LEVERAGING AMARTYA SEN'S CAPABILITY APPROACH FOR ADVANCING MATHEMATICS TEACHER DEVELOPMENT: A DATA-DRIVEN PERSPECTIVE

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ABSTRACT

This study utilizes Amartya Sen's Capability Approach to evaluate and enhance the mathematical skills of teachers. The primary objective is to identify deficiencies in teachers' mathematical knowledge and instructional skills. A quantitative methodology was employed to analyze diagnostic assessments administered to 300 ninth-grade mathematics teachers from the Free State province in South Africa, revealing significant inadequacies in areas such as the sequence of operations, the distributive law, and the integration of mathematical concepts. The Capability Approach reframes these deficiencies as limitations in teachers' capabilities rather than mere knowledge gaps. The findings suggest the need for targeted professional development programs designed to improve both mathematical comprehension and instructional abilities. These programs, grounded in Sen's approach, aim to expand teachers' capabilities, thereby enhancing their effectiveness in the classroom. The study recommends further longitudinal and comparative research to evaluate the impact of these capability-enhanced professional development initiatives.

Keywords: Capability approach, teacher professional development, mathematical competency, pedagogical effectiveness, educational assessment.

1. Introduction

Mathematics plays a crucial role in education, significantly influencing both students' academic performance and teachers' professional growth (Egara & Mosimege, 2023e, 2023b, 2023a, 2024b, 2024a, 2023c, 2023d, 2024c). Effective mathematics education is fundamental not only for developing students' analytical and problem-solving skills (Egara et al., 2023; Mosimege & Egara, 2023b, 2023a; Okeke et al., 2022, 2023; Osakwe et al., 2022, 2023) but also for preparing them for various professional fields (Egara & Mosia, 2024; Mosia & Egara, 2024a,2024b; Mosimege & Egara, 2022). For teachers, continuous improvement in teaching methodologies and subject matter expertise is essential to meet the evolving educational standards and diverse student needs.

This study explores the application of Amartya Sen's Capability Approach in mathematics teachers' professional development. Continuous Professional Development (CPD) literature emphasizes key aspects such as self-improvement and lifelong learning. Scholars like Wahjusaputri et al. (2023) highlight critical drivers of CPD, notably self-assessment and professional profiling. Their work underscores the importance of examining how CPD programs foster self-assessment and professional profiling, which are essential for meaningful professional growth. Shi (2022) supports this notion, placing emphasis on the intrinsic motivation for personal and professional advancement. Both perspectives converge on empowering teachers to take control of their professional development.

This paper examines how the Capability Approach can provide a normative theoretical lens to address the challenges of mathematics teachers' professional development. The Capability Approach offers a broader understanding of professional development beyond pedagogical and subject matter competencies. It considers capacity development as an intersection of human development, agency, well-being, and freedom, as articulated by Walker (2006). This approach emphasizes self-awareness, critical reflection, and intrinsic motivation, which can help teachers understand how professional development contributes to their continuous growth. While the subject of teachers' professional development has garnered interest from numerous scholars, the literature reviewed in this paper reveals that many existing models primarily aim to improve student grades in mathematics. Although this is important, the Capability Approach provides a more comprehensive perspective. It enables us to ask, as Walker (2006) does, how CPD enables mathematics teachers "to do and to be." Although Walker originally posed these questions about the value of education, this paper argues that they are equally relevant for CPD programs.

The ideal starting point for CPD might actually be what others consider an end goal: utilizing data to guide staff capacity development. Targeted CPD programs can address specific needs and opportunities identified through data analysis. Naila's (2022) evaluation of CPD activities supports the emphasis on evidence-based analysis for staff capacity development. Additionally, Yuso's (2023) stages of professional growth align with Walker's (2006) questions on how CPD programs enable teachers "to do and to be." Recent literature underscores the importance of CPD even during significant disruptions, such as the shift to online CPD platforms during the COVID-19 pandemic noted by Sakulprasertsri et al. (2022). This shift aligns with our focus on the Capability Approach, exploring the quality of opportunities provided for teachers to respond to changing circumstances and utilize resources for growth and development.

The literature reviewed in this paper (Hidayah et al., 2022; Supriyono, 2019) illuminates the elements and methods that enable teachers to continuously grow professionally. It highlights the importance of ongoing professional development, emphasizing self-improvement, self-awareness, critical reflection, intrinsic motivation, evidence-based analysis, systematic knowledge acquisition, and adaptability to changing contexts (Alves & Cidrão, 2021; Adhikari, 2021; Pallotta & Bondani, 2022; Shi et al., 2022). By considering these factors, scholars (Maamin et al., 2020) argue that teachers can implement meaningful and impactful CPD activities that foster their growth and ultimately improve educational performance and effectiveness in the classroom. Many scholars (Burns, 2023; Ahmad, 2023; Yoon, 2021; Velástegui, 2020; Saito, 2003) have utilized Amartya Sen's Capability Approach in various contexts and fields, arguing that it helps understand and address complex societal concerns. This normative approach emphasizes personal capacities and freedoms in judging welfare and progress, aligning with this paper's focus on empowerment and staff capacity development. Yoon (2021) notes that the Capability Approach prioritizes opportunities for individuals to achieve their life goals and well-being, which resonates with our focus on addressing power imbalances and systemic inequalities in teaching and learning, particularly in mathematics education. The Capability Approach directs educational policymakers toward fairness and social justice. Administrators can develop and implement strategies that enhance staff development and uphold justice and diversity. By improving educators' capacity and opportunities, the Capability Approach fosters a more equitable educational landscape, examining staff needs and potential beyond standard performance indicators.

2. Theoretical Bases

In this study, the Capability Approach serves as the primary theoretical framework, providing a comprehensive lens to examine and enhance the professional development of mathematics teachers. Developed by Amartya Sen in the 1980s, the Capability Approach emphasizes expanding individuals' capabilities—what they are able to do and be—beyond mere economic or material outcomes. This framework prioritizes capabilities, functionings, agency, well-being, and freedom, offering a holistic perspective on professional growth. To operationalize this approach in the context of mathematics teachers' professional development, a data-driven methodology is integrated. This involves systematically collecting and analyzing data to inform decisions and strategies. Key components include self-assessment, professional profiling, and evidence-based interventions, all aimed at personalizing and enhancing CPD programs based on teachers' specific needs and development goals. Complementing the Capability Approach, Self-Determination Theory (SDT) is incorporated to address the intrinsic motivation and psychological needs of teachers. SDT, developed by Edward Deci and Richard Ryan in 1985, emphasizes autonomy, competence, and relatedness—factors that are crucial for fostering an environment where teachers feel motivated and empowered to pursue their professional development.

Additionally, principles from Adult Learning Theory, introduced by Malcolm Knowles in 1968, are utilized to ensure that CPD programs are tailored to the specific characteristics of adult learners. This includes fostering self-directed learning, leveraging teachers' experiences as valuable learning resources, and focusing on practical application. Finally, Transformative Learning Theory, developed by Jack Mezirow in 1978, is integrated to highlight the importance of critical reflection and perspective transformation. This theory supports the notion that professional development should not only enhance teachers' skills but also encourage them to critically examine their beliefs and practices, leading to meaningful and lasting change. In conclusion, the integration of the Capability Approach with these complementary theories provides a robust and multidimensional theoretical framework. This framework emphasizes the holistic development of mathematics teachers, focusing on expanding their capabilities, fostering intrinsic motivation, addressing the needs of adult learners, and encouraging transformative learning. Through this comprehensive approach, the study aims to promote a more personalized, effective, and empowering professional development experience for mathematics teachers.

The primary objectives of this study were twofold. Firstly, it aimed to enhance educators' mathematical knowledge and skills, specifically in the areas of numbers, operations, and relationships. This focus was intended to directly address the core competencies required for effective mathematics instruction at the ninth-grade level. Secondly, the study sought to align professional development efforts with the principles of the Capability Approach. This alignment emphasized the development of teachers' talents and the expansion of their instructional capabilities, going beyond mere content delivery to fostering a deeper, more holistic growth.

3. Methods

This study examined the abilities and knowledge of ninth-grade mathematics teachers, specifically focusing on the areas of numbers, operations, and relationships. Conducted in one of South Africa's nine provinces, the research employed a quantitative approach to gather and analyze data. A total of 300 mathematics teachers from five education districts in the Free State province participated in this study. The participants of this study were mathematics teachers who taught ninth grade in the Free State during the period of the

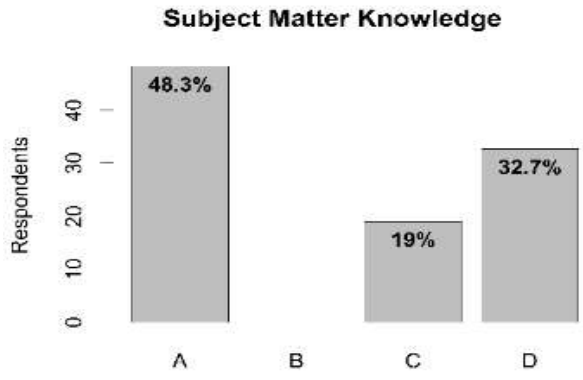
inquiry. Their selection aimed to provide a representative sample of educators responsible for teaching mathematics at this crucial educational level.

A key aspect of the data gathering process was the diagnostic assessment, meticulously developed by four mathematics teacher educators and other education professionals. These assessments were designed to test the teachers' mathematical skills related to teaching, with a particular emphasis on numbers, operations, and relationships. The primary goal of these diagnostic tests was to identify and address common errors and misconceptions among the teachers, thereby providing a clear picture of their current capabilities. The diagnostic assessments consisted of multiple-choice questions specifically crafted to reveal the teachers' mathematical knowledge and practical application skills. This method allowed for a thorough evaluation of the participants' competencies, highlighting both their strengths and areas needing improvement.

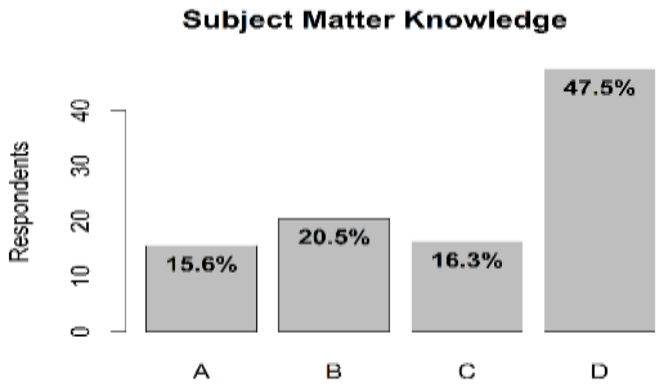
Each response from the diagnostic exams was analyzed individually from the outset. This detailed analysis aimed to uncover the participants' strengths and instructional skills, offering valuable insights into their current level of proficiency. By identifying specific areas where teachers excelled or encountered difficulties, the study aimed to provide targeted recommendations for professional development. The data was then broadly categorized to facilitate a comprehensive understanding of the teachers' potential for development and capacity expansion in the domains of numbers, operations, and relationships. This categorization process was crucial in identifying opportunities for enhancing the teachers' capabilities and tailoring professional development initiatives to meet their specific needs.

4. Results and Discussion

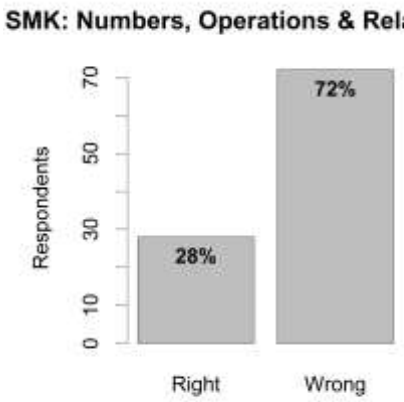
In the South African Curriculum Assessment Policy Statement, known as CAPS policy (DBE, 2011, p. 10), learners are expected to represent numbers in various forms and to convert between these representations. Additionally, they should be able to work flexibly with different number systems, solve a range of problems using various numbers, and accurately perform various operations. In response to these expectations, the diagnostic assessment items were grouped into constructs. The following graphic illustrates teachers' responses to these construct concerns, namely the subject matter knowledge.



This chart depicts participants' responses to a question about calculating the area of a triangle, where one side is described as being one-third of the length of another side. As presented in the chart, 48.3% of the participants selected option A, indicating that these participants interpret "one-third" as equivalent to 0.5. Additionally, 19% of the participants chose option C, suggesting that these participants equate "one-third" with 1/2. The remaining participants selected option D. Choosing option D seems to indicate an understanding that "one-third" can be correctly expressed as the fraction 1/3. The responses were entirely incorrect because "one-third" of a number should be mathematically expressed as 1/3 of that number. Selecting this option suggests that the Department of Basic Education has an opportunity to provide participants lack of understanding in translating word problem fractionations into mathematical expressions. It's important to note that none of the teachers chose option B, which was the correct answer. This misunderstanding was further compounded by subsequent responses related to the same construct.



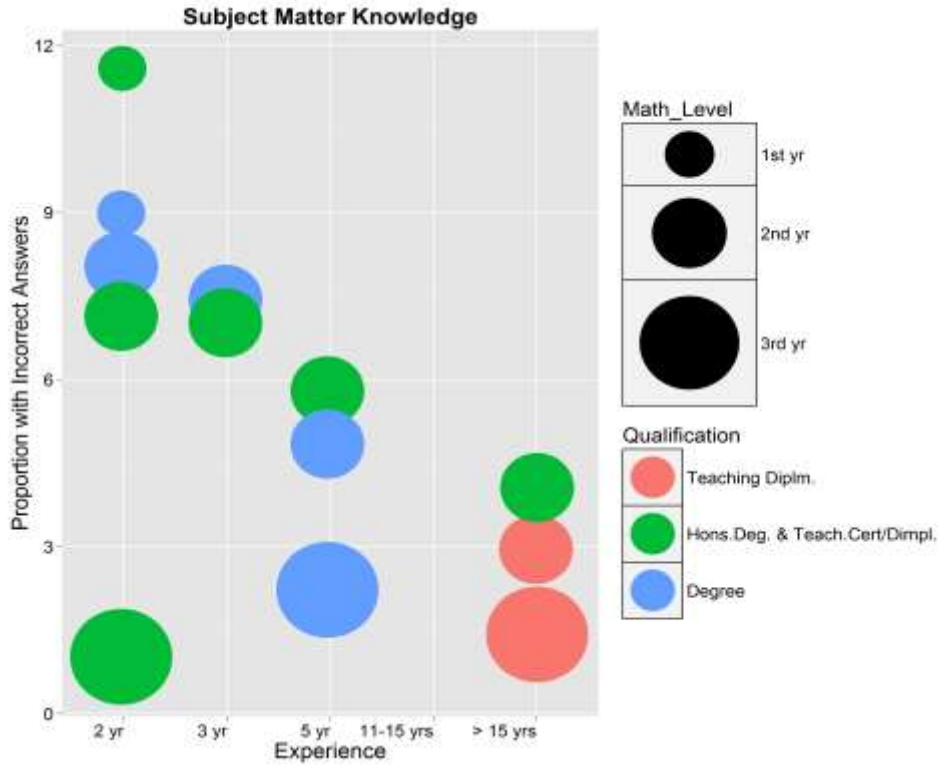
The chart illustrates teachers' responses to the task of simplifying the expression $\frac{4a-8}{4}$. It shows that 47.5% of participants selected option D, interpreting $\frac{4a-8}{4}$ as $a - 4$. This choice suggests a specific area for further development in understanding the distributive property of division over subtraction. Meanwhile, 20.5% chose option B, viewing the expression as $4a - 8 \div 4$, which indicates an opportunity to reinforce concepts regarding the order of operations. Interestingly, 15.6% of the respondents demonstrated a correct understanding of the expression. These insights into the various approaches and misconceptions can guide targeted instructional strategies to enhance subject matter knowledge.



The responses to questions about Subject Matter Knowledge in the context of the CAPS policy (2011) provide valuable insights into specific areas of numeracy where further development could be beneficial. For instance, while approximately 28% of teachers displayed proficiency in areas such as number representation in various forms and understanding multiplication operations, there is an opportunity for professional growth among the 72% in these aspects. The assessment focused on key skills like the order of operations in complex fractional expressions, the ability to simplify fractions by dividing multiple terms in the numerator by a single term in the denominator and translating fractional expressions in word problems to mathematical expressions. Particularly noteworthy was the challenge in applying these skills to practical problems, such as calculating the area of a triangle using fractions. These findings are not just reflective of areas needing improvement but also serve as a diagnostic tool to pinpoint specific topics where targeted training and resources could enhance understanding and teaching efficacy. Such diagnostics are crucial for tailoring professional development programs to meet the actual needs of educators, thereby aligning their competencies more closely with the expectations of the CAPS policy.

The Biographical Data Analysis

The provided figure offers a detailed visual representation of the correct responses, further segmented by the highest level of mathematics studied by the participants and their professional experience. This chart is instrumental in illustrating the relationship between teachers' qualifications, their level of mathematical education, and their work experience. The circles' colours signify credentials, while their sizes indicate math proficiency. The figure shows that instructors with higher circle placements perform worse in mathematics and credentials. The graphic shows the number of instructors that answered subject matter knowledge incorrectly. With two years of experience, instructors with level one mathematics and an honours degree had more inaccurate replies than those with bachelor's degrees. This visualisation shows trends and connections between academic background, teaching experience, and subject matter expertise, providing significant insights for targeted educational development and training initiatives.



The figure shows that even after taking postgraduate degrees, participants who only gained basic subject matter knowledge in their first year of undergraduate study still struggle with the topic. The depth and intensity of subject matter engagement in the early years of undergraduate study may build the groundwork for future teaching abilities. This pattern emphasises the need for thorough subject-matter instruction in early higher education. It proposes educational policy and curriculum development to increase undergraduate subject matter expertise, especially for those who want to teach and work in higher education. Education professionals' long-term effectiveness and confidence may improve with this method.

Capability Approach to Subject Matter Knowledge

The results of the study suggest that the level of subject matter engagement that occurs during the first few years of undergraduate education is quite important. In line with Sen's focus on the significance of foundational education in extending future capacities, this is consistent with his position. It is possible that a solid foundation for the development of successful teaching abilities might be provided by enhancing subject-matter learning at an earlier stage in higher education. For the purpose of producing educational programmes that are specifically tailored to the needs of the target audience, the visual portrayal of the link between academic background, teaching experience, and subject matter competence gives significant insights. Not only should these courses attempt to increase teachers' knowledge, but they should also aim to strengthen teachers' capacities to successfully use this information in their teaching practices. This is in keeping with Sen's capacity approach.

According to the findings of the study, enhancing students' subject-matter knowledge in undergraduate

courses, particularly for those who are interested in pursuing a career in education, can have a significant and lasting influence on the efficiency and self-assurance of education professionals. This reflects Sen's view that enhancing capabilities is a key factor in long-term development and empowerment. The results revealed by the study emphasise the need for educational policy and curriculum development focused on increasing subject matter expertise at the undergraduate level. Sen's approach would advocate for policies and curricula that not only impart knowledge but also foster the ability to use this knowledge effectively in real-world teaching scenarios.

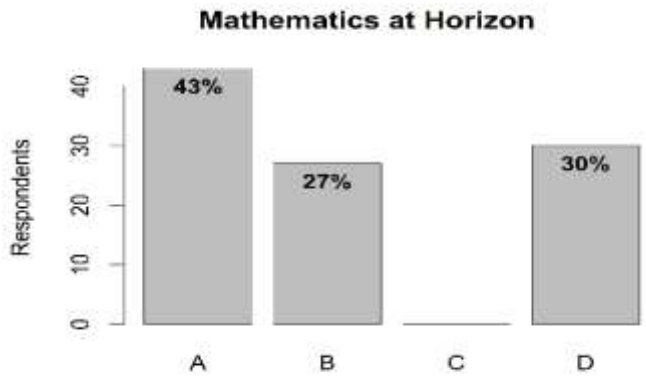
The diagnostic assessment shows that teachers need to improve their mathematics capabilities as framed by Amartya Sen. Focused and targeted continuous professional development can help mathematics teachers teach difficult mathematics concepts. This helps improve teachers' capabilities, and misunderstandings of concepts such as "one-third" as 0.5 or (4a-8)/4 as a-4 should be seen as opportunities to develop capacity. Teacher education can address these misconceptions to enhance teachers' mathematics comprehension and communication with learners (Gogoi et al., 2021). CAPS sets high standards for learners, and thus, teacher competencies must match these goals. This alignment demands instructors to improve their mathematical knowledge and pedagogical skills, which are needed to teach effectively (Hassine & Sghairi, 2021). Instructors' subject matter expertise and teaching skills affect students' ability. A teacher with a solid mathematics background and good teaching methods may help students improve their mathematics skills, which improves educational outcomes. The evaluation results can be used to diagnose teacher needs and thus, individualised professional development courses should follow the capability approach using these diagnostics. These initiatives should focus on improving instructors' skills based on their unique needs (Alaoui, 2019).

The research shows that instructors' qualifications, mathematical education, and professional experience are linked. It is noteworthy that higher degrees do not always lead to better arithmetic skills (Santi & Dwikurnaningsih, 2022; Mensah et al., 2022; Stahi-Hitin & Yarden, 2022). According to Sen's Capability Approach, qualifications do not always lead to teaching competencies. The study suggests that subject matter involvement in the first several years of college education is crucial. This supports Sen's emphasis on foundational education's role in future capacity. Enhancing subject-matter instruction sooner in higher education can lay the groundwork for solid teaching abilities (Elfadil & Ibrahim, 2022; Mbokazi et al., 2022). Visualising the relationship between academic background, teaching experience, and subject matter expertise helps create educational courses that meet target audience demands. According to Sen's philosophy, these courses should expand knowledge and strengthen instructors' abilities to use it in the classroom. According to the study, improving subject-matter knowledge in undergraduate courses, especially for those interested in teaching, can significantly and permanently improve education professionals' efficiency and self-confidence. Sen believes that building capacities is essential to long-term development and empowerment. During the study, the tendency revealed the need for educational policy and curricular reform to improve undergraduate subject matter competency for teacher education (Sin, 2023; Miyazaki, 2019).

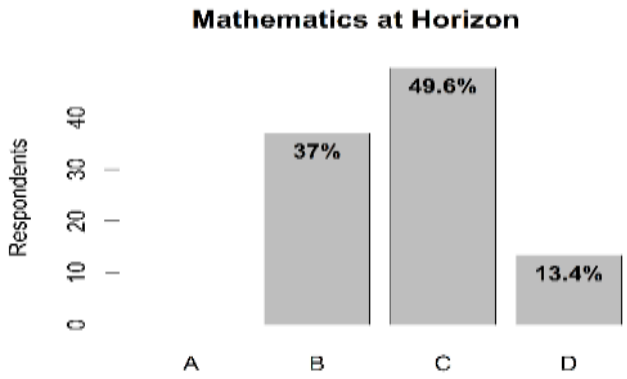
Mathematics at Horizon

The concept of "mathematics at the horizon" emphasises the importance of organising mathematical content as building blocks that cumulatively develop specific skills and knowledge (Jablonski & Ludwig, 2022). This approach aids teachers in considering the diverse prior knowledge and skills of learners, ensuring that the prerequisite knowledge needed for teaching a new concept is adequately addressed (Bekiroğlu &

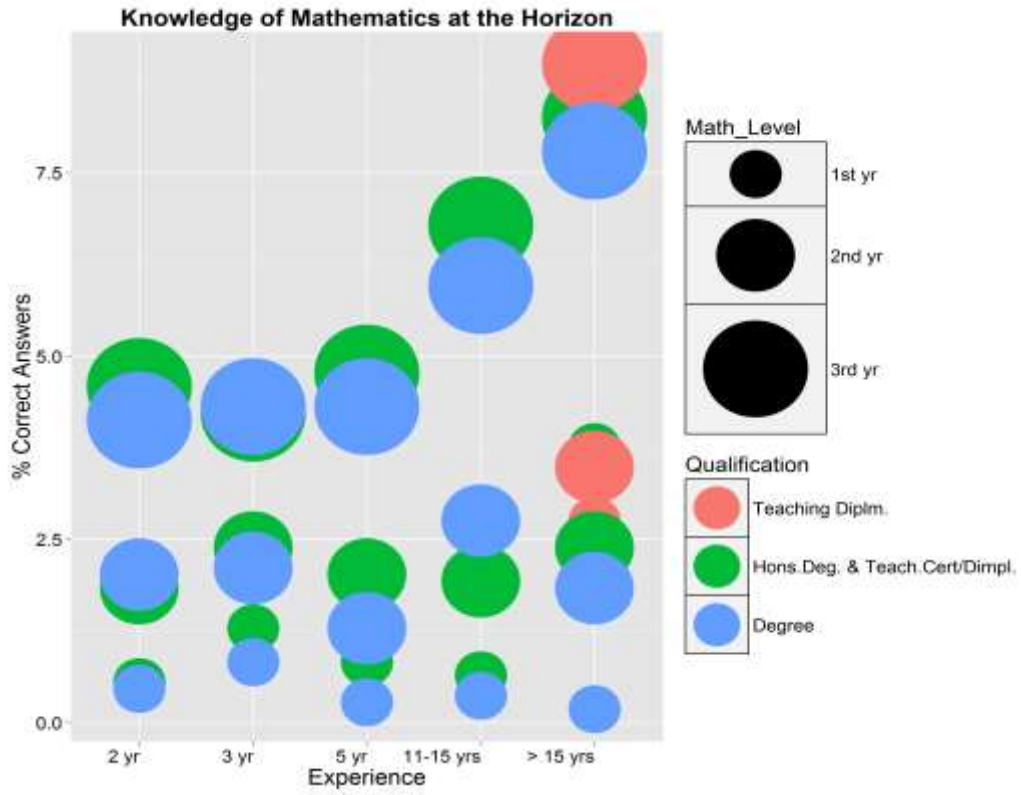
Güllühan, 2023). This aligns with learner-centred pedagogy, as advocated by Kumru et al., (2022), which posits that effective learning occurs when new information or knowledge about mathematical concepts is connected with students' existing knowledge. However, empirical evidence suggests that many teachers do not possess this anticipated level of mathematical horizon. For example, solving question 8 requires prior knowledge of the order of operations, distributive law, and principles of solving equations (applying the same operations on both sides of an equation). The study found that 11% of teachers lacked an understanding of the order of operations, as they did not recognise the need to apply the distributive law before collecting like terms and performing addition or subtraction. This indicates a challenge in transitioning from understanding the order of operations to applying the distributive law.



Participant replies show areas for mathematical improvement. Interestingly, 27% of participants did not apply the notion that equation modifications must be mirrored on the opposite side. This suggests a need to further study equation solving, notably the distributive law and equation balancing. This instruction might improve their capacity to assimilate mathematical concepts. 38% of participants were unable to link mathematical concepts such sequence of operations, distributive law, and problem-solving principles, indicating the need for more integrated and comprehensive mathematics teaching. Consider tactics that stress mathematical ideas' interconnectedness rather than handling them separately. 32% of teachers also pointed out a distributive law error in forgetting to multiply the second word in a bracket by the number outside of it. The distributive law was difficult to apply to equations for 16% of this sample and the sequence of operations for 16%. This identifies professional development options that expand awareness of how mathematical rules and concepts work in diverse circumstances. These needs can be addressed to improve instructors' mathematical ability and ensure a deeper comprehension of mathematical ideas.



By examining question 14 on triangular area, we may see where instructors thrive and where they need to improve. Using ratios to build connections between parameters, substituting them into the triangle area formula, and converting ratios to recurring decimal fractions, 37% of instructors showed high procedural fluency and conceptual grasp. Their capacity to integrate mathematical concepts and techniques shows a thorough understanding of mathematics at the horizon. Instead of the triangle height, 49.6% of teachers used the slant height. Geometry comprehension needs improvement, especially in realising that a triangle's height is perpendicular to its base. Geometric property and relationship-focused training or resources may address this misperception. Also, 13.4% of instructors had trouble using ratios to translate statements into maths. This underlines a need for professional development in reading and turning verbal problems into mathematical expressions, a key ability for maths teachers. Although many instructors can accurately calculate the area of a triangle, the comprehensive analysis identifies particular areas for professional improvement. The concentration on these areas can improve instructors' procedural fluency and conceptual comprehension, boosting their mathematics instruction.



Analysing instructors' performance by credentials, years of experience, and mathematics education reveals tendencies in their mathematical comprehension and instructional effectiveness. Two-year teachers who have taken third-year maths courses show a tendency. An Honours Degree plus Teaching Certificate or Diploma surpass a degree alone. This shows that greater degrees, especially when paired with specific teaching training, lead to a stronger comprehension of mathematics up ahead. This insight helps these teachers recognise students' different backgrounds and existing knowledge while teaching new topics.

Two-year instructors with second-year maths and first-year maths show a similar pattern. Those with an Honours Degree and Teaching Certificate or Diploma outperform those with merely a Degree. This steady pattern supports the assumption that improved academic and teaching certifications improve mathematics teaching ability. With experience, teachers with an Honours Degree and Teaching Certificate or Diploma in third-year mathematics demonstrate exponential improvement in mathematical comprehension and application. Even individuals with first and second-year mathematics education show an increased tendency, notably after five years, showing that actual teaching experience improves mathematical knowledge. Teachers having only a first- and second-year mathematics degree show a distinct pattern. Their maths skills vary at the horizon. Inconsistency may be caused by a lack of continual professional development, which can lead to complacency and reluctance to adjust to curricular changes. Mizell (2010: 18) observes that instructors without comprehensive professional development may not improve, which may impair student learning. Teachers with a Teaching Diploma outperform their counterparts in second- and third-year maths, supporting these findings. This suggests that concentrated teacher training and

ongoing professional development improve maths knowledge and teaching skills.

Related to Sen, "mathematics at the horizon" reflects his focus on individual abilities and backgrounds. Teachers must link new math concepts to student knowledge. This skill boosts teaching and student talents. Many participants don't understand mathematical principles like the distributive rule and sequence of operations, according to the study. These discrepancies imply knowledge gaps and limits in participants' ability to educate, according to Sen. Sen believes these disparities reflect restrictions on their ability to teach effectively and efficiently, not knowledge deficits. Since many participants struggle to cover a variety of mathematical themes in their lectures, a more holistic and integrated method to teaching mathematics is advised. This research suggests a broader strategy. Sen thought that to develop one's talents, one must first grasp the subject and then apply it. Comprehensive research on geometry and problem translation instructors showed the need for tailored therapy to improve their skills. Accordingly, Sen emphasises overcoming personal constraints to develop universal skills. The study on teachers' credentials, experience, and mathematical knowledge showed how continual learning and professional growth may improve teaching skills. Higher degrees, teaching certificates, and teaching experience may increase mathematics knowledge and instruction, according to Sen's method.

The capacity approach emphasises well-being (Yoon, 2021), hence the research examined numerical operations and interpersonal obstacles to improve competence. Austin and Holloway (2022), and Eichsteller (2021) agree that enabling conditions foster competence. Teacher competences must be strengthened through professional development programmes, and policymakers should advocate for capability-based changes. Teachers' degrees, mathematics education, and job experience show the necessity for customised professional growth. In accordance with the CA's emphasis on enhancing individuals' capabilities and liberties (Yoon, 2021; Austin & Holloway, 2022), policy changes should encourage the creation of individualised professional development programmes that account for educators' needs and experiences. Instructor credentials, experience, and mathematical aptitude correlated, the study found. This means governments must ensure equitable resource distribution, collaborative learning environments, and regular evaluation and feedback. California-based adjustments highlight a full, individualised, and enabling approach to mathematics teacher competency, supporting the research and educational policy and teacher development literature.

The notion of "mathematics at the horizon" fits Sen's emphasis on considering people's diverse backgrounds and capabilities. Teachers must be able to connect new mathematical concepts to learners' existing knowledge, a skill that not only improves teaching effectiveness but also expands the learners' capabilities. The study shows that many participants lack a basic understanding of mathematical concepts like the distributive rule and sequence of operations. Sen believes these discrepancies imply restrictions in participants' ability to teach effectively and efficiently, not merely knowledge gaps. According to Sen's approach, these gaps are not just deficits in knowledge but are limitations in their capability to provide effective education. The fact that many participants struggle to combine a variety of mathematical subjects into their lectures suggests a more holistic and integrated approach to teaching mathematics. According to this research, a more holistic strategy is needed. Sen believed that developing individual skills required both a thorough comprehension of the subject and its practical application. A detailed study of teachers' performance in geometry and problem translation has shown the need for tailored treatments to improve teachers' skills. Sen emphasises the need to overcome individual constraints to develop universal skills, which fits with this idea. The study's findings on teachers' credentials, experience, and mathematical knowledge provide light on how continual learning and professional growth improve teaching skills. Sen's

method suggests that greater degrees, teaching certificates, and teaching experience can improve math knowledge and teaching.

The capability approach's focus on personal well-being (Yoon, 2021) led the research to analyse numerical operations and relationship difficulties as a chance to improve competence. Austin and Holloway (2022) and Eichsteller (2021) agree that enabling environments are crucial for competence development. Targeted professional development programmes that improve teacher competences are needed, and policymakers should advocate for capability-based changes. The link between instructors' degrees, mathematics education, and job experience emphasises the need for customised professional growth. According to the CA's focus on increasing individual skills and freedoms (Yoon, 2021; Austin & Holloway, 2022), policy changes should promote personalised professional development programmes that reflect educators' different needs and backgrounds. The study linked teacher credentials, experience, and math ability. Therefore, governments must ensure fair resource allocation and accessibility, collaborative and supportive learning environments, and regular evaluation and feedback.

These CA-based changes reinforce the study's results and educational policy and teacher development literature by highlighting a comprehensive, personalised, and enabling approach to mathematics teacher competency. This supports Velástegui (2020)'s argument for comprehensive teacher education in educational policy. Capability approach promotes cumulative skill and knowledge development with "mathematics at the horizon". The study's capacity gap supports a capability approach for social justice and clarity by integrating professional growth. This necessitates reevaluating professional development programmes to provide a full strategy that aligns with the CA's focus on interconnected competencies. Finally, the study's findings on degree and experience-based teacher performance underline the capability approach's emphasis on professional progress (Gorbunova et al., 2021; Ogunode et al., 2021; Dultra, 2022). This validates the capability approach's statements and illustrates how long-term professional development enhances teaching. Policy implications are clear: Skilled teachers need ongoing professional development.

5. Conclusions

This study applied Amartya Sen's Capability Approach to analyze and enhance the mathematical knowledge and instructional skills of ninth-grade mathematics teachers in the Free State province of South Africa. The diagnostic assessments revealed significant gaps in teachers' understanding of fundamental mathematical concepts, particularly in interpreting fractions, applying the order of operations, and understanding the distributive property. These findings highlight critical areas for targeted professional development. The Capability Approach, which emphasizes enhancing individuals' freedoms and abilities to live meaningful lives, provides a valuable framework for identifying and addressing teachers' mathematical and pedagogical inadequacies. By focusing on expanding teachers' capabilities, this approach not only aims to improve their mathematical knowledge but also their ability to effectively apply this knowledge in classroom settings.

This research underscores the importance of professional development courses that go beyond knowledge delivery. These courses should help teachers learn and apply mathematical principles in various educational contexts, following scientific teaching and learning principles.

The study's data-driven insights provide a basis for designing targeted professional development programs that directly address identified gaps. Furthermore, the relationship between teachers' qualifications, years of experience, and their mathematical competencies suggests the need for a nuanced approach to teacher training. Educational policies should support the creation of professional development programs tailored to the specific needs of mathematics teachers, promoting their long-term development and ultimately improving educational outcomes for students.

In conclusion, leveraging the Capability Approach for mathematics teacher development offers a comprehensive framework that aligns with the goals of the South African Curriculum Assessment Policy Statement. This study emphasizes the need for continuous and integrated professional development to build teachers' capabilities, thereby enhancing the quality of mathematics education in South Africa and supporting the broader goal of talent development for mental wellness, as advocated by Sen.

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