

the edge e_1 and e_n. Let D'' be any strong edge dominating set in $S-D'_1$ such that $|D''|=\left\lceil \frac{n}{3} \right\rceil$.

Therefore $e_1 \in D''$ and $e_n \in D''$. Consider the edges $e_{\frac{n}{2}-1}$ and $e_{\frac{n}{2}+1}$. Note that

$d_s(e_{\frac{n}{2}-1})=\{e_{\frac{n}{2}-2}, e_{\frac{n}{2}}\} \subset D_1'$ and $d_s(e_{\frac{n}{2}+1})=\{e_{\frac{n}{2}}, e_{\frac{n}{2}+2}\} \subset D_1'$. Therefore the edges $e_{\frac{n}{2}-1}$, $e_{\frac{n}{2}+1} \in S-D'_1$ does not have strong neighbor in $S-D'_1$. Hence $e_{\frac{n}{2}-1} \in D''$ and $e_{\frac{n}{2}+1} \in D''$. The four edges of D'' namely $e_1, e_{\frac{n}{2}-1}, e_{\frac{n}{2}+1}, e_{n-1}$ strongly dominate exactly 5 edges of $P_n(\sigma,\mu)$. The other $\left\lceil \frac{n}{3} \right\rceil - 4$ edges of D'', each can dominate $2(\left\lceil \frac{n}{3} \right\rceil - 4) = 2(\frac{n+1}{3}-4)$ edges. Therefore D'' can dominate $2(\frac{n+1}{3}-4)+5=\frac{2n+2}{3}-3=\frac{2n-7}{3}$ edges. Also $|D''|+|D_s''|=\frac{n+1}{3}+\frac{2n-7}{3}=\frac{3n-6}{3}=n-2<n-1$. This is a contradiction to D'' be a strong sedge dominating set. Therefore if D'' be any strong edge dominating set in $S-D'_1$ then $|D''|>\left\lceil \frac{n}{3} \right\rceil$. Consequently, $\gamma'_a(P_n)=\left\lceil \frac{n}{3} \right\rceil$

Case ii) when n is odd.

Consider the minimum strong edge dominating set of $P_n(\sigma,\mu)$. Let $D_1'=\{e_2, e_5, e_8, ...e_{\frac{n+1}{2}-2}, e_{\frac{n+1}{2}}, e_{\frac{n+1}{2}+2}, e_{\frac{n+1}{2}+5}, ...e_{n-2}\}$. Let D'' be any strong edge dominating set in $S-D'_1$ such that $|D''|=\left\lceil \frac{n}{3} \right\rceil$. The similar argument as above in case i, the set $D=\{e_1, e_{n-1}, e_{\frac{n+1}{2}-1}, e_{\frac{n+1}{2}+1}\} \subset D''$. The set D can dominate exactly 5 edges of $P_n(\sigma,\mu)$. The remaining $\left\lceil \frac{n}{3} \right\rceil - 4$ can dominate $2(\left\lceil \frac{n}{3} \right\rceil - 4) = 2(\frac{n+1}{3}-4) = 2(\frac{n+1}{3})-8$ edges. The total number of edges dominated by $D''=\ 2(\frac{n+1}{3})-8+5=2(\frac{n+1}{3})-3=\frac{2n-7}{3}$. Also $|D''|+|D_s''|=\frac{n+1}{3}+\frac{2n-7}{3}=\frac{3n-6}{3}=n-2<n-1$. Therefore D'' could not be a strong edge dominating set of $P_n(\sigma,\mu)$. This is a contradiction. Therefore $|D''|>\left\lceil \frac{n}{3} \right\rceil$ and hence $\gamma'_a(P_n)=\left\lceil \frac{n}{3} \right\rceil$.

Theorem 3.26. Let $P_n(\sigma,\mu)$ be a fuzzy cycle with n vertices, then $\gamma'_a(P_n)=\left\lceil \frac{n}{3} \right\rceil$ for any $n>3$

Proof: Proof follows by theorem 3.23, 3.24 and 3.25

Remark 3.27.

In the following table, we present the accurate domination number of some fuzzy paths

$P_n(\sigma,\mu)$	$P_3(\sigma,\mu)$	$P_4(\sigma,\mu)$	$P_5(\sigma,\mu)$	$P_6(\sigma,\mu)$	$P_7(\sigma,\mu)$	$P_8(\sigma,\mu)$	$P_9(\sigma,\mu)$
$\gamma_a'(P_n)$	2	2	2	2	3	3	3

Conclusion

The concept of domination in graphs is very rich both in theoretical developments and applications. This paper presents the accurate edge domination parameter using strong arcs. The accurate edge domination number of fuzzy cycles and fuzzy paths are obtained. We obtained the condition that the accurate edge domination number and the strong edge domination number of a fuzzy graph are equal. We shall work on this accurate domination parameter in the union of fuzzy graphs and join of fuzzy graphs in our forthcoming papers

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