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Industrial and Trade Policies in a Developing Country Timing of Trade Policy

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Abstract

This paper investigates how many firms the home government should establish in order to foster domestic industry when home and foreign governments set the timing of their trade policies in a two-country model. Our main conclusions are as follows: Suppose that the home government increases the number of home firms, given the number of foreign firms. [1] A cost difference between home and foreign firms restricts feasible timing of trade policy. [2] The equilibrium tariff rate tends to decline and the subsidy rate tends to rise. [3] The home government should establish the home firms at most to three in order to foster the domestic industry.

Keywords:* *Industrial Policy, Endogenous Timing, Import Tariff, Export Subsidy, Number of Firms, Market Structure

JEL Classifications: F12, F13, L13

1 Introduction

Collie (1994) wrote a pioneering paper examining endogenous timing of trade policy in a two-country model. He considered whether a home government should impose countervailing duties when faced with an export subsidy by a foreign government in a two-country duopoly model. He showed that the home government has no incentive to impose countervailing duties. That is, in equilibrium the home government imposes an import tariff first, and then the foreign government pays an export subsidy. Wong and Chow (1997) introduced demand uncertainty into Collie's model. They showed that the home country adopts countervailing duties if demand uncertainty is high. This result is in contrast to that of Collie (1994). Hayashibara (2002) extended Collie's duopoly model to a Cournot oligopoly and established that a country with less concentrated industry tends to act as a Stackelberg leader. Toshimitsu (1997) considered the same problem with and without free entry. Supasri and Tawada (2007) also investigated it in an economy where both countries had their own markets.[1] These papers investigated endogenous timing in two-country models, and showed that the timing of trade policy depends on a combination of the numbers of home and foreign firms.

These researchers did not explicitly address viable conditions for firms, although they constructed their models with cost differences between home and foreign firms. They also were not concerned with how market structure affects import tariffs and export subsidies, nor with social welfare, although market structure plays a crucial role in timing.

In reality, governments of developing countries have often implemented both industrial and trade policies that allowed only large firms to remain in the market to protect the industry. This brings us to the question of why the governments of developing countries implement not only import tariff policy but also domestic industrial policy (i.e., controlling the number of domestic firms).

The purpose of this paper is to investigate how many firms the home government should establish in order to foster domestic industry when home and foreign governments set the timing of their trade policies in a two-country model.

Our main conclusions are as follows. Suppose that the home government increases the

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[1] Collie (1994) and Hayashibara (2002) considered the case where the home government could employ an import tariff as well as a production subsidy for home firms, whereas Toshimitsu (1997) and Supasri and Tawada (2007) considered the case where the home government could use only an import tariff policy.

number of home firms according to the number of foreign firms. [1] A cost difference between home and foreign firms restricts the feasible timings of trade policy. [2] The equilibrium tariff rate tends to decline and the subsidy rate tends to rise. [3] The home government should establish a maximum of three home firms to foster domestic industry.

The rest of the paper is organized as follows. In Section 2 we formulate the basic model. In Section 3 we present preliminary results, which coincide with the conclusions of Toshimitsu (1997) and Supasri and Tawada (2007). Our main results are presented in Section 4. Conclusions and some directions for further studies are provided in Section 5.

2 The Model

We consider a world economy in which two countries (home and foreign) exist, and where n_h and n_f firms are located in the home and foreign country, respectively. The number of firms is fixed. Each firm produces a homogeneous commodity and serves only the home country's market. We assume that foreign firms have a cost advantage over home firms: a constant marginal cost for home firms c_h is higher than that for foreign firms, c_f. That is, $c_h \geq c_f$. Let an inverse demand function be $p = A - Q$, where Q is the demand for a commodity, p is the market price, and A indicates market scale, which is assumed to be greater than c_h. The home government imposes an import tariff of t per unit, while the foreign government pays an export subsidy of s per unit. The timing of trade policy is determined endogenously. Both governments implement these trade policies to maximize their respective national welfare.

Our model is described as a three-stage game. In the first stage, each government decides the timing of trade policy (that is, an import tariff and an export subsidy). In the second stage, the home (foreign) government decides on a level of tariff (subsidy, respectively) according to predetermined timing.[2] In the third stage, all firms compete in the home market *à la* Cournot.

3 Preliminary Analysis

We solve this game by backward induction. Let us begin with the third-stage game. The firms simultaneously and independently choose their outputs so as to maximize profit given an import tariff and an export subsidy set by each government. The profit of a home firm is given by:

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[2] This type of timing game is called an extended game with observable delay. See Hamilton and Slutsky (1990).

$$\pi_h = (p - c_h)q_h , \tag{1}$$

where q_h is the output of the home firm. The profit of a foreign firm is given by:

$$\pi_f = (p - c_f - t + s)q_f , \tag{2}$$

where q_f is the output of the foreign firm. We assume that $A = 1$, $c_f = 0$, and $c_h = c$ for simplicity. Thus, c shows a cost difference between home and foreign firms. From equations (1) and (2), the symmetric Nash equilibrium output of each firm is given by:

$$q_h = \frac{1 - c(n_f + 1) - (s - t)n_f}{n_h + n_f + 1}, \quad q_f = \frac{1 + cn_h + (s - t)(n_h + 1)}{n_h + n_f + 1} . \tag{3}$$

From equation (3), the nonnegative output condition for home firms is:

$$c \leq \frac{1 - (s - t)n_f}{n_f + 1} . \tag{4}$$

The total output in the equilibrium becomes:

$$Q = \frac{n_h + (1 - c + s - t)n_f}{n_h + n_f + 1} . \tag{5}$$

Next, we consider the second stage. Given the level of export subsidy, the home government sets its tariff level to maximize its national welfare W_h, defined as the sum of consumer surplus, total profits of home firms, and tariff revenue:

$$W_h = \frac{1}{2}Q^2 + n_h\pi_h + tq_f n_f . \tag{6}$$

To derive an optimal tariff level, maximizing W_h, we substitute equations (1), (3), and (5) into equation (6) and then maximize with respect to *t*. We obtain the optimization condition:

$$t = \frac{2n_h + 1 - c(n_f - n_h)}{2(n_h + 1)^2 + n_f} + \frac{n_h(n_h - n_f + 2) + 1}{2(n_h + 1)^2 + n_f} \cdot s . \tag{7}$$

The foreign government determines the level of subsidy, in response to the level of import tariff, in order to maximize its national welfare W_f, defined as total profits of foreign firms less the expenditure on export subsidies:

$$W_f = n_f \pi_f - sq_f n_f . \tag{8}$$

We also derive the optimization condition:

$$s = \frac{(n_h - n_f + 1)(1 + cn_h)}{2n_f(n_h + 1)} - \frac{n_h - n_f + 1}{2n_f} \cdot t . \tag{9}$$

Equations (7) and (9) define, respectively, the reaction function of the home government against an export subsidy, and that of the foreign government against an import tariff.

In this two-country model, there are three possible policy timings:

S: the home government and the foreign government determine the level of import tariff and export subsidy simultaneously.

H: the home government imposes an import tariff first, and the foreign government then sets an export subsidy.

F: the foreign government sets an export subsidy first, and then the home government imposes an import tariff.

From equations (6), (7), (8), and (9), we can calculate the equilibrium import tariff and export subsidy for each timing of trade policy. Table 1 shows these results.

(Table 1 about here)

From Table 1, we also calculate each country's welfare: Table 2.

(Table 2 about here)

Thus, we obtain the following results.

Lemma 1 (Toshimitsu (1997) and Supasri and Tawada (2007)):

[i] If $n_h \leq n_f - 3$ and $n_h \geq 2$, the foreign government imposes an export tax first and then the home government imposes an import tariff (Timing F).

[i i] If $n_h = n_f - 2$, both governments move simultaneously (Timing S).

[iii] If $n_h \geq n_f$, the home government imposes an import tariff first and the foreign government then pays an export subsidy (Timing H).

Note that timing is indecisive when $n_h + 1 = n_f$ as well as $n_h = 1$ and $n_f = 4$.[3]

[3] Hereafter, we assume that the number of foreign firms is more than four. This is because a sequential move, by which the foreign (home) government becomes the first (second) mover, emerges when $n_h \leq n_f - 3$ except for

These results state that the country with more (fewer) firms tends to act as a first (second) mover. Figure 1 summarizes these results.

4 Analysis

In this section, we investigate three issues: (i) effects of the cost difference on the equilibrium timing of trade policies; (ii) effects of the number of home firms on the levels of import tariff and export subsidy; and (iii) effects of market structure on social welfare and each of its components, i.e., consumer surplus, producer surplus, and tariff revenue. We assume that the home government increases the number of home firms in response to the number of foreign firms.

4-1 Feasible Timing of Trade Policies under Cost Difference

We verify the nonnegative output conditions for home firms at each policy timing using equations (3) and (4) as well as Table 1. We obtain the following results.

Lemma 2: Given *c*, the nonnegative output conditions at each timing of trade policy (S, H, and F) are $c \leq \overline{c}^i$, ($i = S, H, F$), where:

$$\overline{c}^S = \frac{n_h + n_f + 1}{n_h + (n_h + 2)n_f + 1}, \ \overline{c}^H = \frac{2}{3 + n_h}, \text{ and, } \overline{c}^F = \frac{2(n_h + 1)^2 + n_f}{2(n_h + 1)^2 + (n_h^2 + 2n_h + 2)n_f}.$$

By Lemma 2, there is a market structure that guarantees the viability of home firms under the cost difference. Figure 2 shows this market structure, given home firms' constant marginal cost. When home firms' marginal cost is $\hat{c}$, any market structure depicted on the left side of each line $\overline{c}^i (i = S, H, F)$ ensures viability. No home firms can be viable outside that area. Therefore we obtain following results.

Proposition 1: Suppose that the viability condition holds. The equilibrium timing becomes as follows:

$n_h = 1$ and $n_f = 4$.

[i] if the combination of the number of home and foreign firms is in the region RF, then timing F prevails;

[i i] if the combination of the number of home and foreign firms is in the region RS, timing S prevails;

[iii] if the combination of the number of home and foreign firms is in the region RH, timing H prevails,

such that $n_f^{FS} = \frac{(1-c)\left(1-2c+\sqrt{(1-2c)(1-c)}\right)}{(1-2c)c}$ and $n_f^{SH} = \frac{2(1-c)}{c}$.

Proposition 1 states that cost difference restricts the possible timings of trade policy, as shown in Figure 2. Market structure affects the timing of trade policy as follows.

[i] If $n_f > n_f^{FS}$, then equilibrium timing of trade policy is timing F only.

[i i] If $n_f^{FS} > n_f > n_f^{SH}$, then equilibrium timings of trade policy are timings S and F.

[iii] If $n_f^{SH} > n_f$, then all timings of trade policy are realized in equilibrium.

4-2 Effects of Market Structure on Tariff and Subsidy Levels

Because the signs of $\partial t / \partial n_h$ and $\partial s / \partial n_h$ are ambiguous, we focus on two extreme cases: (i) there are no cost differences between the home and the foreign firms; and (ii) there is a single home firm.

First we consider the effects of changing the number of home firms on the levels of tariff and subsidy. From Table 1, we have following results.

Lemma 3:

[i] $\left.\frac{\partial t^S}{\partial n_h}\right|_{c=0} = -\frac{2n_f(n_h-1)(n_h+1)^2+(n_h+1)^4+n_f^2(n_h^2-2)}{(n_h+1)^2((n_h+1)^2+n_f(n_h+2))^2} < 0$,

[ii] $\left.\frac{\partial t^H}{\partial n_h}\right|_{c=0} = -\frac{4n_h(n_h+1)-1}{(2n_h^2+5n_h+3)^2} < 0$, and

[iii] $\left.\frac{\partial t^F}{\partial n_h}\right|_{c=0} = -\frac{n_f^2(n_h-1)+n_f(n_h-3)(n_h+1)^2+2n_h(n_h+1)^2}{2(n_h+1)^3(n_f+(n_h+1)^2)^2} < 0$, if $n_h > 1$.

Note that the tariff level with a single home firm is higher than that with two home firms. Therefore, Lemma 3 indicates that the level of tariff declines with the number of home firms irrespective of policy timing.

We also have following results.

Lemma 4:

[i] $$\left.\frac{\partial s^S}{\partial n_h}\right|_{c=0} = \frac{-2(n_h+1)^2(n_h-n_f+1)+n_f{}^2(2n_h+3)}{(n_h+1)^2((n_h+1)^2+n_f(n_h+2))^2} > 0,$$

[ii] $$\left.\frac{\partial s^H}{\partial n_h}\right|_{c=0} = -\frac{n_f(4n_h+5)-2(n_h+1)^2}{n_f(n_h+1)^2(2n_h+3)^2} > 0, \text{ if } n_h < n_f - 1 + \sqrt{n_f\left(n_f+\frac{1}{2}\right)}, \text{ and}$$

[iii] $$\left.\frac{\partial s^F}{\partial n_h}\right|_{c=0} = \frac{n_f(n_f+2(n_h+1)^2)}{(n_h+1)^3(n_f+(n_h+1)^2)^2} > 0.$$

Lemma 4 means that the level of subsidy increases with the number of home firms, but it tends to decrease when the number of home firms becomes much greater than that of foreign firms.

Next, we investigate the effects of the number of home firms on the levels of tariff and subsidy when a single home firm operates initially. From Table 1, we have the following results.

Lemma 5:

$$\left.t^F\right|_{n_h=1} > \left.t^F\right|_{n_h=2} \text{ and } \left.s^F\right|_{n_h=1} < \left.s^F\right|_{n_h=2} \text{ where } c < \left.\bar{c}^i\right|_{n_h=1}.$$

Lemma 5 shows that the tariff declines but the subsidy rises when the home government increases the number of home firms from one to two. Note that in this case the timing of trade policy becomes F, that is, the foreign government sets an export tax first, and then the home government imposes an import tariff. The foreign government then decreases the level of export tax.

We summarize lemmas 3 through 5 as follows.

Proposition 2: Suppose that the home government increases the number of home firms in response to the number of foreign firms.

[i] If no cost difference exists, the home government reduces the import tariff level, while the foreign government raises its export subsidy as the number of home firms increases.

[ii] If a single home firm operates initially, the home (foreign) government decreases its tariff (export

tax) level as the number of home firms increases by one.

4-3 Effects of Market Structure on Welfare

Finally, we investigate the effects of a change of market structure on the home country's welfare. Using equations (1), (3), (5), and (6), and Table 1, we have the following results.

Proposition 3: When the home government increases the number of home firms in response to the number of foreign firms, both home-country national welfare and consumer surplus improve for every policy timing.

Proposition 3 states that the home government has an incentive to induce more domestic firms to operate in order to maximize social welfare, and that the resulting consumer surplus increases. As shown in Proposition 2, an increase in the number of home firms lowers the import tariff and export tax at first. When the number of home firms exceeds that of foreign firms, the foreign government changes its export tax to an export subsidy. Therefore, with more home firms, the home tariff rate declines and the foreign export subsidy rises, which facilitates competition. As a result, total output increases and consumer surplus improves.

Next, we consider the effects of market structure on producer surplus, which is defined as $PS = n_h \pi_h$. Using equations (1) and (3), and Table 1, we obtain the following results.

Lemma 6: If there is no cost difference between home and foreign firms, the following inequalities hold.

$$\frac{\partial PS^S}{\partial n_h} < 0, \ \frac{\partial PS^H}{\partial n_h} < 0, \text{ and } \frac{\partial PS^F}{\partial n_h} \gtrless 0 .$$

Lemma 6 indicates that, without a cost difference, the number of home firms that maximizes producer surplus at Timing S (H) is $n_f - 2$ (n_f), although the number is ambiguous at Timing F. We investigate the number of home firms that maximizes producer surplus at Timing F. By comparing producer surplus with the viable number of home firms, given the number of foreign firms, we have the following results.

Lemma 7: If there is no cost difference between home and foreign firms, the number of home firms that maximizes producer surplus at timing F (n_h^{F*}) is at most three.

Lemma 8: If there is no cost difference between home and foreign firms, the following inequalities hold:

$$\max[PS^F] > PS^S\Big|_{n_h=n_f-2} > PS^H\Big|_{n_h=n_f}.$$

From lemmas 6, 7, and 8, we establish the following.

Proposition 4: If there is no cost difference between home and foreign firms, the number of home firms that maximizes its producer surplus (n_h^*) is at most three.

Proposition 4 states that, given the number of foreign firms, the home government can maximize its producer surplus by increasing the number of home firms to a maximum of three, if there is no cost difference between home and foreign firms.

Now, we examine how the cost difference affects the optimal number of home firms necessary to maximize the home country's producer surplus. From equation (3) and Table 1, we obtain the following results.

Lemma 9: Suppose that the home government increases the number of home firms, in response to the number of foreign firms. The following inequality holds.

$\frac{\partial n_h^*}{\partial c} < 0$, where n_h^* is n_h such that $\partial PS / \partial n_h = 0$.

From Lemma 9 together with Proposition 3, we obtain the following results.

Proposition 5: The home government should increase the number of home firms to a maximum of three in order to maximize producer surplus even if there is a cost difference between home and foreign firms.

We explain these results intuitively. Suppose that the home government chooses the number of home firms in order to foster the domestic industry. Note that the home government maximizes its social welfare when policy timing and the level of tariff/subsidy are determined endogenously. A possible real-world situation is as follows. The Department of Trade formulates trade policies while the Department of Industry formulates domestic industry policies in the home country. Both departments make their decisions independently. An increase in the number of home firms has two effects: (i) the home firms' market share rises; and (ii) the market price declines. If the home government raises the number of home firms above that of foreign firms, then the foreign government pays an export subsidy and the home government lowers the level of its tariff. As a result, the market price decreases. However, if the number of home firms is less than that of foreign firms, the foreign government imposes an export tax and the home government imposes a relatively high import tariff. Then, the market price remains high. In this situation, the former effect dominates the latter effect in a particular range. The home government (or Department of Trade) does not impose a prohibitive level of import tariff. Therefore, the number of home firms that maximizes producer surplus may be more than one but does not exceed three.

Next we consider how the number of home firms influences tariff revenue. From equation (3) and Table 1, we have the following.

Proposition 6: Suppose that $c=0$. The tariff revenue declines with the number of home firms irrespective of the timing of trade policy.

Proposition 6, together with propositions 3 and 4, implies that home consumers gain from the operation of more home firms, and this positive effect dominates the reduction of producer surplus and tariff revenue. Therefore, the home country should raise the number of domestic firms as much as possible in order to maximize its social welfare, even though producer surplus increases to a certain point and afterwards declines.

We examine the number of home firms that maximizes producer surplus in a free trade economy (n_h^{FT*}). To obtain n_h^{FT*}, we substitute equations (3), $t=0$, and $s=0$ into equation (1), and find the following.

Proposition 7: Suppose that both the governments follow free trade policy, that is $t = 0$, and $s = 0$. The number of home firms that maximizes producer surplus is:

$n_h^{FT*} = n_f + 1$ where $\bar{c}^{FT} > c \geq 0$,

where $\bar{c}^{FT}$ is the nonnegative output condition under free trade, which is derived from equation (4).

Proposition 7 states that the home government should determine the number of home firms to one more than the number of foreign firms. Propositions 4 and 7 indicate that the number of home firms that maximizes producer surplus under endogenous timing is less than that under free trade. Propositions 3 and 4 suggest that the home government can foster domestic industry by controlling the number of home firms under bilateral interventions instead of free trade policy.

Dixit (1984) considered similar issues in a different two-country model. He investigated how affects the existence of foreign competition on domestic anti-trust policy in an economy where both countries had their own markets, and showed that the home government should not encourage the mergers of home firms under free trade from the domestic welfare viewpoint; it should establish one more home firms than the number of foreign firms to maximize the home firms' profit in the foreign market.

5 Concluding Remarks

In this paper, we investigated how many firms the home government needed to establish in order to foster domestic industry as well as its welfare, under endogenous timing of trade policy in a two-country model. Our main conclusions are as follows. Suppose that the home government raises the number of home firms in response to the number of foreign firms. [1] A cost difference between home and foreign firms restricts the equilibrium timing of trade policy. [2] An increase in the number of home firms tends to lower the level of an import tariff and induce a higher export subsidy. [3] The home government should establish at most three firms in order to foster the domestic industry. Thus, if the home government intends to foster domestic industry, it should limit the number of home firms to a maximum of three. However, to maximize social welfare it should establish as many firms as possible. The number of home firms that maximizes producer surplus is less than the optimal number in the free trade case: $n_h^{FT*} = n_f + 1$.

Appendix

Proof of Lemma 7: From equation (3) and Table 1, the output of a home firm at timing F is as follows.

$$q_h^F = \frac{(2(n_h+1)^2+n_f)}{2(n_h+1)((n_h+1)^2+n_f)} > 0\,. \qquad \text{(A-1)}$$

Using equation (A-1), we have:

$$\begin{aligned}\frac{\partial PS^F}{\partial n_h} &= q_h^F\left(q_h^F + 2n_h\frac{\partial q_h^F}{\partial n_h}\right)\\ &= -q_h^F\left(\frac{\sigma(n_h,n_f)}{2\left((n_h+1)^2+n_f\right)^2(n_h+1)^2}\right).\end{aligned} \qquad \text{(A-2)}$$

Note that $\sigma(n_h,n_f) = \left(2(n_h+1)^4+n_f{}^2\right)(n_h-1)-(n_h+3)(n_h+1)^2 n_f$.

Since $q_h^F > 0$, we can determine the sign of ($\partial PS^F / \partial n_h$) as follows.

$$sign\frac{\partial PS^F}{\partial n_h} = -sign\ \sigma(n_h,n_f).$$

Let us take the above expression as a quadratic expression of n_f, and examine the possibilities when the roots of the equation are real. Using the discriminant of the equation $\sigma(n_f)=0$, we have:

$$D = -\left(7n_h{}^2 - 22n_h - 1\right)(n_h+1)^4\,. \qquad \text{(A-3)}$$

If $n_h < 4$, then equation (A-3) is positive. Therefore, the optimal number of home firms that maximizes producer surplus in the home country at timing F is at most three. *Q.E.D.*

Proof of Lemma 8: First, we compare maximum producer surplus at timing S with that at timing H. From Lemma 6, given n_f, maximum producer surplus in the home country at timings S and H are as follows.

$$PS^S = \frac{(n_f-2)(1-2n_f)^2}{\left((n_f-1)^2+n_f{}^2\right)^2}\text{, and} \qquad \text{(A-4)}$$

$$PS^H\Big|_{n_h=n_f} = \frac{4n_f}{(2n_f+3)^2}\,. \qquad \text{(A-5)}$$

Note that timing S occurs if and only if $n_h = n_f - 2$. From equations (A-4) and (A-5), we have the inequality:

$$PS^S - PS^H\Big|_{n_h=n_f} = \frac{-18+53n_f+8n_f^{\ 2}-104n_f^{\ 3}+32n_f^{\ 4}}{(2n_f+3)^2(2n_f^{\ 2}-2n_f+1)^2} < 0 \text{ for } n_f \geq 5.$$

Next, we show that maximum producer surplus at timing F is greater than that at timing S.

$$\begin{aligned} PS^F\Big|_{n_h=2} - PS^S &= \frac{(n_f+18)^2}{18(n_f+9)^2} - \frac{(n_f-2)(2n_f-1)^2}{((n_f-1)^2+n_f^{\ 2})^2} \\ &= \frac{4n_f^{\ 6}+64n_f^{\ 5}-64n_f^{\ 4}-4414n_f^{\ 3}+17065n_f^{\ 2}-13734n_f+3240}{18(n_f+9)^2(2n_f^{\ 2}-2n_f+1)} > 0. \end{aligned}$$

Noting Lemma 7, the above condition is enough to show that $\max[PS^F] > PS^S$ for $n_f \geq 5$.

Finally, we investigate the number of home firms maximizing producer surplus at timing F, given the number of foreign firms. Noting Lemma 7, we compare producer surplus when $n_h = 1, 2,$ and 3 as follows.

$$\begin{aligned} PS^F\Big|_{n_h=1} - PS^F\Big|_{n_h=2} &= \frac{(n_f+8)^2}{16(n_f+4)^2} - \frac{(n_f+18)^2}{18(n_f+9)^2} \\ &= \frac{n_f^{\ 4}-46n_f^{\ 3}-1127n_f^{\ 2}-3312n_f+5184}{144(n_f+4)^2(n_f+9)^2}. \end{aligned} \quad \text{(A-6)}$$

$$\begin{aligned} PS^F\Big|_{n_h=1} - PS^F\Big|_{n_h=3} &= \frac{(n_f+8)^2}{16(n_f+4)^2} - \frac{3(n_f+32)^2}{64(n_f+16)^2} \\ &= \frac{n_f^{\ 4}-24n_f^{\ 3}-1328n_f^{\ 2}-3072n_f+16384}{64(n_f+4)^2(n_f+16)^2}. \end{aligned} \quad \text{(A-7)}$$

$$\begin{aligned} PS^F\Big|_{n_h=2} - PS^F\Big|_{n_h=3} &= \frac{(n_f+18)^2}{18(n_f+9)^2} - \frac{3(n_f+32)^2}{64(n_f+16)^2} \\ &= \frac{5n_f^{\ 4}-38n_f^{\ 3}-5515n_f^{\ 2}-10944n_f+414720}{576(n_f+9)^2(n_f+16)^2}. \end{aligned} \quad \text{(A-8)}$$

From equations (A-6), (A-7), and (A-8), we have the following results.

[i] $n_h^{F*} = 1$ if $n_f \geq 65$.

[ii] $n_h^{F*} = 2$ if $64 \geq n_f \geq 38$ or $7 \geq n_f \geq 5$.

[iii] $n_h^{F*} = 3$ if $37 \geq n_f \geq 8$.

Using mathematical software *MATHEMATICA v. 4*, we find critical values for the above conditions: [i] $n_f \approx 64.3$; [ii] $n_f \approx 37.2$; and [iii] $n_f \approx 7.7$.

Q.E.D.

Proof of Proposition 4: This can be proved by taking into account lemmas 7 and 8.

Numerical Example of Proposition 4: Suppose that $n_f = 10$. From equations (1) and (3) as well as Table 1, producer surpluses in the home country at each timing are as follows.

$$PS^S = \frac{n_h (n_h + 11)^2}{(n_h{}^2 + 12n_h + 21)^2},\ PS^H = \frac{4n_h}{(2n_h + 3)^2},\ \text{and}\ PS^F = \frac{n_h (n_h{}^2 + 2n_h + 6)^2}{(n_h + 1)^2 (n_h{}^2 + 2n_h + 11)^2}.$$

Figure A-1 represents these results graphically.

(Figure A-1 about here)

We should note that, given $n_f = 10$, when $n_h = 8$ the timing changes from F to S, and when $n_h = 10$ the timing becomes H. Noting that the number of home firms maximizing producer surplus at timing F is at most three, from Lemma 7, we calculate PS^F with at most three home firms:

$$PS^F\Big|_{n_h=1} = \frac{81}{784},\ PS^F\Big|_{n_h=2} = \frac{392}{3249},\ \text{and}\ PS^F\Big|_{n_h=3} = \frac{1323}{10816}.$$

From here, we obtain $n_h{}^* = 3$ when $n_f = 10$.

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Figure 1: Policy Timing Given Combinations of Numbers of Home and Foreign Firms

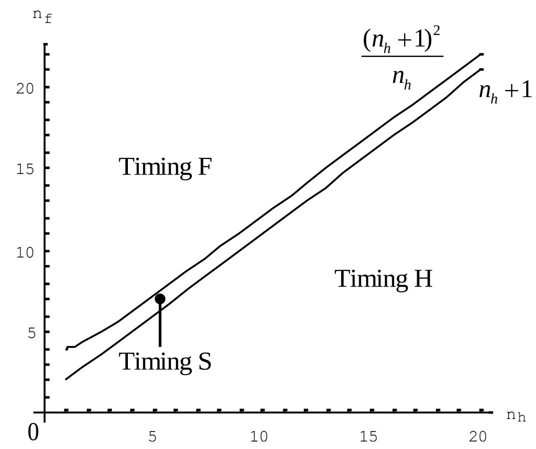

Figure 2: Feasible Policy Timing Given Home Firms' Marginal Cost $\hat{c}$

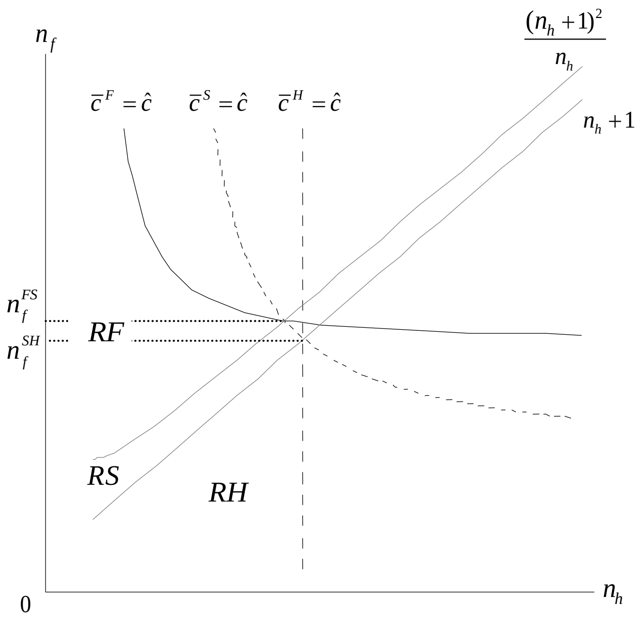

Figure A-1: Producer Surplus in the Home Country when $n_f = 10$.

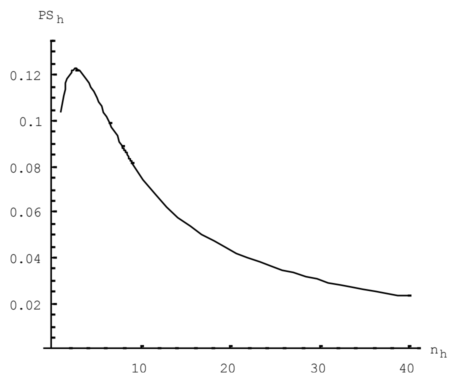

Table 1: Tariff and Subsidy at Each Timing

Timing (i)	Import tariff (t^i)	Export subsidy (s^i)
S	$\frac{1+(2+c+(1-2c)n_f)n_h+(1+2c-cn_f)n_h^2+cn_h^3}{(n_h+1)((n_h+1)^2+n_f(n_h+2))}$	$\frac{(n_h-n_f+1)(1+2cn_h+cn_h^2)}{(n_h+1)((n_h+1)^2+n_f(n_h+2))}$
H	$\frac{(2-c)n_h+1}{(2n_h+3)(n_h+1)}$	$\frac{(n_h-n_f+1)(1+2cn_h+cn_h^2)}{n_f(2n_h+3)(n_h+1)}$
F	$\frac{1+(4+(1-2c)n_f)n_h+(5-(2n_f-1)c)n_h^2+(2-(n_f-1)c)cn_h^3+cn_h^4}{2(n_h+1)^2(n_f+(n_h+1)^2)}$	$-\frac{n_f(1+2cn_h+cn_h^2)}{2(n_h+1)^2(n_f+(n_h+1)^2)}$

Table 2: Welfare of Home and Foreign Countries at Each Timing

Timing (i)	Welfare of Home Country ($W_h{}^i$)
S	$\frac{(1-c)^2 n_h(n_h+1)^3\left(n_h+2\right)+2n_f(n_h+1)^2(1+(3-4c+4c^2)n_h+(1-2c+4c^2)n_h{}^2+c^2n_h{}^3)+n_f{}^2(1+(7-12c+8c^2)n_h+(5-10c+8c^2)n_h{}^2+(1-2c+2c^2)n_h{}^3}{2(n_h+1)((n_h+1)^2+n_f(n_h+2)^2)}$
H	$\frac{1+(5-8c+6c^2)n_h+(2-4c+5c^2)n_h{}^2+c^2n_h{}^3}{2(n_h+1)(2n_h+3)}$
F	$\frac{4(1-c)n_h(n_h+1)^4(n_h+2)+n_f(n_f+2(n_h+1)^2)(1+4(2-3c+2c^2)n_h+2(2-3c+4c^2)n_h{}^2+c^2n_h{}^3(n_h+4)}{8(n_h+1)^2(n_f+(n_h+1)^2)^2}$
	Welfare of Foreign Country ($W_f{}^i$)
S	$\frac{n_f{}^2(1+cn_h(n_h+2))^2}{2(n_h+1)((n_h+1)^2+n_f(n_h+2)^2)}$
H	$\frac{(1+cn_h(n_h+2))^2}{(n_h+1)(2n_h+3)^2}$
F	$\frac{n_f(1+cn_h(n_h+2))^2}{4(n_h+1)^2(n_f+(n_h+1)^2}$