SOME NEW GENERATING FUNCTIONS FOR ONE PROBABILITY DISTRIBUTION

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1. INTRODUCTION.

Golomb [1] defined an information generating function of the probability distribution and also defined all the properties of generating function. He proved that the first moment of self-information is Shannon's [2] measure of entropy. Golomb [1] obtained simple expression, measures of generating function for discrete as well as continuous distribution, geometric distribution, zeta distribution, exponential distribution, pareto distribution and normal distribution. Later Golomb [1] and Guiasu and Reischer [3] have given generating function for Shannon's [2] and Kullback- Leibler's [4] for corresponding measure of information also defined generating function for relative information or cross entropy or directed divergence of P from Q.

In the present paper we have obtained some new generating functions which generates various measures of entropy and directed divergence and generating functions for several measures of inaccuracy are introduced and prove that derivative at 1 and 0 of proposed generating functions includes several well-known results. We have also discussed particular and limiting cases of obtained generating functions.

In section two some preliminaries are presented along with the discussion on the basic concepts, definitions and properties of generating functions for measures of information, generating functions of directed divergence and generating functions of measures of inaccuracy. In section three we proposed new generating functions for various information measures, based on one probability distribution corresponding to Shannon's [2] Havrda Charvat's [5], Renyi's [6], Kapur's [7] [8], Vejda's [9] , Behara-Chawala [10] measure of entropy. We have obtained new two parametric generating functions, also discussed particular and limiting cases of obtained generating functions. In section four concluding remarks of the literature are discussed. The reference of the paper is given in section six.

Key Words: Measure Of Information, Generating Functions, Measure Of Entropy, Generlised Entropy.

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2. PRELIMINARIES

For any probability $P = \{p_1, p_2, \ldots \ldots p_n\}$ and $Q = \{q_1, q_2, \ldots \ldots q_n\}$ be two probability distribution then Golomb [1] defined an information generating function of probability distribution function P as

$$f(t) = -\sum_{i=1}^{n} p_i^t \qquad (2.1)$$

as the generating function having with property that property that

$$f'(t) = -\sum_{i=1}^{n} p_i^t \ln p_i$$

$$f'(1) = -\sum_{i=1}^{n} p_i \ln p_i$$

$$= S(P) \qquad (2.2)$$

Where S(P) in Shannon's [2] measure of entropy for probability distribution P,

Again from (2.1) we get

$$f^r(1) = (-1)^{r-1} \sum_{i=1}^{n} p_i (-\ln p_i)^r, \; r = 1,2,\ldots\ldots \qquad (2.3)$$

Golomb [1] proved that the first moment of self-information is Shannon [2] that is $-\ln p_i$ is the self-information of ith outcomes

Again from (2.3)

$(-1)^{r-1}f'(1)$ (2.4)

The expected value of the rth power of the self-information gives by equation (2.4)

Again Golomb [1] gives the variance of the self-information as $f^2(1)-(f'(1))^2$

Later 1985, Guiasu and Reisher [3] defined the generating function g(t) for relative information or cross entropy or directed divergence of probability distribution P $=\{p_1,p_2,\ldots\ldots,p_n\}$from another probability distribution Q by

$$g(t)=\sum_{i=1}^{n}q_i\left(\frac{p_i}{q_i}\right)^t, \quad (2.5)$$

With property that,

$$g'(t)=\sum_{i=1}^{n}p_i\ln\frac{p_i}{q_i} \quad (2.6)$$

Therefore

$$g^r(t)=\sum_{i=1}^{n}p_i\left(\ln\frac{p_i}{q_i}\right)^r \qquad r = 1, 2, 3 \quad (2.7)$$

by using equation (2.6) and (2.7) Guiasu and Reisher [3] suggested that, we can find variance of change of self-information as well as the moments about the mean of the probability distribution of self – information.

Later 1997, Kapur [7] defined generating function for measure of information based on one probability distribution

$$f_\alpha(t)=\frac{1}{1-\alpha}\left(\sum_{i=1}^{n}(p_i^\alpha)^t-1\right), \quad \alpha\neq 1 \quad (2.8)$$

With the property

$$f_\alpha(1)=\frac{1}{1-\alpha}\left(\sum_{i=1}^{n}p_i^\alpha-1\right), \quad \alpha\neq 1 \quad (2.9)$$

Equation (2.9) gives Havrda –Charvat's [5] measure of entropy

$$f'_\alpha(0)=\frac{1}{1-\alpha}\ln\sum_{i=1}^{n}p_i^\alpha, \quad \alpha\neq 1 \quad (2.10)$$

Renyi [6] has generalized Shannon's measure of entropy by introducing parameter, $(\propto>0,\propto\neq 1)$

As $R_\propto(P)=\frac{\ln\sum_{i=1}^{n}p_i^\propto}{1-\propto}$ (2.11)

With property $R'_\propto(P)=\frac{\ln\sum p_i^\propto \ln p_i}{-1}$

$$\lim_{\propto\to 1}R_\propto(P)=-\ln\sum_{i=1}^{n}p_i\ln p_i$$

$$\lim_{\propto\to 1}R_\propto(P)=S(P)$$

3. GENERATING FUNCTIONS FOR MEASURES OF INFORMATION BASED ON ONE PROBABILITY DISTRIBUTION

MEASURE OF ENTROPY

(I) Now define

$$f_{\propto,\beta}(t)=\frac{1}{\beta-\propto}\left[\left(\sum_{i=1}^{n}p_i^{\propto\beta}\right)^t-\beta\right],$$

$$\beta\neq\propto,\ \beta>0,\propto>1 \quad (3.1)$$

$$f_{\propto,1}(t)=\frac{1}{1-\propto}\left[\left(\sum_{i=1}^{n}p_i^{\propto}\right)^t-1\right], \propto\neq 1 \quad (3.2)$$

Therefore $f_{\alpha,1}(1)$ gives Havrda-Charvat's [5] measure of entropy.

$$f'_{\propto,\beta}(t)=\frac{1}{\beta-\propto}\left(\sum_{i=1}^{n}p_i^{\propto\beta}\right)^t\ln\sum_{i=1}^{n}p_i^{\propto\beta}$$

$$f'_{\propto,1}(t)=\frac{1}{1-\propto}\left(\sum_{i=1}^{n}p_i^{\propto}\right)^t\ln\sum_{i=1}^{n}p_i^{\propto}, \propto\neq 1 \quad (3.3)$$

$$f'_{\propto,1}(0)=\frac{1}{1-\propto}\ln\sum_{i=1}^{n}p_i^{\propto}, \propto>0,\propto\neq 1 \quad (3.4)$$

Therefore $f'_{\propto,1}(0)$ is Renyi's [6] measure of entropy.

$$f_{1,1}(t)=f_{\propto,1}(t)$$

$= \frac{1}{1-\propto}[\sum_{i=1}^{n} p_i^{\propto} - 1]$, $\propto \neq 1$

$= (-\sum_{i=1}^{n} p_i^{\alpha} ln p_i)$, $\alpha \neq 1$ (3.5)

$f_{1,1}(1) = -\sum_{i=1}^{n} p_i ln p_i$ (3.6)

Equ. (3.6) is the Shannon's measure of entropy. Which is a trivial generating function for Shannon's [2] measure of entropy.

(II) Let,

$f_{\alpha,\beta}(t) = \frac{1}{1-\alpha\beta}\left(\sum_{i=1}^{n}\left(p_i^{\alpha\beta}\right)^t - \beta\right)$, $\alpha > 1, \beta > 1$ (3.7)

$f_{\alpha,1}(t) = \frac{1}{1-\alpha}(\sum_{i=1}^{n}(p_i^{\alpha})^t - 1)$, $\alpha \neq 1$

$f_{\alpha,1}(1) = \frac{1}{1-\alpha}(\sum_{i=1}^{n} p_i^{\alpha} - 1)$, $\alpha \neq 1$ (3.8)

Therefore $f_{\alpha,1}(1)$ gives Havrda-Charvat's [5] measure of entropy

Also

$f'_{\alpha,1}(t) = \frac{1}{1-\alpha}(\sum_{i=1}^{n}(p_i^{\alpha})^t \, ln \sum_{i=1}^{n} p_i^{\alpha})$, $\alpha \neq 1$ (3.9)

$f'_{\alpha,1}(0) = \frac{1}{1-\alpha}(\sum_{i=1}^{n} p_i^{\alpha} - 1)$, $\alpha \neq 1$ (3.10)

Therefore $f'_{\alpha,1}(0)$ Renyi's [6] measure of entropy so that $f_{\alpha,\beta}(t)$ is the generating function for Renyi's [6] measure of entropy.

Again we take $\alpha \to 1$

$\lim_{\alpha \to 1} f_{\alpha,1}(1) = f_{1,1}(1)$

$= (-\sum_{i=1}^{n} p_i^{\alpha} ln p_i)$, $\alpha \neq 1$ (3.11)

$f_{1,1}(1) = -\sum_{i=1}^{n} p_i ln p_i$ (3.12)

Which is a trivial generating function for Shannon's [2] measure of entropy.

(III) Again we take (3.7)

$f_{\alpha,\beta}(t) = \frac{1}{1-\alpha\beta}\left(\sum_{i=1}^{n}\left(p_i^{\alpha\beta}\right)^t - \beta\right)$, $\alpha > 1, \beta > 1$

$f_{2,1}(t) = -\left(\sum_{i=1}^{n}(p_i^2)^t - 1\right)$ (3.13)

$f_{2,1}(1) = \left(1 - \sum_{i=1}^{n} p_i^2\right)$ (3.14)

$f_{2,1}(1)$ is the Vajda's [9] measures of entropy

$f'_{\alpha,1}(t) = \frac{1}{1-\alpha}(\sum_{i=1}^{n}(p_i^{\alpha})^t \, ln \sum_{i=1}^{n} p_i^{\alpha})$, $\alpha \neq 1$ (3.15)

$f'_{\alpha,1}(0) = -\frac{1}{1-\alpha} ln \sum_{i=1}^{n} {p_i}^{\alpha}$, $\alpha \neq 1$ (3.16)

$f'_{\alpha,1}(0) = -\frac{1}{1-\alpha}(1-\alpha) ln(\sum_{i=1}^{n} {p_i}^{\alpha})^{\frac{1}{\alpha-1}}$, $\alpha \neq 1$ (3.17)

Equation (3.17) is antilogarithm of (1-α) times Behara-Chawala [10] measure of entropy, so that $f_{\alpha,\beta}(t)$ can be regarded as generating function for Behara-Chawala [10] measure of entropy.

(IV) Again we defined

$f_{\alpha,\beta,\gamma}(t) = \frac{1}{\gamma-\alpha}\left(\sum_{i=1}^{n}\left(p_i^{\alpha\beta}\right)^t - \gamma\right)$ $\alpha \neq \gamma, \gamma > 0$ (3.18)

$f_{1,1,2}(t) = (\sum_{i=1}^{n}(p_i)^t - 2)$ (3.19)

$f'_{1,1,2}(t) = \sum_{i=1}^{n}(p_i)^t \, ln \, p_i$ (3.20)

$f'_{1,1,2}(0) = \sum_{i=1}^{n} ln \, p_i$ (3.21) Equation (3.21) is Burg's measure of entropy, so that $f_{\alpha,\beta,\gamma}(t)$ can be regarded as generating function for Burg's measure of entropy [11]

Again we take expression (3.18) with different values

$f_{2,\frac{1}{2},1}(t) = -(\sum_{i=1}^{n}(p_i)^t - 1)$

$f_{2,\frac{1}{2},1}(t) = (1 - \sum_{i=1}^{n}(p_i)^t)$ (3.22)

$f'_{2,\frac{1}{2},1}(t) = -(\sum_{i=1}^{n}(p_i)^t lnp_i)$ (3.23)

$f'_{2,\frac{1}{2},1}(1) = -\sum_{i=1}^{n} p_i lnp_i$ (3.24)

Equation (3.24) gives Shannon's [2] measure of entropy.

Again (3.22) becomes

$f_{2,\frac{1}{2},1}(t) = 1 - \sum_{i=1}^{n}(p_i)^t$ (3.25)

$f_{2,\frac{1}{2},1}(2) = 1 - \sum_{i=1}^{n}(p_i)^2$ (3.26)

Equation (3.26) gives the Vajda's [9] measure of entropy

$f_{\alpha,1,1}(t) = \frac{1}{1-\alpha}(\sum_{i=1}^{n}(p_i^{\alpha})^t - 1))$

$f_{\alpha,1,1}(1) = \frac{1}{1-\alpha}(\sum_{i=1}^{n}(p_i^{\alpha}) - 1)$

Therefore $f_{\alpha,1,1}(1)$ gives Havrda-Charvat's [5] measure of entropy.

(VI) Now we define

$f_{\alpha,\beta}(t) = \frac{1}{1-\alpha\beta}\left(\sum_{i=1}^{n}\left(p_i^{\alpha/\beta}\right)^t - \beta\right),$ $\alpha > 1 , \beta > 0$ (3.34)

$f_{\alpha,1}(t) = \frac{1}{1-\alpha}(\sum_{i=1}^{n}(p_i^{\alpha})^t - 1), \alpha \neq 1$ (3.35)

$f_{\alpha,1}(1) = \frac{1}{1-\alpha}(\sum_{i=1}^{n} p_i^{\alpha} - 1), \alpha \neq 1$ (3.36)

Therefore $f_{\alpha,1}(t)$ gives Havrda-Charvat's [5] measure of entropy.

$f'_{\alpha,1}(t) = \frac{1}{1-\alpha}(\sum_{i=1}^{n}(p_i^{\alpha})^t lnp_i^{\alpha}), \alpha \neq 1$ (3.37)

$f'_{\alpha,1}(0) = \frac{1}{1-\alpha}(\sum_{i=1}^{n} lnp_i^{\alpha}), \alpha \neq 1$ (3.38)

$f'_{\alpha,1}(0) = \frac{\alpha}{1-\alpha}(\sum_{i=1}^{n} lnp_i), \alpha \neq 1$ (3.39)

Equation (3.39) is $\frac{\alpha}{1-\alpha}$ times of Burg's measure of entropy.

$f_{2,1}(t) = -\left(\sum_{i=1}^{n}(p_i^2)^t - 1\right),$ (3.40)

$f_{2,1}(t) = 1 - \sum_{i=1}^{n}(p_i^2)^t$ (3.41)

$f_{2,1}(1) = 1 - \sum_{i=1}^{n} p_i^2$ (3.42)

$f'_{2,1}(t) = -\sum_{i=1}^{n}(p_i^2)^t \ln p_i^2$ (3.43)

$f'_{2,1}(1/2) = -2\sum_{i=1}^{n} p_i lnp_i$ (3.43)

Thus $f_{2,1}(1), f'_{2,1}(1/2)$ gives Vajda's [9] and two times of Shannon's [2] measure of entropy respectively.

(V) Now we define new two parametric generating function

$\overline{f_{\frac{b}{a}}}(t) = -\sum_{i=1}^{n} p_i{}^t + \frac{a}{b}\sum_{i=1}^{n}(1 + \frac{b}{a}p_i)^t - \frac{a}{b}\sum_{i=1}^{n}(1 + \frac{b}{a})^t p_i ,$ b> 0 , 0 < a ≤ 1 (3.44) So that,

$\overline{f'_{\frac{b}{a}}}(t) = -\sum_{i=1}^{n} p_i{}^t \ln p_i + \frac{a}{b}\sum_{i=1}^{n}(1 + \frac{b}{a}p_i)^t \ln\left(1 + \frac{b}{a}p_i\right) - \frac{a}{b}\sum_{i=1}^{n}\left(1 + \frac{b}{a}\right)^t \ln\left(1 + \frac{b}{a}\right),$ $\overline{f'_{\frac{b}{a}}}(1) = -\sum_{i=1}^{n} p_i \ln p_i + \frac{a}{b}\sum_{i=1}^{n}(1 + \frac{b}{a}p_i) \ln\left(1 + \frac{b}{a}p_i\right) - \frac{a}{b}\left(1 + \frac{b}{a}\right) \ln\left(1 + \frac{b}{a}\right),$ (3.45) the above expression (3.45) is regarded as the generating function for two parametric measures of entropy

$-\sum_{i=1}^{n} p_i \ln p_i + \frac{a}{b}\sum_{i=1}^{n}(1 + \frac{b}{a}p_i) \ln\left(1 + \frac{b}{a}p_i\right) - \frac{a}{b}\left(1 + \frac{b}{a}\right) \ln\left(1 + \frac{b}{a}\right) p_i ,$ (3.46)

Again (3.45) becomes

$\overline{f'_{\frac{b}{1}}}(t) = -\sum_{i=1}^{n} p_i{}^t \ln p_i + \frac{1}{b}\sum_{i=1}^{n}(1 + bp_i)^t \ln(1 + bp_i) - \frac{1}{b}\sum_{i=1}^{n}(1 + b)^t \ln(1 + b)$,(3.47)

$\overline{f'_{\frac{a}{1}}}(t) = -\sum_{i=1}^{n} p_i{}^t \ln p_i + \frac{1}{a}\sum_{i=1}^{n}(1+ap_i)^t \ln(1+ap_i) - \frac{1}{a}\sum_{i=1}^{n}(1+a)^t \ln(1+a)$,

$\overline{f'_{\frac{a}{1}}}(1) = -\sum_{i=1}^{n} p_i ln p_i + \frac{1}{a}(1+ap_i)\ln(1+ap_i) - \frac{1}{a}\sum_{i=1}^{n}(1+a)\ln(1+a),$ (3.48)

Which gives Kapur's [8] [7] measures of entropy, so that (3.44) can be regarded as generating function for Kapur's [8] [7] measure of entropy,

$\sum_{i=1}^{n} p_i ln p_i + \frac{1}{a}(1+ap_i)\ln(1+ap_i) - \frac{1}{a}\sum_{i=1}^{n}(1+a)\ln(1+a)p_i,$ a> 0

Again (3.48) becomes

$\overline{f'_{\frac{1}{1}}}(1) = -\sum_{i=1}^{n} p_i ln p_i + (1+p_i)\ln(1+p_i) - 2ln\,2$

which is Bose- Einstein's [7] measure of entropy.

$\overline{f'_{\frac{-1}{1}}}(1) = -\sum_{i=1}^{n} p_i ln p_i - (1-p_i)\ln(1-p_i)$

which is Fermi-Dirac [7] measure of entropy

(VI) now we define three parametric bi-measure of entropy

$\overline{f_{a,b,k}}(t) = -\sum_{i=1}^{n} p_i{}^t + \frac{b}{a^k}\left[\sum_{i=1}^{n}\left(1+\frac{a}{b}p_i\right)^t - p_i\left(1+\frac{a}{b}\right)^t\right]$, b> 0 , 0 < a ≤ 1 Where a,b,k are parametric, (3.49)

with the property that,

$\overline{f'_{a,b,k}}(t) = -\sum_{i=1}^{n} p_i{}^t ln p_i + \frac{b}{a^k}\left[\sum_{i=1}^{n}\left(1+\frac{a}{b}p_i\right)^t ln\left(1+\frac{a}{b}p_i\right) - p_i\left(1+\frac{a}{b}\right)^t ln\left(1+\frac{a}{b}\right)\right]$, b> 0 , 0 < a ≤ 1 (3.50)

$\overline{f'_{a,b,k}}(1) = -\sum_{i=1}^{n} p_i ln p_i + \frac{b}{a^k}\left[\sum_{i=1}^{n}\left(1+\frac{a}{b}p_i\right) ln\left(1+\frac{a}{b}p_i\right) - p_i\left(1+\frac{a}{b}\right) ln\left(1+\frac{a}{b}\right)\right]$ (3.51)

the above expression (3.46) can be regarded as the generating function for two parametric bi-measures of entropy,

$H_{a,b,k}(P) = -\sum_{i=1}^{n} p_i ln p_i + \frac{b}{a^k}[\sum_{i=1}^{n}(1+ap_i)ln(1+ap_i) - p_i(1+a)ln(1+a)]$, a ≥ −1, b > 0

(I) If We take k=1,a = b and b= a in (3.49) approached to (3.44)

(II) If we take k = 1and b=1, in generating function (3.44) we get generating function for Kapur's [8] measure of entropy (3.48)

(III) By taking k= 1 , b =1 and a=1 in (3.49) we get generating function for Bose- Einstein's [7] measure of entropy.

(IV) If k=1, b=1 and a= -1 we get a generating function for Fermi-Dirac [7] measure of entropy

(V) If we put k = 2 and b =1 in (3.49) we get

$\overline{f_{a,1,2}}(t) = -\sum_{i=1}^{n} p_i{}^t + \frac{1}{a^2}[\sum_{i=1}^{n}(1+ap_i)^t - p_i(1+a)^t]$ (3.52)

$\overline{f'_{a,1,2}}(t) = -\sum_{i=1}^{n} p_i{}^t ln p_i + \frac{1}{a^2}[\sum_{i=1}^{n}(1+ap_i)^t ln(1+a\,p_i) - p_i(1+a)^t ln(1+a)]$, a ≥ −1 (3.53)

$\overline{f'_{a,b,k}}(1) = -\sum_{i=1}^{n} p_i ln p_i + \frac{1}{a^2}[\sum_{i=1}^{n}(1+ap_i)ln(1+a\,p_i) - p_i(1+a)ln(1+a)],$ a ≥ −1

Which is Kapur's [8] measure of entropy , therefore (3.49) can be regarded as a regarded as generating function for Kapur's measure of entropy [8].

4. CONCLUDING REMARKS

- In the existing literature of information theory, we find various measures of information, directed divergence and inaccuracy, each with its own merits, demerits and limitations but our best interest lies in the development of those measures which can find applications in a variety of disciplines.

- In biological, science, we have observed that researchers frequently use Shannon's [2] or Renyi's [6] measures for measuring diversity in different species but, if we have a variety of information measures then we shall be more flexible in applying a standard measures depending upon the situation keeping this idea in mind we have developed some measures depending upon the situations.

- In this paper we obtained generating functions for several measures of information, directed divergence and inaccuracy corresponding to the Shannon's [2] measures of entropy, Kapur's [7] , [8] measures of entropy, Renyi's [6] measures of entropy, Kerridge's [12] measure of entropy, Bose-Einstein's [7], Vejda's [9] , Behara-Chawala [10] ,Havrda Charvat's [5] measure of entropy We prove that derivative at 1 and 0 of proposed generating functions includes several well-known results.
- We have discussed the results for a random variate takes a finite set of discrete values. The discussion can be easily extended to cases when the number of values taken is infinite or variate is continues, provided the series and integrals which arise all are convergent. In the present discussion we discuss for one probability distribution, two probability distribution but the discussion can be easily extended to multi-variate distribution.

- Almost all the measures of entropy has one, two or more parameter/parameter(s). Here, the suggested measure of entropy has two and three parameters. These parameters provided certain flexibility in applications; and their values can be verified from the data itself.

- The domain of Information theory is the development of probabilistic, non-probabilistic, weighted parametric and non-parametric measures of information. However, here we strive to develop those information theoretic measures which can be effectively applied to the diverse disciplines.

- The amount of uncertainty of a probability distribution is measured by a measure of entropy; however, the discrimination of information or discrepancy between two given probability distributions is measured by a measure of directed divergence

- The merits and advantages of the new two parametric divergence measures is that it is a distance measure; and the value of divergence can be regulated by adjusting the value of parameter a and b. With the intent to increase the flexibility of application of this divergence measure, two parametric generalizations may be discovered.

- As the parameter b tends to zero, it approaches Shannon's [13] entropy.
  It is verified that $\left(1+\frac{b}{a}x\right)$is the concave function of x and $\sum_{i=1}^{n}(1+\frac{b}{a}p_i)$ is a concave function of $p_1, p_2, \ldots, p_n$ of which the minimum value is $\left(1+\frac{b}{a}\right)$.

$H_{\frac{b}{a}}(P)$ is decreasing function of b which is decreasing from $-\sum_{i=1}^{n} p_i \ln p_i$ to b and increasing function of b increasing from 0 to ∞.

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