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PREDICTORS OF STUDENT SUCCESS IN MATHEMATICS: HIERARCHICAL BAYESIAN APPROACH

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ABSTRACT
*This study employs a mixed-effects model to investigate the determinants of mathematics success among first-year students at a South African university. By formulating a regression model, it explores the interaction between academic disciplines and demographic factors, modeling mathematics scores against fixed effects (e.g., Physical Sciences and English First Additional Language scores, Gender, and Age) and random effects (grouped by secondary school quintile and student postal code). Drawing from data collected from 1600 first-year students spanning various faculties and aged between 18 and 20, the analysis reveals a significant positive correlation (Corr: 0.646***) between mathematics and physical sciences competency, indicating mutual reinforcement between these subjects. However, while statistically significant, the correlation between English language proficiency and mathematics achievement was weak (Corr: 0.064*), challenging conventional assumptions about linguistic proficiency's importance in STEM education. Moreover, age did not emerge as a significant predictor of mathematics success, contrary to traditional beliefs about age-related improvements in academic performance. These findings underscore the value of integrated STEM education, emphasizing the need to reinforce concepts across disciplines to enhance overall student understanding and achievement. The implications suggest a reevaluation of educational priorities, shifting focus towards strengthening domain-specific skills and knowledge rather than overemphasizing linguistic proficiency. Additionally, the study advocates for educational strategies prioritizing cognitive readiness over chronological age and recommends fostering cross-disciplinary approaches in STEM education, addressing gender disparities, and conducting further research into the complex interplay between academic specialties and demographic variables.*

Keywords: *Mathematics success, predictors, hierarchical Bayesian approach, STEM education, academic disciplines, demographic factors.*

1. Introduction

In contemporary education, mathematics plays a pivotal role in shaping student success, significantly influencing both academic performance and professional growth for educators. Proficiency in mathematics is essential, as it serves as a foundational skill that impacts various aspects of students' futures, preparing them for numerous professional fields (Bui et al., 2021; Egara & Mosimege, 2023a, 2024a; Mosimege & Egara, 2022). Effective mathematics education is fundamental for cultivating students' analytical, critical thinking, and problem-solving skills, which are crucial for their overall academic and professional opportunities (Egara et al., 2023; Mosimege & Egara, 2023a, 2023b; Okeke et al., 2022, 2023a; Osakwe et al., 2022, 2023). Understanding the intricate dynamics that contribute to mathematics proficiency is paramount, as it not only enhances student success but also supports teachers' professional development (Egara & Mosimege, 2023a, 2023b, 2023c, 2023d, 2023e, 2024a, 2024b).

The journey towards mastering mathematics is influenced by a multitude of factors, including individual aptitude, instructional strategies, socio-economic backgrounds, and motivational factors (Egara et al., 2021; Mosimege et al., 2024; Nzeadibe et al., 2019, 2020, 2023; Okeke et al., 2023b; Pitsia et al., 2017). Recent educational research has focused on the interplay between socioeconomic variables and mathematical aptitude. In accordance with this focus, Abalde and Oco (2023) state that teacher interactions shape student cognition and behavior, thereby influencing learning outcomes. Conversely, Singh et al. (2002) found a weak association between students' interest in mathematics and their performance. Their research on factors influencing student performance in mathematics, including motivation, self-efficacy, and teacher enthusiasm, revealed that teachers' enthusiasm boosted Grade 7 learners' interest in mathematics. This juxtaposition of findings underscores the complexity of educational processes and the necessity for various pedagogical strategies.

Contributing to this dialogue, Eduwem et al. (2017) cite Sikwari (2014), who found a strong correlation between academic achievement and motivation, noting that female students exhibit higher motivation levels. Tella (2007) further corroborates these findings, emphasizing the impact of motivation on secondary school mathematics achievement, with observed gender differences. Additionally, Eduwem et al. (2017) address test anxiety, a phenomenon identified by Tobias (1993), which can impair mathematical performance. They suggest that educational interventions could mitigate this anxiety, emphasizing the importance of addressing psychological obstacles alongside motivational factors.

Torres et al. (2020) investigated variables influencing secondary mathematics success and noted that unsustainable external motivators may lead to short-term expectations but limited academic gains. They highlight the significance of intrinsic motivation in driving students' attitudes and achievements in mathematics. Hasanah et al. (2022) introduced a unique method using Python for predicting math students' academic achievement, achieving high accuracy through linear regression. This approach aligns with the findings of Setyoningrum and Rahimma (2022), highlighting the potential of data-driven methods in academic research.

In contrast to qualitative approaches, Iddrisu et al. (2023) examine how mathematics class attention affects academic success. Qualitative research (Iddrisu et al., 2023) on Gang South kids' low academic achievement linked class attentiveness to mathematics performance. Quantitative research (Iddrisu et al., 2023) also found that students' personal discipline and concentration in mathematics sessions predict their interest in the subject and academic achievement. This study concluded that disciplined and attentive mathematics students perform better.

Despite the plethora of studies on the factors influencing math performance indicated by the studies referenced above and others, there remains a vacuum in understanding how subject achievement influences or correlates with a student's math ability. This line of study is critical because it may reveal linkages and factors that determine a student's math abilities and achievements. Examining these linkages may provide insights for educators and policymakers in developing well-rounded and integrated educational programs. Thus, the current study seeks to answer the question: How do successes in subjects such as Physical Sciences and English as a First Additional Language, as well as demographic characteristics, predict students' mathematical achievement?

Although studies have examined various factors that affect math proficiency, less is known about how students' success in other subjects may affect their math performance especially using the Hierarchical Bayesian Model. The Hierarchical Bayesian Approach has emerged as a powerful analytical framework in educational research, particularly in studying the predictors of student success in mathematics. This approach enables researchers to account for the hierarchical structures within educational systems and simultaneously model the influence of multiple predictors on student outcomes (Yao et al., 2022). By integrating Bayesian techniques, researchers can incorporate prior knowledge, uncertainty, and variability into their models, resulting in more accurate and robust estimates of the factors impacting student performance (Kruschke & Vanpaemel, 2015). This research is significant because it may reveal disciplinary impacts on a student's mathematical skills and successes. Exploring these relationships may help educators and policymakers design pedagogical approaches. Therefore, the purpose of this study is to answer: How do successful outcomes in subjects like Physical Sciences and English First Additional Language, as well as socioeconomic and demographic characteristics, predict mathematics success?

2. Theoretical Framework

This study examines the predictors of student success in mathematics using a Hierarchical Bayesian approach. The theoretical framework integrates Hierarchical Bayesian Theory with relevant educational theories, offering a comprehensive perspective on the multifaceted nature of mathematical success among students.

Hierarchical Bayesian Theory

Hierarchical Bayesian Theory provides the primary framework for this research, offering a robust statistical approach to analyze complex educational data. This theory facilitates multi-level modeling, essential for assessing data organized hierarchically, such as students within classrooms and classrooms within schools. By accommodating the nested structure of educational settings, this approach allows for a detailed examination of both individual and contextual factors influencing mathematical success. A significant advantage of Bayesian methods is their ability to incorporate prior information or expert knowledge into the analysis. This feature enhances the estimation process, leading to more informed and precise conclusions. Furthermore, Bayesian analysis provides posterior distributions that offer a probabilistic interpretation of the results, reflecting the uncertainty and variability in the estimates of predictors. By applying Hierarchical Bayesian methods, this study aims to uncover the nuanced relationships between various predictors and student success in mathematics, offering a more precise and reliable understanding than traditional statistical methods.

Educational Theories

To contextualize the findings within the educational landscape, the study draws upon several educational theories that explain how different factors contribute to student success in mathematics.

Constructivist Theory: Constructivist Theory, proposed by Jean Piaget (1972) and further developed by Jerome Bruner (1960), posits that learners construct new knowledge by building upon their existing cognitive structures, emphasizing the importance of prior knowledge and active engagement in the learning process. This theory helps explain how students' prior achievements and learning experiences influence their current success in mathematics.

Motivation Theories: Motivation theories, such as Self-Determination Theory (SDT) by Deci and Ryan (1985) and Expectancy-Value Theory by Eccles et al. (1983), highlight the role of intrinsic and extrinsic motivation in educational outcomes. SDT focuses on the need for autonomy, competence, and relatedness, while Expectancy-Value Theory emphasizes the importance of students' expectations for success and the value they place on the task. These theories provide insight into how students' attitudes, beliefs, and motivation levels impact their mathematical performance.

Sociocultural Theory: Sociocultural Theory, developed by Lev Vygotsky (1978), underscores the significance of social interactions and cultural contexts in the learning process. This theory suggests that learning is inherently social and greatly influenced by interactions with peers, teachers, and the broader community. Understanding the sociocultural context helps identify how classroom dynamics and school environments affect student success in mathematics.

Cognitive Load Theory: Cognitive Load Theory, proposed by John Sweller (1988), examines the mental effort required to process new information and solve problems, suggesting that excessive cognitive load can hinder learning and performance. This theory is particularly relevant for understanding how instructional strategies and task complexity influence students' ability to succeed in mathematics.

By integrating Hierarchical Bayesian Theory with these educational theories, this study adopts a comprehensive approach to examining the predictors of student success in mathematics. The hierarchical Bayesian model allows for the precise estimation of the effects of various predictors while accounting for the nested structure of educational data. Simultaneously, educational theories provide a rich interpretive framework to understand the underlying mechanisms driving these effects.

In this study, data was collected on multiple levels, including individual student characteristics (e.g., prior achievement, motivation), classroom variables (e.g., teaching methods, classroom environment), and school-level factors (e.g., resources, policies). The hierarchical Bayesian model analyzed the data, estimating the effects of predictors at each level. The findings from the Bayesian analysis were interpreted through the lens of educational theories, offering insights into how and why certain factors influenced student success in mathematics. This integrative approach provides a holistic understanding of the predictors of mathematical success, contributing to both statistical methodology and educational practice.

3. Related Studies

Researchers have shown interest in exploring the factors that impact student success in mathematics. For instance, Benő (2022) delved into how socio-political elements affect students' emotional characteristics like self-perception, confidence, interest, curiosity and outlook. The research revealed that various socio-political settings had influences on student growth. It was also noted that emotional aspects such as self-perception and attitudes significantly influenced math performance. Similarly, Ouda et al. (2021) carried out a study to examine the relationship between the level of mathematics courses taken and the academic performance of 12th grade students from marginalized ethnic and linguistic backgrounds in urban schools. They utilized hierarchical linear models to identify correlation between students' English proficiency, ethnic backgrounds, participation, in college mathematics classes, school environment and math accomplishments. This research underscored the interconnectedness of students' backgrounds, academic choices, urban setting dynamics and educational outcomes.

Some researchers, like Prinsloo et al. (2018) delved deeper into languages. Explored how language differences relate to success and vary across different subjects. Through regression analysis their study showed that language equivalence had an impact on science achievement. They concluded that individual student factors play a role in success with language equivalence particularly influencing Science performance. In a study by Farrell (2011) supporting this idea, it was argued that proficiency in both languages in an environment could impact a student's cognitive abilities and academic performance. Farrell's research involved examining the exam results of 1,262 13-year students, in Physics and Mathematics compared to their Maltese performance. The study findings seemed to align with the threshold hypothesis proposed by Jim Cummins indicating that students who excelled in both languages tended to perform in Physics and Mathematics exams. It is also around that same time that a study by Gilleece et al. (2010) examined learners and school background characteristics associated with low and high achievement in mathematics and science on the Programme for International Student Assessment. Based on the results of a multilevel multinomial model of achievement for each domain, findings indicated that a greater number of the variables examined are associated with low rather than high achievement. At learner level, home language, intention to leave school early, socioeconomic status, grade level, cultural capital, and books in the home are significantly associated with achievement in mathematics and science.

Similarly, Kunnari et al. (2023) conducted a study focusing on the transition, from school to university. Their research explored how matriculation examination (ME) results correlated with university performance, grades and credit accumulation among educational sciences students pursuing a three-year bachelor's degree from 2007 to 2017. The study findings revealed that ME results accounted for 15% of the variation in student success and then 1% in study progress. Interestingly student achievements in subjects like mother tongue, mathematics, psychology, and health education during the ME exam showed associations with success over the three years of study. However, the relationship between ME results and third year progress were more nuanced, with differences observed compared to years. Additionally, Hu et al. (2018) research is noteworthy for its analysis of math achievement across 51 countries based on large scale assessments and the influence of culture, on students' performance. Through the application of Two level hierarchical linear modelling the study findings highlighted the influence of culture, on students' mathematical success. Precisely it was noted that 23.89% of variations in math performance across countries can be attributed to culture with long term orientation demonstrating a notably positive correlation with achievements, in mathematics. Consequently, this study tends to use the Hierarchical Bayesian approach to understand how successful outcomes in subjects like Physical Sciences and English First Additional Language, as well as socioeconomic and demographic characteristics, predict mathematics success.

Hierarchical Bayesian models are versatile and have roots in many study fields. These models thrive in complicated, multilayer data structures, enabling meaningful analysis in several disciplines (Gelman, 2006; Bakker & Heskes, 2003; Vehtari et al., 2017). Yao et al. (2022) formalised optimization-based stacking into a Bayesian hierarchical model, a major change. This methodological innovation integrates generative and algorithmic modelling, as described by Breiman (2001) and Wolpert (1992), improving predictive model interpretability and theoretical robustness in diverse areas (Yao et al., 2022). Bunnin and Smith (2021) demonstrate Bayesian hierarchical models' use in criminal investigations. They analyse visible data, preparation actions, and assault modalities using organised domain expertise. This approach has been used in law enforcement and security applications such as terrorist cell network analysis, bio-terrorist attack detection, and counterterrorism measures (Bunnin & Smith, 2021). Kruschke and Vanpaemel (2015) use hierarchical Bayesian estimation to cognitive modelling.

Hierarchical Bayesian models can replace self-report techniques in psychological and cognitive investigations, as shown by Treat et al. (2002) in (Kruschke & Vanpaemel, 2015). König et al. (2023) provides a full Bayesian hierarchical response time modelling lesson that advances methodology. They use Stan and its R package rstan to describe and estimate models using Bayesian hierarchical techniques, demonstrating the usefulness of Hamiltonian Markov Chain Monte Carlo (MCMC) sampling (König et al., 2023). Researchers seeking advanced statistical modelling advice can benefit from this training resource. In 2020, Wang and Lan proposed a Bayesian model for semi-supervised multi-view anomaly detection, a major data science advance. They use a generative model for normal data, presuming a single latent component generates several perspectives of a normal instance. Assigning priors on model parameters and then over these priors deepens this hierarchical Bayesian model and allows for nuanced data interpretation (Wang & Lan, 2020).

Despite the versatile applications and methodological advancements of Hierarchical Bayesian models across various fields, there is a notable gap in their application to educational research, particularly in analyzing the predictors of student success in mathematics. While these models have been extensively utilized in fields such as criminal investigations (Bunnin & Smith, 2021), cognitive modeling (Kruschke & Vanpaemel, 2015), and anomaly detection in data science (Wang & Lan, 2020), their potential in educational settings remains underexplored. Existing research primarily focuses on the technical and methodological aspects of Hierarchical Bayesian models (Gelman, 2006; Bakker & Heskes, 2003; Vehtari et al., 2017), with limited emphasis on their practical application in understanding educational outcomes.

This study aims to fill this gap by leveraging the robust analytical capabilities of Hierarchical Bayesian models to investigate the predictors of student success in mathematics. By applying this advanced statistical approach to educational data, the study seeks to provide a more nuanced understanding of how various academic and demographic factors interact to influence mathematical achievement among first-year university students. This approach will not only enhance the precision and reliability of the findings but also offer valuable insights for educators and policymakers in designing effective strategies to improve student performance in mathematics.

Therefore, the main purpose of the study is to investigate and identify the key academic and demographic factors that predict mathematics success among first-year students at a South African university. The study employs a Hierarchical Bayesian approach to analyze the interaction between these factors, aiming to provide a more precise and reliable understanding of the predictors of student success in mathematics. Specifically, it seeks to determine how achievements in subjects such as Physical Sciences and English First Additional Language, as well as demographic characteristics, influence students' mathematical performance.

4. Methods

The problem is formulated in mathematical notation using standard symbols and notations for statistical modelling, particularly for a mixed-effects model. The problem was framed to represent a linear mixed effects model where the outcome variable "mathematics" is being modelled as a function of fixed effects (Physical Sciences Score, English First Additional Language Score, Gender, and Age) and random effects (grouped by secondary school quintile and student postal code).

The regression model is presented below:

$$\mathrm{maths}_i = \beta_0 + \beta_1 \mathrm{Science}_i + \beta_2 \mathrm{English}_i + \beta_3 \mathrm{Gender}_i + \beta_4 \mathrm{Age}_i + \beta_5 u_{j[i]} + \beta_6 v_{k[i]} + \epsilon_i$$

where:

maths_i represents the mathematics score of the i^{th} student.

$\beta_0, \beta_1, \beta_2, \beta_3$ and β_4 are the fixed effect coefficients for the Intercept, Physical Sciences Score, English First Additional Language Score, Gender, and Age, respectively.

$u_{j[i]}$ is the random effect for the jth school quintile to which the i^{th} student belongs.

- $v_{k[i]}$ is the random effect for the k^{th} postal code to which the ith student belongs.
- To be more explicit, the Bayesian hierarchical model is defined as follows:

- $y_i \sim \text{Normal}(\mu_i, \sigma^2)$,
- $\mu_i = \beta_0 + \beta_1 \text{Science}_i + \beta_2 \text{English}_i + \beta_3 \text{Gender}_i + \beta_4 \text{Age}_i + \beta_5 u_{j[i]} + \beta_6 v_{k[i]}$,
- $u_j \sim \text{Normal}(0, \tau_u^2)$,
- $v_k \sim \text{Normal}(0, \tau_v^2)$,
- With priors,
- $\beta_{0:6} \sim \text{Normal}(0, 5)$,
- $\tau_u, \tau_v, \sigma \sim \text{Normal}(0,10)$.

The research involved 1600 first-year student at a South African university. The sampling process aimed to have a mix of students from Faculty of Economic and Management Sciences, Education and Natural and Applied Sciences however, this were all the first-year students that took Mathematics, Physical Science and English as subjects from the school. The participants were aged between 18 and 20 which's common for first year university students in South Africa.

5. Results

A thorough pairs plot study illuminates the complex relationship between academic specialties and demographic factors. The substantial positive correlation (Corr: 0.646***) between mathematics and physical sciences competency shows a substantive relationship and highlights common abilities between the two subjects. A strong relationship between physical and mathematics has attracted academic discussion on how improvements in one may improve performance in the other. The link between mathematical success and English language ability is statistically significant (Corr: 0.064*), albeit weak. This suggests that while linguistic proficiency is important in mathematical cognition, domain-specific knowledge and abilities may be more important.

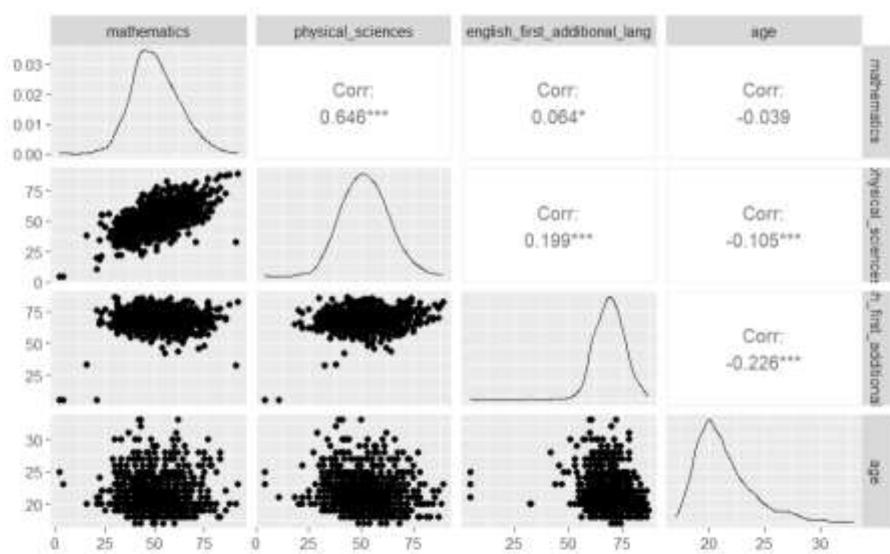

Figure 1: Pairs Plot Analysis: Relationship Between Academic Specialties and Demographic Factors.

The lack of a significant correlation between mathematics ability and age (Corr: -0.039) contradicts maturity and learning presumptions, suggesting that chronological development does not inevitably improve mathematical accomplishment. This unexpected finding joins a growing scientific debate over age's impact on secondary schooling. The significant negative correlation between English language and physical sciences with age (Corr: -0.226*** and Corr: -0.105***, respectively) advances this discussion, suggesting generational changes in pedagogical effectiveness or curriculum receptiveness. The diagonal distributions show additional educational phenomena, such as the skewness in mathematics and physical sciences scores, which suggests a concentration of low performers and a tail of high performers, suggesting student abilities or instructional quality. The more symmetric distribution of English language scores may indicate standardised competence, while the age distribution's skewness reflects a younger cohort in the dataset.

Central to findings from the above Bayesian formalising is the pronounced variability in mathematics achievement across different secondary school quintiles, as highlighted by the random effect in our model. This unpredictability, reflected by a 2.79 standard deviation, suggests a systemic educational deficit. The broad credible range of 0.83 to 6.98 highlights the variability of educational experiences among schools and raises important concerns regarding educational fairness and resource allocation. It shows that a school's quintile might be a proxy for unmeasured elements including socioeconomic status, educational resources, and institutional support systems, which greatly affect math ability. This was supported by the random effect linked with student postal code, which had a standard deviation of 2.49, confirming that geographical and contextual variables shape educational performance. Physical sciences competency increases mathematics success by 0.66 points per unit, confirming its connection.

Table 1: Parameter Estimates for Mixed-Effects Model

Parameter	Estimate	Est. Error	Lower 95% CI	Upper 95% CI	Rhat
Group-Level Effects					
secondary school quintile (intercept)	2.82	1.80	0.71	7.66	1.00
student postal code (intercept)	2.49	0.60	1.26	3.64	1.00
Population-Level Effects					
Intercept	22.82	4.10	14.57	30.74	1.00
Physical Sciences	0.66	0.03	0.61	0.71	1.00
English First Additional Lang	-0.06	0.04	-0.13	0.02	1.00
Gender1-famale	-3.29	0.62	-4.48	-2.11	1.00
Age	0.02	0.11	-0.20	0.23	1.00
Family Specific Parameters					
Sigma	8.56	0.22	8.14	9.01	1.00

This highlights the necessity for an integrated STEM education, where reinforcing concepts across topics may improve student understanding and accomplishment. In contrast, the influence of English language proficiency, particularly in the context of English as a First Additional Language, on mathematics performance is intriguingly marginal. The minimal negative effect, estimated at -0.05, within a credible interval that includes zero, suggests that language proficiency, contrary to popular belief, may not play a significant role in mathematics achievement. This finding invites a re-examination of the emphasis placed on language proficiency in mathematics education and potentially points to the universality of mathematical concepts beyond linguistic barriers. However, the revelation of a gender gap in mathematics achievement, indicated by the negative impact of the gender 1 category (females), is a cause for concern.

The decrease of 3.40 points in mathematics scores relative to the baseline (males) is a stark indicator of existing gender disparities in STEM fields. This finding echoes the broader discourse on gender inequality in education, emphasising the need for gender-responsive teaching practices, curricular reforms, and policy interventions that actively work towards closing this gap. Interestingly, age appears to be a less significant factor in our model, suggesting that within the secondary school age range, developmental or age-related differences might not be as influential on mathematical proficiency as other variables. This outcome aligns with the understanding that individual cognitive and learning differences, rather than chronological age per se, are more critical determinants of academic success in secondary education. The model's residual standard deviation (sigma) of 8.89, indicating the variability of scores around the predicted values, reminds us of the inherent complexity and multifactorial nature of educational achievement. It underscores the limitations of any analytical model in fully capturing the vast array of influences on academic performance.

6. Discussions

The results of the current study revealed that there is a positive correlation (Corr: 0.646***) between competencies in mathematics and physical sciences. This suggests that an improvement in one may result in an improved performance on the others. The substantial positive correlation observed between mathematics and physical sciences competency reaffirms the overlap in skills and knowledge required for success in these subjects, echoing the findings of Torres et al. (2020) and Hasanah et al. (2022). This suggests that an integrated STEM education approach, as advocated by previous scholars, may indeed enhance student understanding and accomplishment across academic specialties.

Despite a statistically significant correlation (Corr: 0.064*), the weak correlation between English language proficiency and mathematics achievement challenges common assumptions. This finding suggests that language proficiency may not be as critical to mathematical understanding as previously believed, refuting previous research by Farrell (2011) supporting the 'threshold hypothesis' indicating a strong link between proficiency in both languages and success in Physics and Mathematics exams. However, in this study, English language skills showed minimal impact on mathematical performance, sparking discussions on the relative importance of language skills in STEM education and whether the focus should shift towards developing domain-specific skills and knowledge. Moreover, the marginal effect observed implies that domain-specific knowledge and abilities may be more critical determinants of mathematical success, aligning with the findings of Prinsloo et al. (2020) and Farrell (2011). This suggests that while linguistic proficiency may play a role in mathematical cognition, its impact is relatively minor, challenging the notion that language skills significantly influence mathematical achievement.

The gender gap in mathematics achievement revealed in this study echoes the broader discourse on gender inequality in education, as highlighted by previous scholars such as Eduwem et al. (2017) and Tella (2007). The significant decrease in mathematics scores among female students relative to their male counterparts underscores the urgent need for gender-responsive teaching practices and policy interventions aimed at narrowing this gap, as advocated by these researchers.

Turning to age as a possible predictor of student success in mathematics, the current study found that, there was a minimal negative effect, estimated at 0.02, within a credible interval that includes zero, suggests that age, contrary to popular belief, may not play a significant role in mathematics achievement. The finding that age has a relatively minor impact on mathematical proficiency within the secondary school age range aligns with previous research by Kunnari et al. (2023) and Hu et al. (2018). This suggests that individual cognitive and learning differences may outweigh chronological age in influencing academic success, as proposed by these researchers.

The variability in mathematics achievement across different secondary school quintiles and geographical locations highlights systemic educational disparities, as discussed by Benő (2022) and Mosqueda et al. (2021). Factors such as socioeconomic status, educational resources, and institutional support systems significantly impact mathematical ability, as identified by these researchers, underscoring the need for equitable resource allocation and educational policies aimed at addressing these disparities.

Overall, this study contributes to the existing literature by providing empirical evidence supporting the complex interplay between academic specialties, demographic characteristics, and institutional contexts in shaping student success in mathematics. It emphasizes the importance of holistic approaches to education that address systemic inequalities and promote equitable access to high-quality learning experiences for all students.

7. Conclusion

In conclusion, the current study has substantiated the substantial positive correlation between competencies in mathematics and physical sciences, confirming a robust correlation. This finding aligns with existing research, which identifies a mutual reinforcement between these academic specialties, suggesting that an enhancement in one is likely to bolster performance in the other. The implication for educational strategy is clear: A cross-disciplinary approach that integrates these subjects could potentially elevate overall student performance in STEM fields.

Contrary to traditional expectations, our study also challenges the presumed importance of English language proficiency in mathematics achievement. Despite the statistical significance of the correlation, its weak magnitude questions the threshold hypothesis by Cummins, which posits that language mastery is crucial for success in technical subjects like mathematics and physics. The minimal impact of English language skills on mathematics observed in this study suggests a need for a shift in educational focus towards strengthening domain-specific skills and knowledge, rather than overemphasizing linguistic proficiency.

Furthermore, our analysis revealed that age does not significantly predict success in mathematics, with a minimal negative effect. This finding disrupts conventional beliefs about the progressive improvement in mathematical ability with age, thereby challenging the appropriateness of age-based curricular designs in education systems. This suggests that educational strategies and curricula might better serve students by focusing on cognitive and learning readiness rather than on chronological age.

Therefore, to enhance student success in mathematics and related disciplines, it is recommended to adopt interdisciplinary approaches that integrate mathematics with other STEM subjects. This cross-disciplinary approach fosters a deeper understanding of concepts and promotes overall student achievement.

Additionally, there should be a reassessment of the emphasis placed on English language proficiency in mathematics education, with a shift towards prioritizing domain-specific skills and knowledge. Individualized learning approaches tailored to students' diverse needs should be implemented, along with continuous professional development for educators to refine their pedagogical skills. Addressing systemic educational disparities through equitable resource allocation and inclusive policies is also essential for ensuring equal opportunities for all students.

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