International Journal of Applied Engineering & Technology

TWO-STAGE EVACUATION MODEL CONSIDERING UNCERTAINTY OF STORM TRACKS IN THE CARIBBEAN

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Abstract
Hurricanes cause a lot of damages if adequate protection measures are not taken in time. In the last decades the frequency and intensity of this natural phenomenon has increased a lot in the Caribbean area. So, it is important to be efficiently prepared for this class of events. Evacuation of populations in danger is usually done 48 hours in advance when the trajectory and intensity of the hurricane is still uncertain. In this study, we propose a two-stage stochastic model designed to address this uncertainty. The solution proposes the amount of people that should go from each population to each evacuation center. A tradeoff between saving human lives and economic efficiency is obtained. According to each scenario, the decision maker knows how many people are left behind and have to propose another safety alternative and how many were evacuated without need. An illustrative example based on simulated data and based on the forecasts of Hurricane Ian 48 hours before affecting Cuba in 2022 is presented.

Keywords*: Meteorological emergency, evacuation, robust stochastic model, decision problem.*

INTRODUCTION

Climate change and its profound impacts are undeniable [1]. Remarkable catastrophic events, such as the 2003 European heat wave and the 2004-2005 Atlantic Ocean cyclone season, were highlighted news. However, they are becoming more common as climate change continues to affect the world [2]. In the last decades, an increasingly number of hurricanes have affected the Caribbean, causing big damages. Severe storms, rainfalls, and similar phenomena are climatological events. Measures for protecting human lives and critical assets have to be taken [3].

This paper discusses how to handle people evacuation during hurricanes. The main difficulty of the model, from a mathematical viewpoint is that decision-makers must decide which vulnerable people have to be evacuated before having accurate information about the storm's path and intensity.

Considering the uncertainty of these phenomena, a two-stage stochastic model is proposed. The scenarios are constructed as a combination of possible trajectories, affected areas and intensity of the event, and have a probability of occurrence. We assume that, for each scenario and each location, the number of vulnerable people is known. This is not a strong assumption because the local governments have, through censured data, estimations of the number of people and the status of their houses. From an economical viewpoint, the model minimizes the costs associated to accommodation of the centers and the transportations from the areas in danger. It proposes the number of people that should go to each center, considering the capacity of the habilitated ones. Vulnerable people are uncertain. For each scenario the difference between the number of evacuated people and the actual need is multiplied by the probability of the scenario and a penalty term. This penalty is high if people are left behind because it is important to protect human lives. A smaller value is taken for the case of evacuating people without need. It is also important because evacuating more people than needed for highly probable scenarios is inefficient from an economic viewpoint. The proposed approach supports the robust decision-making process in tropical storms

A. State of the Art on Under Uncertainty Optimization Models in Meteorological Emergencies

There is a lot of research related to mathematical modeling to assist decision makers in evacuating people in the threat of a hurricane.

For instance, [4] presents a deterministic model framework for scenario-based evacuation with hazard modeling. In contrast, this work highlights an ensemble and physical model-based approach to flood and wind forecasting, which requires dependence on meteorological accuracy and while addressing uncertainty, the applicability and concrete results in realistic evacuation situations are not highlighted.

Also, a conceptual model for hurricane forecasting and evacuation dynamics is proposed in [5] analyzing the influencing factors and their interaction with real or synthetic scenarios. The work highlights the importance of representing uncertainty in evacuation planning using an approach based on physical and ensemble models. However, the use of multiple models and the need for perturbations to generate storm solutions can increase prediction uncertainty, which could complicate decision making in emergency situations. In contrast, a simpler, more robust model could provide more straightforward and applicable results in realistic evacuation scenarios.

Additionally, [6] discusses mass evacuation using public transportation before a hurricane's impact, considering stochastic arrival of evacuees, some points of analysis in the given response suggest considerable challenges; for example, the strategy is based on rigid assumptions, such as the predetermination of evacuation zones, shelter locations and the exact timing of hurricane impact, which may not be realistic or adaptable to dynamic and changing situations.

In addition, the use of a multi-stage planning approach can result in a slow and inefficient response to emergencies that require quick and decisive action. The simulation developed to evaluate the strategy lacks empirical validation and may not adequately capture the complexity and uncertainty inherent in disaster evacuations.

In [7], authors employ various software and deterministic time windows to simulate hurricane emergency evacuations. Although the study addresses an important issue in emergency management, the methodology and results presented raise several concerns. First, the reliance on retrospective data from post-storm surveys and newspaper archives may introduce biases and limitations in understanding the decisions of emergency managers. While the study provides a valuable attempt to understand evacuation decisions, its methodological approach and scope could benefit from greater rigor and consideration of the complexity of the emergency decision-making process.

Lastly, [8] emphasize the significance of algorithms for evacuee numbers and evacuation times in real-life emergency scenarios. The paper provides an uncritical review of emergency evacuation planning approaches, lacking a comprehensive analysis of the methodologies reviewed and their applicability in various evacuation situations. The assessment of the strengths and weaknesses of the models and algorithms reviewed is limited, which reduces the practical utility of the review.

This contribution proposes an evacuation model that incorporates robustness under the uncertainty in determined parameters [9] and stochastic parameters (with known probabilistic distribution) [10] .

For literature review on robustness see [11], [12], [13], [14] and [15], [16] for the management of the uncertainty. Stochastic scenarios can be found in [17], [18]. Applications to hurricanes can be found in [19].

The models include deterministic parameters (their values are known) [20], stochastic (parameters with probabilistic distribution), or robust (uncertainty in at least one parameter).

Although this research focused on hurricane-related evacuation, the modeling process can be applied to other meteorological events like heavy rains [21], tropical storms [22], typhoons [23], flood hazards [24], landslides [25], and more [26].

B. Basics Concepts dealing Uncertain Parameters with Stochastic Scenarios

Optimization under uncertainty computes solutions that are nearly optimal in various scenarios. In this work we consider a stochastic discrete linear programming model:

$$s.t.\ \begin{array}{l} \min c^T x \\ \begin{array}{ccc} Ax & \leq & b \\ Bx & = & e \\ x & \geq & 0, \\ x & \in & Z^n \end{array} \end{array} \qquad (1)$$

where $c \in R^n, A \in R^{m\times n}, B \in R^{p\times n}, d \in R^p$ and e is a random vector taking the values $e_1, \dots e_S$ with respective probability $p_1, \dots p_S$. Each occurrence of e is a scenario. As the value of e is the combination of different factors sharing common information, the set of scenarios is described by a tree and a scenario is a path from the root to the leaves, [9], [27], [28]. This concept is applied in our model since scenarios are a combination of trajectories and intensity of the hurricanes. This determines how many persons should be evacuated. So, it is the uncertain parameter of the situation we are addressing in this paper.

As the uncertainty is in the set of feasible solutions, the model achieves robustness if it maintains feasibility across diverse scenarios, balancing optimality, and feasibility goals. The scenario method enables decision makers to link uncertainties and input parameters within a decision model. [9]. The solution approach we will use is based on adding variables that control the infeasibility.

In [27], the solution of (Equation. 2) is proposed as a robust solution of problem (Equation 1)

$$\begin{cases} \mathrm{P}(\xi): \min\{\sigma(x, y_1, y_2, \dots, y_s) + \omega\rho(z_1, z_2, \dots, z_n)\} \\ \quad (i) \qquad\qquad s.t.\ Ax \leq b \\ (ii) \qquad B_s x + C_s y_s + Z_s = e_s, \qquad s = 1, \dots S \\ (iii) \qquad\qquad x \geq 0, y_s \geq 0, \qquad s = 1, \dots S \end{cases} \qquad (2)$$

Here $\{y_1, y_2, \dots y_s\}$ and $\{z_1, z_2, \dots., z_s\}$ are the control variables and the error variables respectively. In (1), for $\sigma(x, y_1, y_2, \dots, y_s)$ measures the optimality of robustness, the second term for $\rho(z_1, z_2, \dots, z_n)$ is a function for penalizing violations of control constraints in some scenarios, A is a deterministic matrix, b is a deterministic resource and the other Matrix B, C and resource e are noise – affected depend - scenarios, ω is the goal programming weight used to derive a range of compensatory responses for model robustness.

P(ξ) prevents single choices for an objective function with several ξ scenarios with $\xi = c^T x + d^T$ becoming a random variable $\xi_s = c^T x + d_s^T\, y_s$ with probability p_s and $\sigma(\cdot) = \sum_{s\in\Omega} p_s\, \xi$ is the aggregation function of the problem.

The main objective of this contribution is to introduce an optimization model for evacuating population in danger during a hurricane emergency. The uncertainty associated to the meteor is modelled using finitely many scenarios, whose probability is computed. Based on that, the number of persons that should be evacuated are given as a random discrete vector with known probability distribution. The resulting problem has the structure given in (Equation 1) and it is solved using the approach that leads to (Equation 2).

This contribution is organized in three sections: methodological framework, practical simulation, and interpretation of results.

PROCEDURE METHODOLOGY: TWO-STAGE EVACUATION MODEL FORMULATION FOR METEOROLOGICAL EVENTS PROBLEMS

A. Problem Statement

The Saffir-Simpson wind scale is used to classify hurricanes and tropical cyclones based on their sustained wind speed. This scale consists of five categories, numbered from 1 to 5, each with a specific range of wind speeds and a description of the expected damage. Table 1 presents the Saffir-Simpson wind scale categories, corresponding sustained wind speeds in knots and kilometers per hour, and descriptions of the expected damage in each category.

The use of this information is vital for a proper understanding of the potential impact of a hurricane on the socio-economic and environmental sustainability of a given vulnerable population.

Table 1. Saffir-Simpson Wind Scale categories

Category	Wind Speed (Knots)	Wind Speed (Km/h)	Damage
Cat – I	74 – 95	119 – 153	Minimal
Cat – II	96 – 110	154 – 177	Moderate
Cat – III	111 - 129	178 – 208	Extensive
Cat – IV	130 – 156	209 – 251	Severe
Cat – V	> 157	> 252	Catastrophic
Other Classifications			
Tropical Storm	39 – 63	63 – 118	Moderate
Tropical Depression	< 34	< 63	Minimal

Tropical cyclones, categorized from 1 to 5, bring varying degrees of damage.

Category 1 causes minimal harm with limited wind speeds and minor flooding in low-lying areas.

Category 2 inflicts moderate damage, resulting in power outages and structural harm due to stronger winds and heavy rainfall in vulnerable regions.

Category 3 leads to extensive damage as it can destroy roofs and buildings, accompanied by widespread flooding and potential landslides.

Category 4 is severe, with catastrophic winds causing widespread structural destruction, power failures, significant flooding, and dangerous storm surges.

Category 5 is catastrophic, devastating entire communities with extreme winds, long-term power outages, life-threatening storm surges, and flash floods.

Tropical storms have stronger winds than tropical depressions, and this is reflected in the expected damage level. Tropical storms can cause moderate damage, while tropical depressions typically result in minimal damage.

Cyclones intensify or weaken along their path due to factors like sea temperatures, topography, and atmospheric conditions. Wind force relates to the square of sustained wind intensity, with brief gusts reaching 1.5 times that speed.

B. Model Formulation

In our modelling approach, we utilize a discrete uncertainty set based on forecasted trajectories and intensity models. Each scenario combines predicted meteorological paths and estimated intensities. This data helps the Civil Defense forces to identify vulnerable populations based on the damage associated with the behavior of a hurricane with respect to its intensity.

A robust stochastic model with a Two-Stage approach is described below to address the problem of evacuation of vulnerable people under meteor uncertainty.

1. The indexes

Affected Locations: $i \in AL$.

Eligible Centers: $j \in EC$.

Emergency Damage: $d \in ED$.

Meteor Intensities: $e \in MI$.

Meteor Trajectories: $q \in MT$.

The scenarios corresponding to the possible hurricane development may characterized by the indexes q (possible trajectories), e (intensity of meteor) and d (damage)

2. The parameters

$\boldsymbol{C}_{(\boldsymbol{i},\boldsymbol{j})}$: Unit cost of transportation from location (i) to eligible center (j).

$\boldsymbol{\theta}_{+}$: Constant penalty cost for non - evacuee people.

$\boldsymbol{\theta}_{-}$: Constant penalty cost for over - evacuee people.

$\boldsymbol{A}_{(j)}$: Cost of accommodating the eligible center (j).

$\boldsymbol{K}_{(j)}$: Capacity of the (j) Eligible Center.

$\boldsymbol{v}_{(q,e,d,i)}$: Vulnerable population for scenario (q,e,d) in affected locality (i)

$\boldsymbol{\pi}_{(\boldsymbol{q},\boldsymbol{e},\boldsymbol{d})}$: Probability of scenario (q,e,d).

3. The integer decision variables

$\mathbf{Y_j}$: {1 if eligible center is selected; 0 otherwise}

$\mathbf{X}_{(i,j)}$: Number of persons to evacuate from Locality (i) to Candidate Evacuation Center (j).

$\boldsymbol{\delta}_{+(i,q,e,d)}$: Number of non – evacuated vulnerable people from location (i) in scenario (q,e,d).

$\boldsymbol{\delta}_{-(i,q,e,d)}$: Number of over – evacuated vulnerable people from location (i) in scenario (q,e,d).

4. The objective

$$Min\ Z = \sum_{(i,j)} \boldsymbol{C}_{(i,j)} \cdot \boldsymbol{X}_{(i,j)} + \sum_{(j)} \boldsymbol{A}_{(j)} \cdot \boldsymbol{Y}_{(j)} + \sum_{(i,e)} \boldsymbol{\pi}_{(q,e,d))} \cdot \cdot \left[\boldsymbol{\theta}_{+} \cdot \boldsymbol{\delta}_{+(i,q,e,d)} + \boldsymbol{\theta}_{-} \cdot \boldsymbol{\delta}_{-(i,q,e,d)}\right] \tag{2}$$

5. The constraints

Accommodation Capacity:

$$\sum_{(i)} \boldsymbol{X}_{(i,j)} \leq \boldsymbol{K}_{(j)} \cdot \boldsymbol{Y}_{(j)} \qquad \forall\, j \in EC \tag{3}$$

Evacuation: For each scenario (q,e,d)

$$\sum_{(j)} [\boldsymbol{X}_{(i,j)}] + \boldsymbol{\delta}_{+(i,q,e,d)} - \boldsymbol{\delta}_{-(i,q,e,d)} = \boldsymbol{v}_{(q,e,d,i)} \qquad \forall\, i \in AL, q \in MT, e \in MI, d \in AL \tag{4}$$

Variable Type:

$$\boldsymbol{X}_{(i,j)}, \boldsymbol{\delta}_{+(i,q,e,d)}, \boldsymbol{\delta}_{-(i,q,e,d)} \in Z^{+},\ \ \boldsymbol{Y}_{(j)} \in \{0,1\} \qquad i \in AL, j \in EC, q \in MT, e \in MI, d \in AL \tag{5}$$

C. Meteorological Robust Emergency Framework Proposal

This contribution proposes a simple two – frameworks methodology to address the evacuation problem, Framework A is based on 4 key phases (Figure 1). It is complemented by the mathematical part described in Framework B (Figure 2) which synthesizes the steps of the two-stage stochastic robust model. The methodological process covers the entire duration of the decision-making process by establishing the following basic rules:

- Once the emergency is unleashed, social data should begin to be collected from the security or civil defense department in charge in correspondence with the national meteorological emergency department.
- The data must be available at least 48 hours before the meteorological emergency contacts the first possibly affected population.
- The final decisions are made by the decision-maker once the solutions provided by the mathematical model have been obtained.

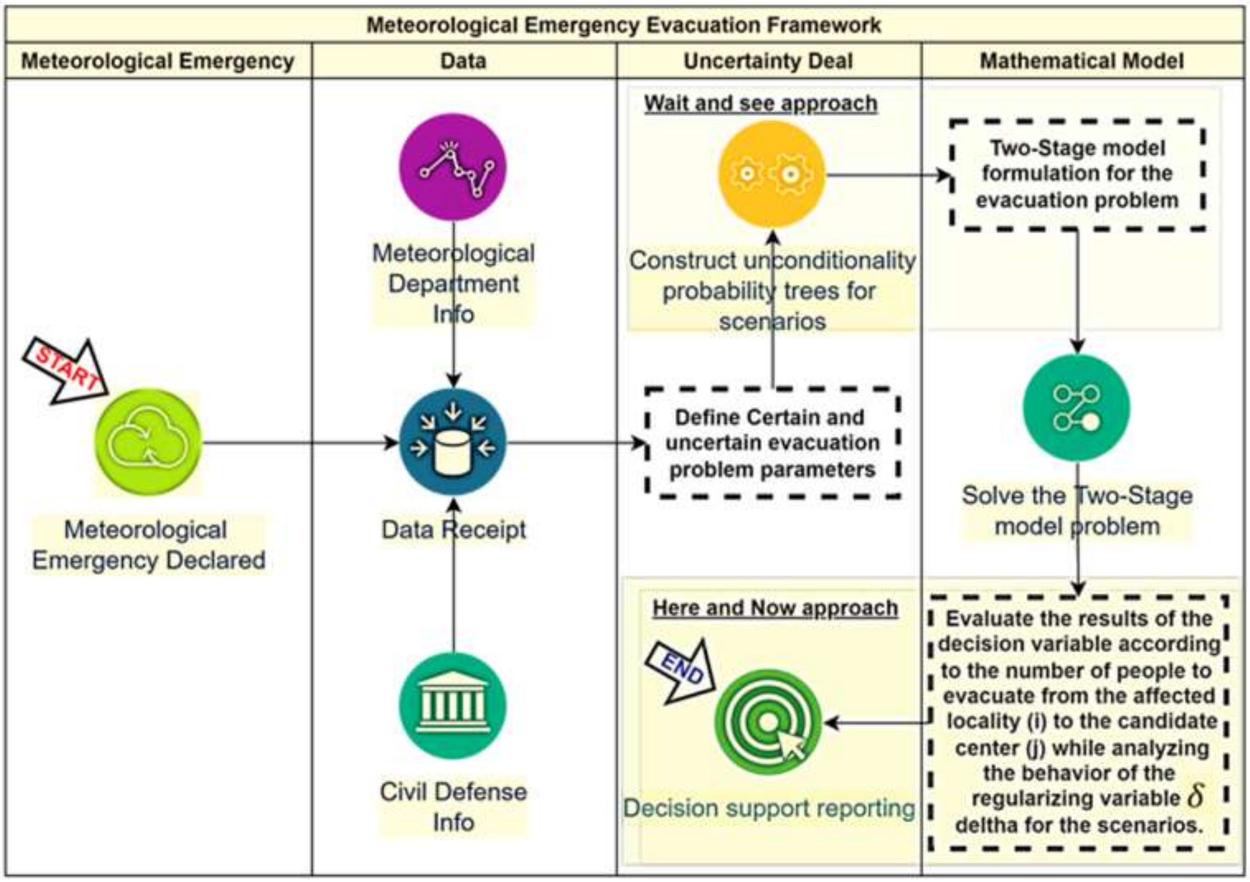

Figure 1. Framework A: Modeling Evacuation Process

Phase I: Meteorological Emergency Declaration

- Activated upon meteorological emergency declaration.

Phase II: Data Capture

- Collection of meteorological data (trajectories, wind speed, damage, etc.) and demographic data (vulnerable population, housing conditions and others).

Phase III: Uncertainty Management

- Detection of uncertain parameters and their nature (e.g., storm trajectory, intensity). In this case parameters are stochastic optimization, and the probability distribution of the associated random variables are also needed. They are computed using information provided by specialists working with advanced forecasting models.

Phase IV: Mathematical Model

- Proposes a two-stage stochastic optimization approach. Model configuration depends on specific characteristics. Components include scenario parameters, integer decision variables for evacuation, binary variables for accommodation, control variables for uncertainty, and corresponding constraints. This part is solved in Framework B, Figure 2.

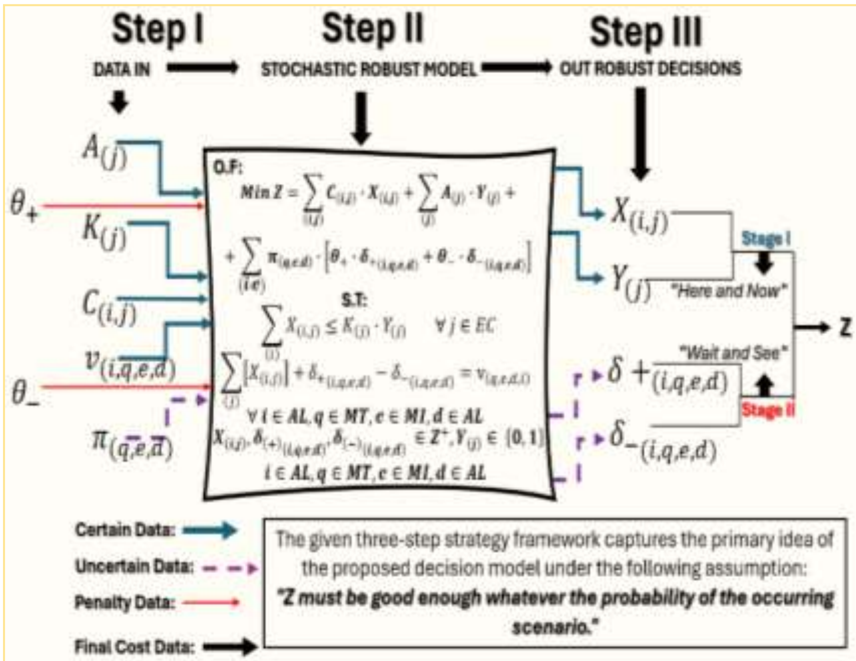

Figure 2. Framework B: Two Stage Stochastic model - steps for hurricane vulnerable people evacuation

After inputting the data in Step I, the model described in (3)-(6) is solved. Note that the Here and Now variables are$\boldsymbol{X}_{(i,j)}, \boldsymbol{Y}_{(j)}$. Indeed, they are decision taken before the realization of the events, hence they are independent of the scenarios. The "wait and see" approach is also applied because the $\boldsymbol{\delta}_{+(i,q,e,d)}, \boldsymbol{\delta}_{-(i,q,e,d)}$ adjust the difference between the evacuation proposed by the Here and Now variables and the actual values. Note that the penalty costs are different: $\boldsymbol{\theta}_{+}$ is usually larger than $\boldsymbol{\theta}_{-}$ because the cost of losing a human life is much larger than evacuating people without need.

Decision makers can manage these two parameters to obtain different evacuation policies and their costs in terms of human life and evacuation costs. The following section demonstrates this framework using Hurricane Ian in September 2022 as a real-world case.

SIMULATING EVACUATION UNDER UNCERTAINTY IN HURRICANE EMERGENCIES: A REAL CASE

The North Atlantic hurricane season is from June 1 to November 30, with peak activity from mid-August to late October. In 2022, Hurricane Ian struck the Antilles and Florida during this active period, resulting in fatalities and billions in damage. [29].

A. Simulation case: Hurricane Ian 2022 (Emergency – Impact on Occident of Cuba)

Using Ian's forecasted path and intensity, we present an evacuation model for this case by feeding the parameters of the mathematical model with information provided by National Institute of Meteorology of Cuba (INSMET) and estimated data from public information published in Granma [30].

Meteorological services monitor cyclones, initiating an information phase. At least 48 hours before landfall, a cyclone alert is issued, and 24 hours prior, a cyclone alarm.

Forecast models identify municipalities vulnerable to hurricane or tropical storm winds, heavy rain, and coastal inundation. Municipal Civil Defense authorities use this data to select evacuation populations. [31].

B. Known Case Parameters (Data)

As emphasized in the proposed methodology, data collection is of vital importance for decision making. It is proposed to have Meteorological and demographic data.

Because of the security required for this type of information, it is necessary that it be provided by the competent authorities under the appropriate permits.

1). Meteorological data

This proposal simulates the predicted trajectories using almost eight predictive models at 48 hours prior to the arrival of the hurricane to Cuban territory with information from NHC-display [32]. Based on the data matrix obtained by the predictor models for hurricane Ian, a probability analysis of the information on the trajectories and their possible intensities was generated, comparing the forecast at 48 hours.

The (Figure 3) shows the trajectories for each of these models for our case study.

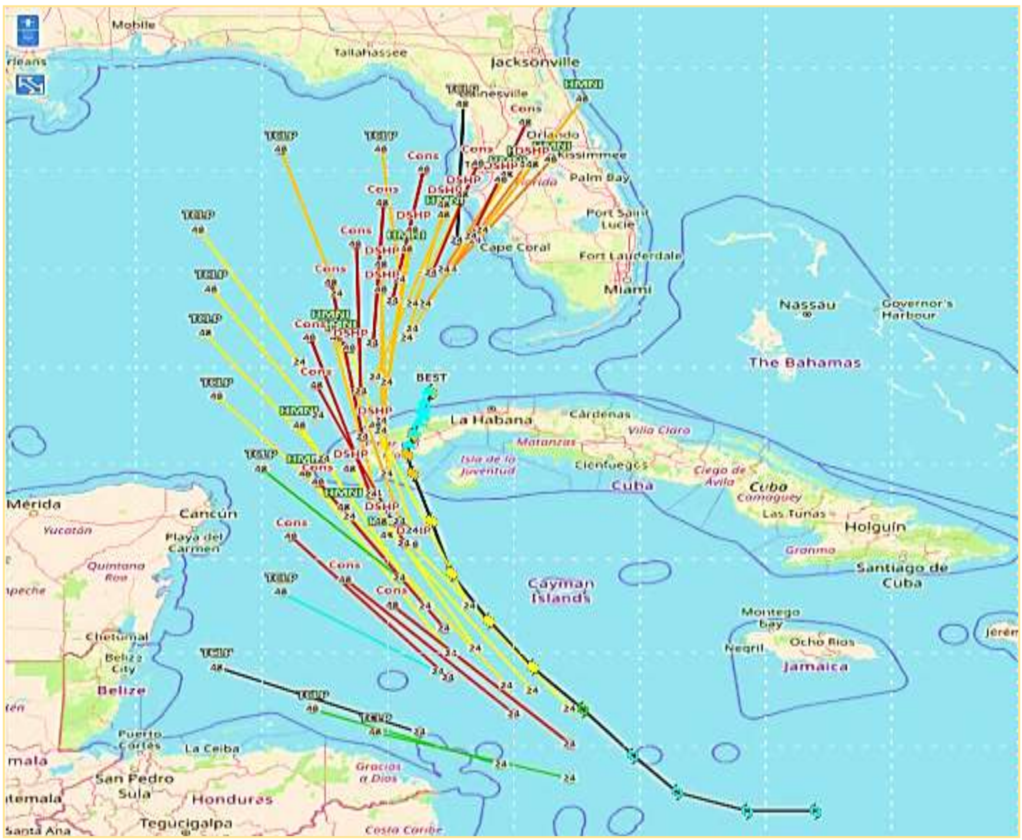

Figure 3. Trajectory models from NHC-display: Case Ian-2022, Cuba

For the simulation problem we will assume expected damages in sub localities of the province of Pinar del Río that coincide with the trajectories predicted by the predictors (Figure 4).

Figure 4. Pinar del Rio possible affected localities

Table 2 shows the areas that will be affected by the hurricane. Each locality is included in an area. Area *i* will be affected by scenario *i*. The population of each locality and an estimation of vulnerable people are also given.

Table 2. Vulnerable population case Ian-2022, Pinar del Rio, Cuba

Code	Area	Affected Localities	Population	Vulnerable
Al1	A4	San Luis	32,393	5000
Al2	A4	La Coloma	7,000	4000
Al3	A1	Guane	36,172	1200
Al4	A4	Viñales	27,972	3000
Al5	A1	Mantua	25,391	3350
Al6	A4	La Palma	35,487	3000
Al7	A1	Los Palacios	38,636	3000
Al8	A2	San Juan y Martinez	44,969	3800
Al9	A5	Sandino	37,891	2000
Al10	A4	Pinar del Rio	190,337	8000
Al11	A3	Minas de Matahambre	33,733	2720
Al12	A2	Consolación del Sur	88,950	930

The values were simulated generating random numbers whose sum is 40 000, the number of vulnerable people reported by GRANMA in [30] reported that 54 centers were used.

Their capacity was generated randomly, taking a into account that their sum will be 2 500, the number of total evacuated people reported in GRANMA (Figure 5). We created an extra center that includes people that are hosted by friends or relatives.

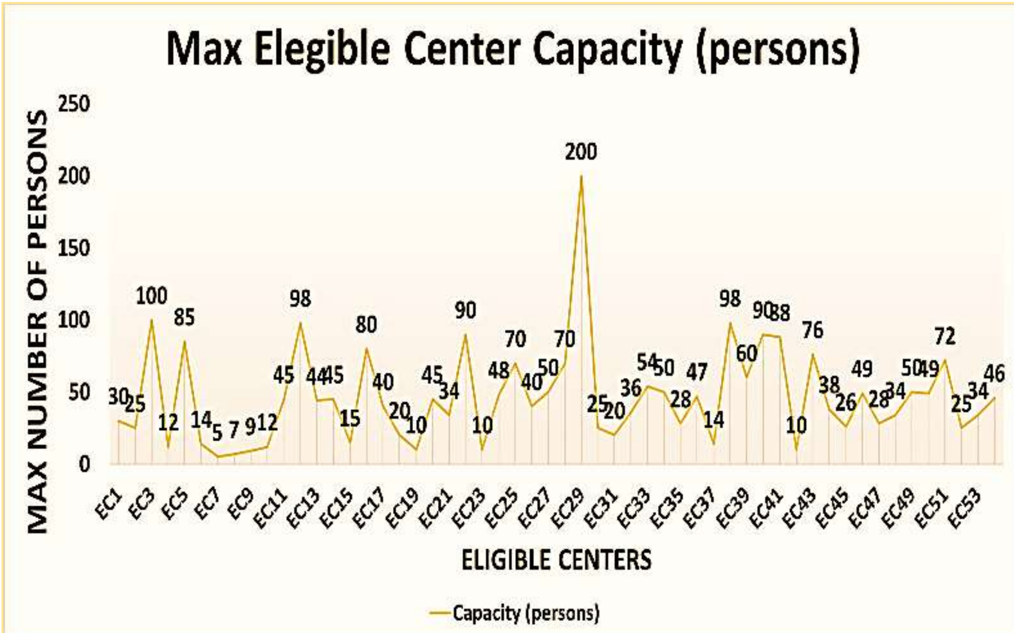

Figure 5. Max eligible centers capacity (referential)

The accommodation costs in USD (Figure 6) are simulated.

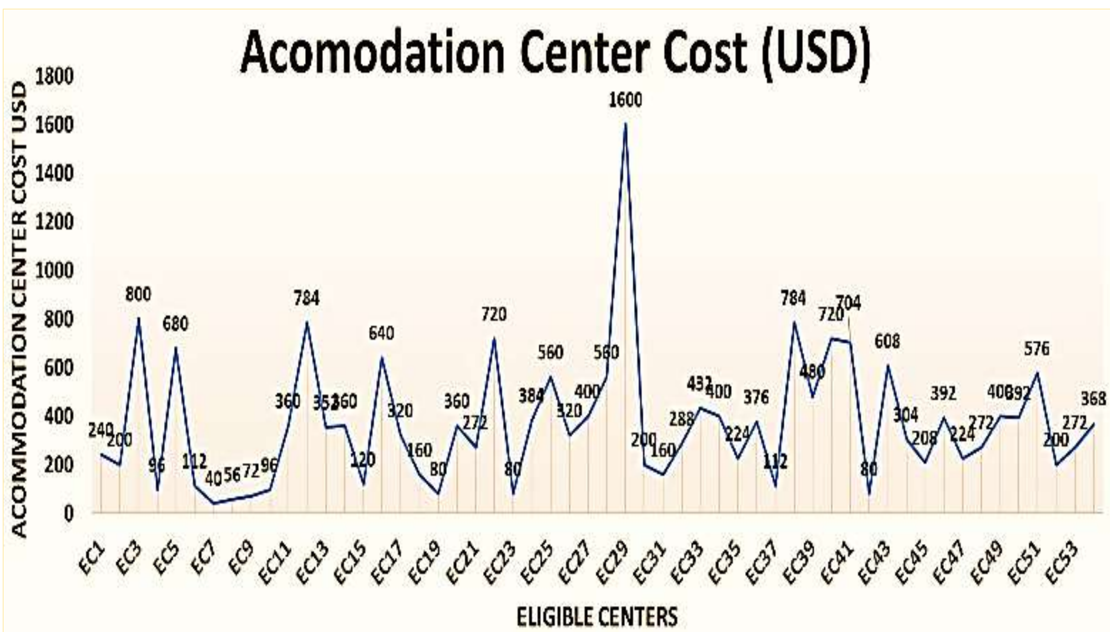

Figure 6. Eligible Centers Cost (USD) (Referential)

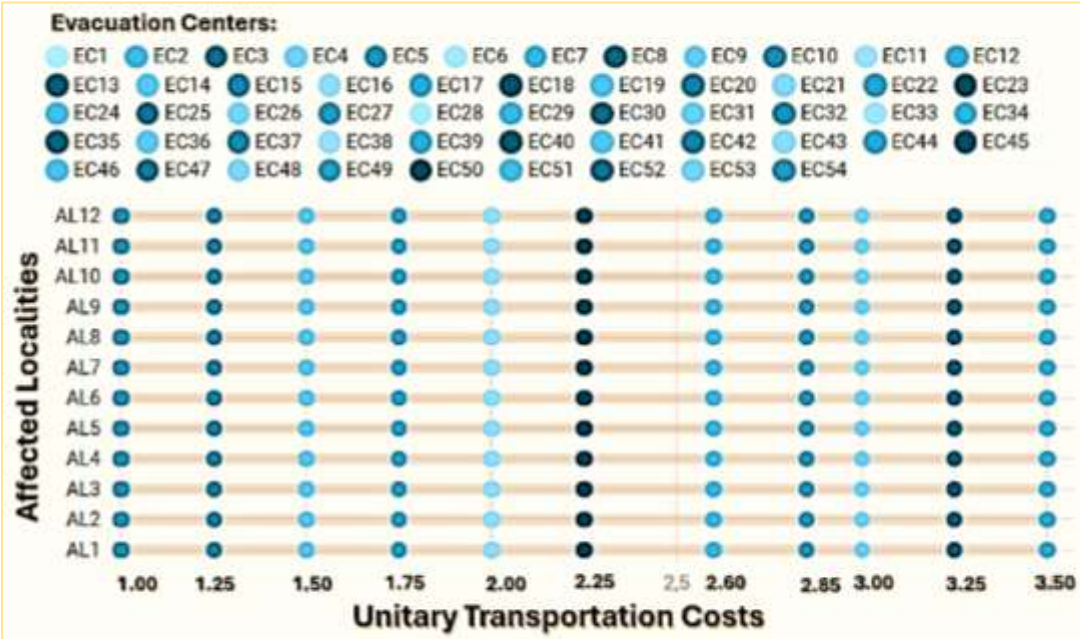

Figure 7. Transportation cost (USD) per evacuated person per unit (referential)

The unit transportation cost per evacuee is shown in (Figure 6).

In the following, the information previously provided will be used to simulate the mathematical model proposed in the evacuation process of the province Pinar del Rio in Cuba.

2). Stochastic Scenarios Approach (Uncertainty Deal)

To predict possible scenarios for Hurricane Ian-2022 in Cuba, we employed trajectory and intensity predictors like:

- **TCLP:** Tracks Consensus and Model Guidance Plot (TCLP) - This model combines consensus and model guidance to provide a more comprehensive picture of hurricane tracks.
- **DSHP:** Dynamical Statistical Hurricane Prediction Model (DSHP) - This model utilizes both statistical and dynamical approaches to predict hurricane behavior.
- **HMNI:** Hurricane Multi-Model NHC (HMNI) - This is a multi-model consensus approach used by the NHC for hurricane prediction.
- **UKMI:** United Kingdom Meteorological Office (UKMI) - While not specifically an NHC model, the UKMI's weather prediction models may be considered by the NHC for forecasting hurricanes, especially for storms affecting regions in or near the United Kingdom.
- **AEMI:** Atlantic Ensemble Prediction System (AEMI) - This is a probabilistic hurricane forecast system that uses multiple ensemble members to generate forecasts.
- **Cons:** Consensus Model - This is a model that combines forecasts from multiple individual models to generate a consensus forecast.
- **AVNI:** Aviation Model - This model is primarily used for aviation-related forecasts but may also be consulted for hurricane prediction, especially when considering the potential impact on aviation operations.
- **HWFI:** Hurricane Weather Research and Forecasting Model (HWRF) Intensity - This model specifically focuses on forecasting the intensity of hurricanes.

ISSN: 2633-4828 Vol. 5 No. S6, (Oct - Dec 2023)

These models, along with others, are used by the National Hurricane Center to generate forecasts and advisories to help prepare and warn the public about potential hurricane threats.

Each provided multiple trajectories at 48 hours. (Table 3) summarizes result probability hurricane track data from these predictors, including average trajectories and wind-intensity relationships based on [32] database queries before Ian's arrival in western Cuba.

Table 3. Probabilities of hurricane Ian's trajectories constructed from recognized predictive models for 48 h.

P***	***Intensity at 48h***	***Area***	***Probability***
TCLP; UKMI	Tropical-Storm	A1	0.106
AEMI; TCLP; UKMI; HMNI	Cat-I	A2	0.152
Cons; AVNI; HMNI; TCLP; UKMI	Cat-II	A3	0.197
DSHP; HWFI; HMNI; AEMI; Cons; AVNI	Cat III	A4	0.455
HMNI; DSHP; HWFI	Cat IV	A5	0.090

$\boldsymbol{P}^{*}: Predictors$ [33]

The forecast area of the models according to the intensity of hurricane Ian shows diverse affectations in the western zone of Cuba. The high probabilities are for Cat-II and Cat - III with probability of 0.197 and 0.455 respectively (Figure 8).

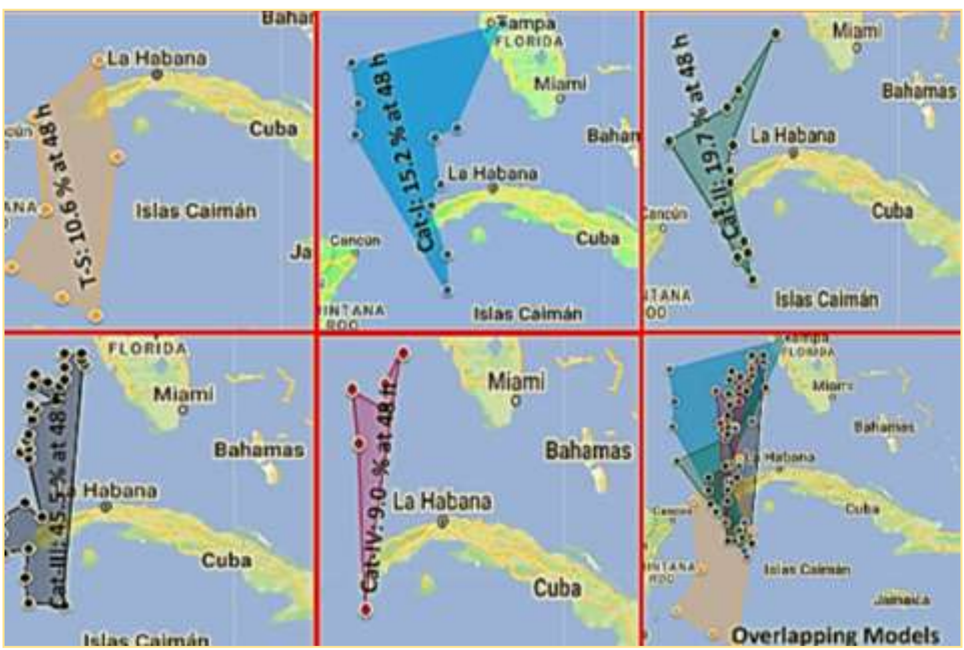

Figure 7. Affected areas and probabilities

The following section will formulate and provide a solution to the proposed problem of evacuation of vulnerable population by simulating what happened in Cuba during the passing of Hurricane Ian in the year 2022, according to the proposed methodology and with the support of the AIMMS 4.96.3.1 tool (under an educational license from AIMMS Licensing Center).

C. Robust decision for Case Ian-2022, Pinar del Rio, Cuba

Figure 9 shows the proposed solution computed to distribute the evacuation for Ian -2022 at the province of Pinar del Río in Cuba.

The capacity of the centers is large. So, as expected, independently of the scenario the 40,000 vulnerable people have had a safe evacuation center. The wait and see variables $\delta^{s}_{+i} = 0$, for all scenario s and all locality i, i.e. all person in need is evacuated. In the case of δ^{s}_{-i}, they are equal to the difference between the largest amount of people to be evacuated and the real number for each scenario and locality, see Table 4.

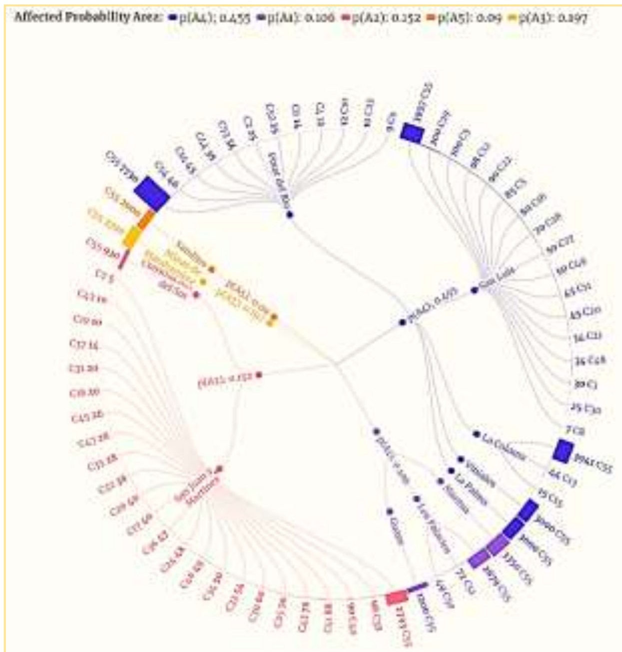

Figure 8. Evacuation decisions about vulnerable population in hurricane Ian - 2022, Pinar del Río, Cuba

Figure 10 shows the relationship among the here and now and the wait and see variables introduced in Figure FRAMEWORK B. For example, the people evacuated under the scenario of a "category 3 hurricane" (Cat III) with a probability of 0.45 in the affected area A4 were are 23 000. As 40 000 were evacuated, $\sum_i \delta^{3}_{i-}$ =17,000.

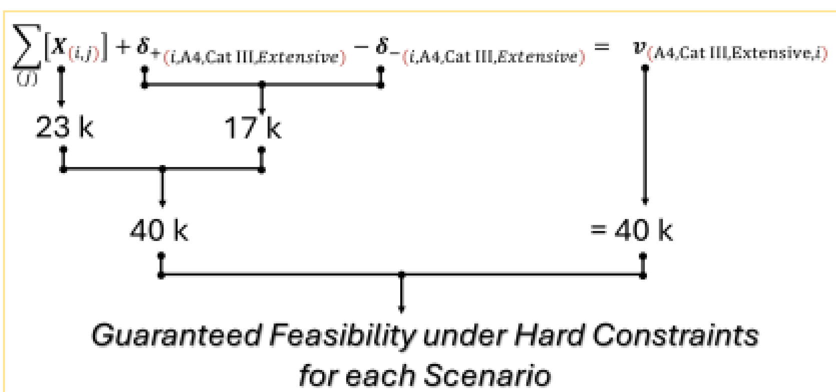

Figure 9. Decision interpretation feasibility under Scenario III

Although it is expensive, no risk is taken. Lower values of $\boldsymbol{\theta}_{+}$ should lead or higher accomodation consts can lead to lefting people behind at some scenarios, see Figure 11.

Table 4. Explanation of (δ_{-}) performance data

Affected Locality	Affected Area	Cat I-TS	Cat I- TD	Cat II	Cat III	Cat IV
San Luis	A4	5000	5000	5000	0	5000

La Coloma	A4	4000	4000	4000	0	4000
Guane	A1	0	1200	1200	1200	1200
Viniales	A4	3000	3000	3000	0	3000
Mantua	A1	0	3350	3350	3350	3350
La Palma	A4	3000	3000	3000	0	3000
Los Palacios	A1	0	3000	3000	3000	3000
San Juan y Martinez	A2	3800	0	3800	3800	3800
Sandino	A5	2000	2000	2000	2000	0
Pinar del Rio	A4	8000	8000	8000	0	8000
Minas de Matahambre	A3	2720	2720	0	2720	2720
Consolación del Sur	A2	930	0	930	930	930
Gran Total	-	32450	35270	37280	**17000**	38000

In other words, the main constraint (Equation 4) of the proposed model make possible a trade-off between the decision variables and the control variables be provided.

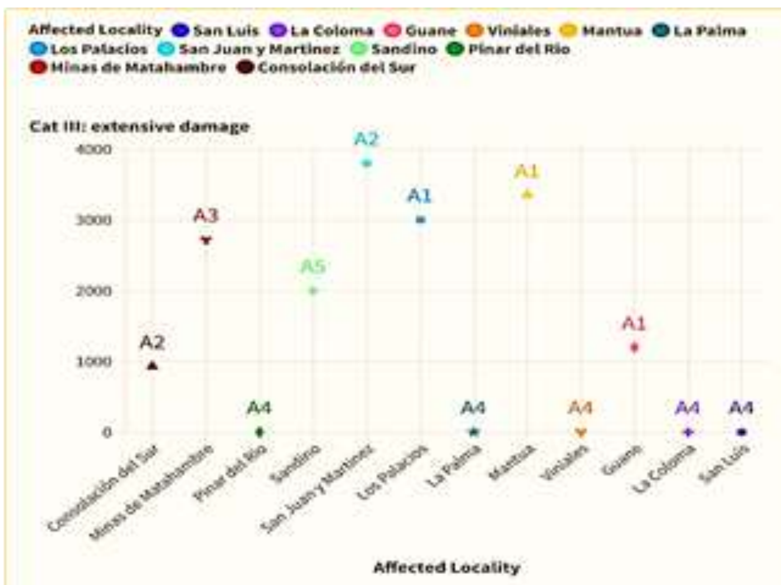

Figure 10. Cat III - Affected Area 4 (δ_+,δ_-)

To carry out the operations to evacuate 40,000 vulnerable people to the different evacuation centers had an estimated transportation cost of USD 22,245 and an accommodation cost of USD 20,000 according to the proposed model. These values reflected in the result of the objective function and robust decisions made.

The final section of this contribution discusses the benefits of using linear stochastic optimization to handle difficult decisions under partial uncertainty in the evacuation problem in front hurricane emergencies.

DISCUSSION AND FINAL REMARKS

A. Interpretation results

The use of robust models to solve complex decision problems is a useful tool for analyzing how to handle inherent uncertainties of the evacuation process when a hurricane is approaching.

The decisions taken according to official sources of national information ended with the evacuation of about 40 thousand people before the meteor made landfall. Protection of people was guaranteed.

The model Framework B presented in Figure 11 is a two-stage model proposed to manage the uncertainty in the problem. The combination of here and now and wait and see variables models the situation, because the decision is taken before knowing what is going to happen and the values of the losses for all scenarios is known. Of course, to have accurate solution, the quality of the data is very important

The proposed mathematical model can handle this situation by alerting the decision makers that it is necessary to cover the supply and mitigate the demand with adjustments that minimize the risk to the vulnerable population.

Since the proposed model is a two-stage stochastic optimization model, it is interesting to note that the uncertainty inherent in the decision system lies in the probability of impact of the meteorological emergency in one of the vulnerable areas of a region.

With this probability, discrete scenarios can be designed that allow. For simplicity we only presented the case in which for each scenario only one area is affected. The case in which for one or more scenarios the hurricane affects several populations can be also treated.

There are other approaches to deal with this problem that are being addressed. Robust approaches considering the total uncertainty or the contemplation of the measurement of such total uncertainty through a possibilistic approach [34], [35], will be addressed in other research spaces.

B. Final remarks

The proposed research is part of a group of studies that the research team is conducting to support decision making during a severe climatological emergency.

The main contributions achieved with this part of the research are listed below:

I. The proposed model with two-stage stochastic approach for the evacuation problem can solve the decision problem under various conditions of resource availability if the data are handled with serious enough reliability in the complex decision context before the climatological emergency occurs.

II. This model was used to simulate a real case of meteorological emergency in Cuba in the year 2022 caused by the passage of hurricane Ian, where an evacuation of some 40,000 people in the province of Pinar del Río was estimated before the event.

III. The results of the simulation corresponded to the reality.

IV. In the present research, two methodological frameworks (Figure 1 and Figure 11) are proposed. They complement each and provide a better answer to the addressed problem

V. The first methodological Framework A is based on 4 general phases that go from obtaining the data to identifying the uncertainty in the emergency; the second one Framework B is based on the steps in which the stochastic optimization model must be proposed in two stages to respond to the emergency.

For future work in this area, several mathematical approaches (robust optimization for total uncertainty, possibilistic optimization and constrained optimization) for modeling with uncertainty treatment have been contemplated, which have not been addressed for reasons of objectivity and context in the present research

ACKNOWLEDGMENT

The present contribution tributes to:

- **Doctoral thesis:** "Mathematical optimization models for the management of meteorological emergencies in the presence of uncertainty" of the Department of Mathematics and Computation of the University of Havana, Havana, Cuba.

- **Project:** Smart DATALAB UPEC of the Universidad Politécnica Estatal del Carchi, Tulcán, Ecuador.
- **Project:** 10-0049 Desarrollo de modelos y métodos matemáticos y computacionales para la solución de problemas específicos de toma de decisión.
- **The authors want to thank the fruitful discussions with Drs Elia N. Cabrera Alvarez.**

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