A STUDY OF THE COUPLE-STRESS ON VISCO-ELASTIC FLUID FLOW SATURATING A POROUS MEDIUM IN THE PRESENCE OF HALL CURRENT

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Abstract

We have studied the effect of couple-stress on the visco-elastic fluid layer heated from below in the presence of Hall current through porous media. The dispersion relation has been analyzed by the normal mode method. The problem has been investigated the analytically and numerically. We have obtained the effects of porosity, couple-stress, magnetic field, and Hall current and the impact of Hall current is a very significant result of this model. We compare these results with previous work and they agree with each other.

Keywords: *Hall Current, Magnetic Field, Couple-stress, Visco-elastic Fluid, Porous Medium.*

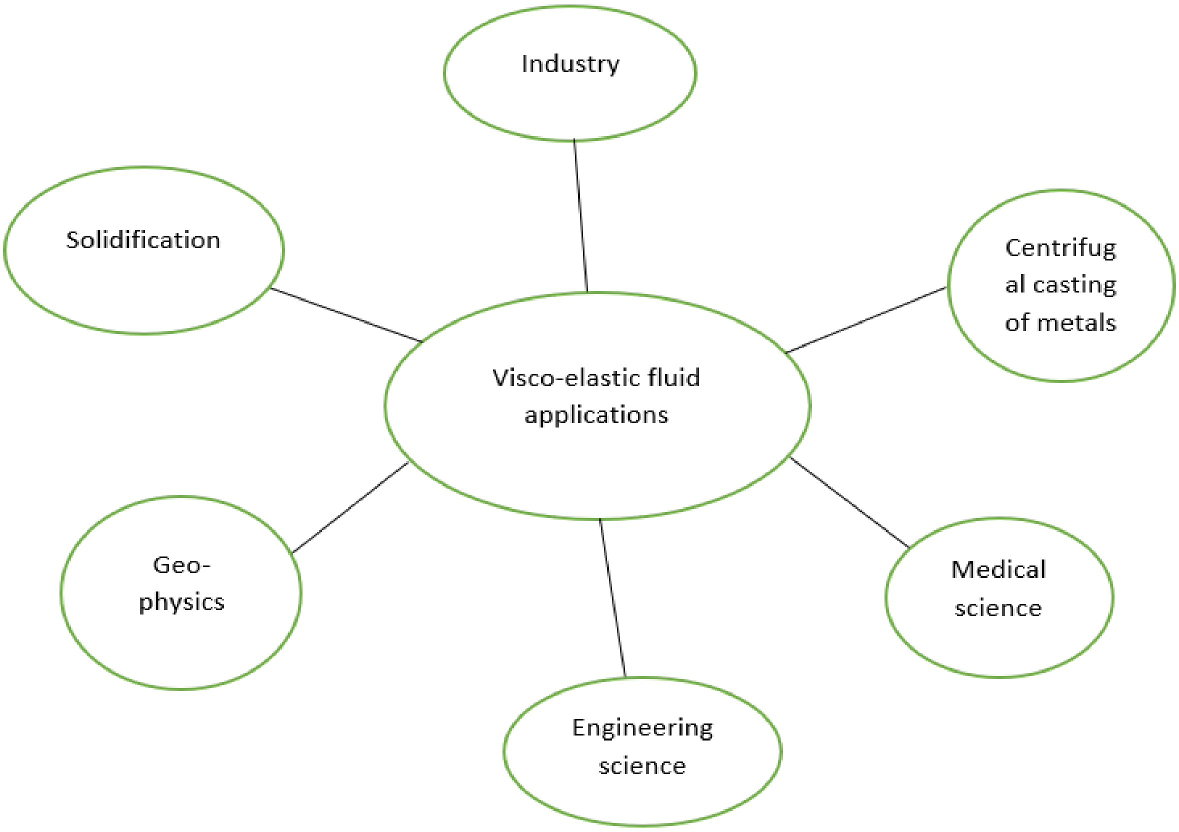
2020 Mathematical Science Classification: 76A10, 76D05, 76D10

1. Introduction

The visco-elastic fluids are gaining considerable interest and research focus due to their applications in geophysics, medical science, metal plate cooling and engineering sciences. The standard theory of viscous elastic fluid and viscous elastic boundary layer flow have been discovered by [1, **2**]. The concepts of hydromagnetic two-phase flow, micropolar fluid and viscous fluid flow in a vertical channel were investigated by [3,4]. Thermal convection and dust particles on the viscous elastic fluid flow via a porous media analyzed by [5,6]. The effects of porosity, magnetic and other parameters on viscous and viscous fluid flow through porous media with Hall effect have been analyzed by [7,8]. The theory of Rayleigh instability on the thermal boundary layer flow in a porous media examined by [9]. Entropy study of MHD fluid flow with thermal effects was covered by [10]. Electro-elastic instability and EHD instability on visco-elastic fluids in porous media with heat transfer were investigated by [11, 12].

The general theory of couple-stress fluids was discovered by [13]. The effects of rotation, thermal convection and couple tension on viscoelastic fluid flow have been discussed by [14]. The introduction of convection in couple-stress fluid flow saturating a porous medium by the Galerkin method was developed by [15]. The effects of couple-tension, magnetic field, porosity, rotation and suspended particles on microporous and viscous elastic fluid flows have been analyzed analytically and numerically [16–18]. Numerical and analytical analysis of the effects of porosity, magnetic field, microscale parameters and various parameters on microscale fluid flow through porous media have been investigated by [19-22].

The applications of this theory are as follows:



In the paper, we discuss the effects of porosity, couple-stress, magnetic field and Hall current on viscoelastic fluid flow in the presence of porous media. To our knowledge, this problem has not been currently investigated using the Darcy's generalized model.

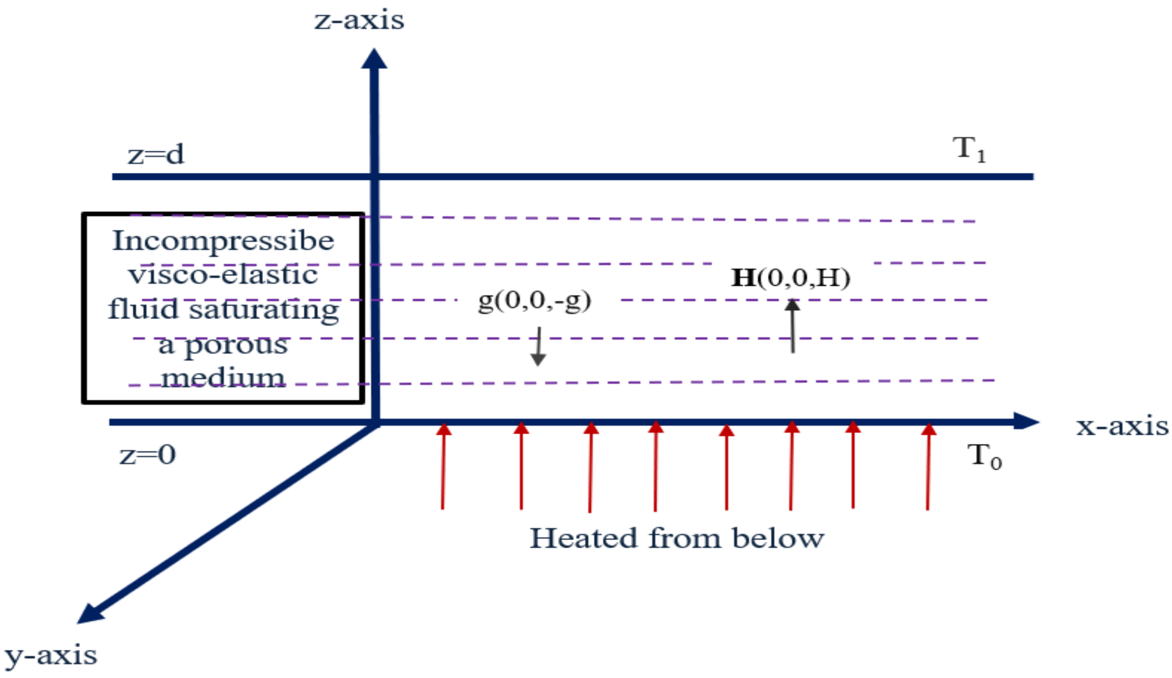
2. Mathematical Formulation

A horizontal, infinite, and incompressible electrically non-conducting fluid layer of thickness d is assumed, has porosity $\in$ and medium permeability k_1. The upper $z = d$ and lower limits $z = 0$ are maintained at constant but varying temperatures T_0 and T_1 such that a study adverse temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ has been continued. The magnetic field and gravity are applied along with the z-axis of the system.

The equations of continuity, motion, angular momentum, temperature and basic's state are:

$$\nabla.\vec{q} = 0 \tag{2.1}$$

$$\frac{\rho_0}{\in}\left[\frac{1}{\in}(\vec{q}.\nabla)\vec{q} + \frac{\partial\vec{q}}{\partial t}\right] = -\nabla P - \rho g\hat{e}_z + \left(\mu - \frac{\nu}{\rho_0}\nabla^2\right)\nabla^2\vec{q} - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\vec{q} + \frac{\mu_e}{4\pi}\left(\nabla\times\vec{H}\right)\times\vec{H} \tag{2.2}$$



$$\left[\in\rho_0 C_v+(1-\in)\rho_s C_s\right]\frac{\partial T}{\partial t}+\rho_0 C_v\left(\vec{q}.\nabla\right)T=\chi\nabla^2 T \tag{2.3}$$

$$\rho=\rho_0\left[1-\alpha\left(T-T_a\right)\right] \tag{2.4}$$

The Maxwell's equations are:

$$\in\frac{\partial\vec{H}}{\partial t}=\nabla\times\left(\vec{q}\times\vec{H}\right)+\in\gamma_m\nabla^2\vec{H}-\frac{\in}{4\pi e n_e}\nabla\times\left[\left(\nabla\times\vec{H}\right)\times\vec{H}\right] \tag{2.5}$$

$$\nabla.\vec{H}=0 \tag{2.6}$$

where, $\vec{q}$ – Velocity, ρ – Fluid density, P – Pressure, ρ_0 – Reference density, ρ_s – Density of solid matrix, T – Temperature, ν – Couple-stress Viscosity, μ - Viscosity, μ' - Viscoelasticity, α – Coefficient of thermal expansion, $\hat{e}_z$ – Unit vector, C_v – Specific heat at constant volume, χ – Thermal conductivity, C_s – Specific heat of solid, T_a – Average temperature is given by $T_a=\frac{(T_0+T_1)}{2}$, t – time, $\vec{H}=(0,0,H_z), H_z$ – Constant, n_e – Electron density, e – Charge on electron and γ_m – Magnetic viscosity.

3. Basic's State

The basic's state of the problem is taken as:

$\vec{q} = \vec{q}_b(0,0,0)$, $\rho = \rho_b(z)$ and $P = P_b(z)$

Using the above conditions in equations (2.1) to (2.4), we get

$$\frac{dP_b}{dz} + \rho_b g = 0 \tag{3.1}$$

$$T = -\beta z + T_a = T_b(z) \tag{3.2}$$

$$\rho_b = \rho_0 + \alpha\beta z \rho_0 \tag{3.3}$$

4. Linearized Perturbation Equations

Now, we the linearize equations (2.1) to (2.6), we obtained

$$\nabla.\vec{q}' = 0 \tag{4.1}$$

$$\frac{\rho_0}{\in}\frac{\partial \vec{q}'}{\partial t} = -\nabla P' - \rho' g\hat{e}_z + \left(\mu - \frac{\nu}{\rho_0}\nabla^2\right)\nabla^2\vec{q}' - \frac{1}{k_1}\left(\mu + \mu'\frac{\partial}{\partial t}\right)\vec{q}' + \frac{\mu_e H_z}{4\pi}\left(\nabla \times \vec{h}\right)\times\hat{e}_z \tag{4.2}$$

$$E\frac{\partial \theta}{\partial t} = k_T \nabla^2\theta + \beta\,\vec{q}.\hat{e}_z \tag{4.3}$$

$$\in\frac{\partial \vec{h}}{\partial t} = H_z \times\left(\vec{q}'\times\hat{e}_z\right) + \in\gamma_m\nabla^2\vec{h} - \frac{\in H_z}{4\pi e n_e}\nabla\times\left[\left(\nabla\times\vec{h}\right)\times\hat{e}_z\right] \tag{4.4}$$

$$\nabla.\vec{h} = 0 \tag{4.5}$$

$$\rho' = -\rho_0\alpha\theta \tag{4.6}$$

Converting the equations (4.1) to (4.6) by the following transformations:

$x = dx^*$, $y = dy^*$, $z = dz^*$, $\vec{q}' = \frac{k_T}{d}\vec{q}^*$, $\nabla = \frac{\nabla^*}{d}$, $P' = \frac{\mu k_T}{d^2}P^*$, $t = \frac{\rho_0 d^2}{\mu}t^*$, and $\theta = \beta d\,\theta^*$, we have

$$\nabla.\vec{q} = 0 \tag{4.7}$$

$$\frac{1}{\in}\frac{\partial \vec{q}}{\partial t} = -\nabla P + R\theta\hat{e}_z + \left(1 - F\nabla^2\right)\nabla^2\vec{q} - \frac{1}{K_1}\left(1 + F_1\frac{\partial}{\partial t}\right)\vec{q} + Q\left(\nabla\times\vec{h}\right)\times\hat{e}_z \tag{4.8}$$

$$EP_r\frac{\partial \theta}{\partial t} = \nabla^2\theta + \vec{q}.\hat{e}_z \tag{4.9}$$

$$\in P_r \frac{\partial \vec{h}}{\partial t} = \frac{\partial \vec{q}}{\partial z} + \frac{\in P_r}{P_m} \nabla^2 \vec{h} - \in \beta_e^{1/2} \frac{\partial}{\partial z} \left(\nabla \times \vec{h} \right) \tag{4.10}$$

where, $P_r = \frac{\mu}{k_T \rho_0}$ – Prandtl number, $F = \frac{\nu}{\rho_0 d^2}$, $Q = \frac{\mu_e H_z^2 d^2}{4\pi \mu k_T}$ – Chandrasekhar number, $k_T = \frac{\chi}{\rho_0 C_v}$, $F_1 = \frac{\mu'}{\rho_0 d^2}$ – Viscoelastic Parameter, $P_m = \frac{\mu}{\rho_0 \gamma_m}$ – Magnetic Prandtl number, $R = \frac{\rho_0 \alpha \beta g d^4}{k_T \mu}$ – Rayleigh number, $\beta_e = \left(\frac{H_z}{4\pi k_T e n_e} \right)^2$ – Hall parameter, $E = \in + \frac{(1-\in)\rho_s C_s}{\rho_0 C_v}$, $K_1 = \frac{k_1}{d^2}$, and $W = \vec{q}.\hat{e}_z$.

5. Boundary conditions

The boundary conditions are:

$$W = \frac{d^2 W}{dz^2} = 0, \theta = 0 \; at \; z = 0 \; and \; z = d. \tag{5.1}$$

6. Dispersion Relation

Taking the curl on both sides in equation (4.6), we have

$$\left[\frac{1}{\in} \frac{\partial}{\partial t} + \frac{1}{K_1} \left(1 + F_1 \frac{\partial}{\partial t} \right) - \left(1 - F \nabla^2 \right) \nabla^2 \right] \left(\nabla \times \vec{q} \right) = \left[R \left(\frac{\partial \theta}{\partial y} \hat{e}_x + \frac{\partial \theta}{\partial x} \hat{e}_y \right) \right] + Q \frac{\partial}{\partial z} \left(\nabla \times \vec{h} \right) \tag{6.1}$$

Assume, $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $D = \frac{\partial}{\partial z}$.

Again, applying the curl and z-component on both sides in equations (6.1) and (4.10), we have

$$\left[\left\{ \frac{1}{\in} \frac{\partial}{\partial t} + \frac{1}{K_1} \left(1 + F_1 \frac{\partial}{\partial t} \right) - \left(1 - F \nabla^2 \right) \nabla^2 \right\} \right] \nabla^2 W = R \nabla_1^2 \theta + Q D \left(\nabla^2 \vec{h}_z \right) \tag{6.2}$$

$$\in P_r \frac{\partial m_z}{\partial t} = D \zeta_z + \frac{\in P_r}{P_m} \nabla^2 m_z + \in \beta_e^{1/2} D \left(\nabla^2 \vec{h}_z \right) \tag{6.3}$$

Taking the z-component on the both sides in equations 6.1), (4.9) and (4.10), we have

$$\left[\left\{\frac{1}{\in}\frac{\partial}{\partial t}+\frac{1}{K_1}\left(1+F_1\frac{\partial}{\partial t}\right)-\left(1-F\nabla^2\right)\nabla^2\right\}\right]\zeta_z = QDm_z \tag{6.4}$$

$$P_r E\frac{\partial\theta}{\partial t} = \nabla^2\theta + W \tag{6.5}$$

$$\in P_r\frac{\partial\vec{h}_z}{\partial t} = DW + \frac{\in P_r}{P_m}\nabla^2\vec{h}_z - \in\beta_e^{1/2}Dm_z \tag{6.6}$$

Thus, the boundary conditions (5.1), becomes

$$W = D^2W = \theta \text{ at } z=0 \text{ to } z=1 \tag{6.7}$$

Normal Mode Analysis

Let $\left[W,\theta,\zeta_z,\vec{h}_z,m_z\right] = \left[W(z),\Theta(z),X(z),B(z),M(z)\right]\exp.\left[ik_xx+ik_yy+\sigma t\right]$

Apply normal modes in equations (6.2) to (6.6), we get

$$\left[\frac{\sigma}{\in}+\frac{1}{K_1}(1+F_1\sigma)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]\left(D^2-a^2\right)W = -Ra^2\Theta + QD\left(D^2-a^2\right)B \tag{6.8}$$

$$\left[\frac{\sigma}{\in}+\frac{1}{K_1}(1+F_1\sigma)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]X = QDM \tag{6.9}$$

$$\left[EP_r\sigma-\left(D^2-a^2\right)\right]\Theta = W \tag{6.10}$$

$$\left[\in P_r\sigma-\frac{\in P_r}{P_m}\left(D^2-a^2\right)\right]M = DX + \in\beta_e^{1/2}D\left(D^2-a^2\right)B \tag{6.11}$$

$$\left[\in P_r\sigma-\frac{\in P_r}{P_m}\left(D^2-a^2\right)\right]B = DW - \in\beta_e^{1/2}DM \tag{6.12}$$

where, $\sigma = \sigma_r + i\sigma_r$ - stability parameter and $a^2 = k_x^2 + k_y^2$ - wave number.

Thus, the boundary conditions are:

$$W = D^2W = X = DX = 0, \Theta = 0 \text{ at } z=0 \text{ to } z=1 \tag{6.13}$$

$D^{2n}W = 0$ at $z=0$ to $z=1,\ n>0$.

The exact explanation of the equation (6.9), we get

$W = W_0 \sin\pi z$, W_0 - Constant.

Eliminating the Θ and B from equations (6.8) to (6.12), putting the value of W and $b=\pi^2+a^2$, we have

$$\left[\left\{\left(\in P_r\sigma+\frac{\in bP_r}{P_m}\right)^2+\in^2\beta_e\pi^2 b\right\}\left\{\frac{\sigma}{\in}+\frac{1}{K_1}(1+F_1\sigma)+Fb^2+b\right\}+Q\pi^2\left(\in P_r\sigma+\frac{\in bP_r}{P_m}\right)\right]$$

$$\left[(-b)(EP_r\sigma+b)\left\{\frac{\sigma}{\in}+\frac{1}{K_1}(1+F_1\sigma)+Fb^2+b\right\}+Ra^2\right] \tag{6.14}$$

$$=Q\pi^2 b(EP_r\sigma+b)\left[\left(\in P_r\sigma+\frac{\in bP_r}{P_m}\right)\left\{\frac{\sigma}{\in}+\frac{1}{K_1}(1+F_1\sigma)+Fb^2+b\right\}+Q\pi^2\right]$$

7. Stationary Convection

Putting $\sigma=0$ in equation (6.14), we have

$$R=\frac{b^2}{a^2}\frac{\in^2\beta_e\pi^2 b^3\left\{\frac{1}{K_1}+Fb^2+b\right\}^2+\left[\left\{\frac{1}{K_1}+Fb^2+b\right\}\left(\frac{\in bP_r}{P_m}\right)+Q\pi^2\right]^2}{\left[\left\{\left(\frac{\in bP_r}{P_m}\right)^2+\in^2\beta_e\pi^2 b\right\}\left\{\frac{1}{K_1}+Fb^2+b\right\}+Q\pi^2\left(\frac{\in bP_r}{P_m}\right)\right]} \tag{7.1}$$

Neglecting the couple-stress (i.e. $F=0$), we have

$$R=\frac{b^2}{a^2}\frac{\in^2\beta_e\pi^2 b^3\left\{\frac{1}{K_1}+b\right\}^2+\left[\left\{\frac{1}{K_1}+b\right\}\left(\frac{\in bP_r}{P_m}\right)+Q\pi^2\right]^2}{\left[\left\{\left(\frac{\in bP_r}{P_m}\right)^2+\in^2\beta_e\pi^2 b\right\}\left\{\frac{1}{K_1}+b\right\}+Q\pi^2\left(\frac{\in bP_r}{P_m}\right)\right]}$$

It is derived by Kumar and Mehta [8].

Neglecting the magnetic field (i.e. $\beta_e=0$ and $Q=0$), we have

$$R=\frac{b^2}{a^2}\left\{\frac{1}{K_1}+b\right\} \tag{7.2}$$

Now, we take the non-porous media $(K_1\to\infty)$ in equation (7.2), become

$$R=\frac{b^3}{a^2}$$

It is discovered by G. Lebon and C. Perez-Garcia [22].

Result and discussion

Now, we discuss the behaviour of medium permeability, magnetic field, Hall current and couple-stress analytically and numerically from equation (7.1).

$$\frac{dR}{dK_1} = -\frac{b^2}{K_1^2 a^2} \frac{\left[\left\{\left(\frac{\in bP_r}{P_m}\right)^2 + \in^2 \beta_e \pi^2 b\right\}\left\{\frac{1}{K_1} + Fb^2 + b\right\}^2 \left\{\in^2 \beta_e \pi^2 b^3 + \left(\frac{\in bP_r}{P_m}\right)^2\right\}\right] + \left[Q\pi^2 \in^2 \beta_e \pi^2 b \left\{2\left\{\frac{1}{K_1} + Fb^2 + b\right\}\left(\frac{\in bP_r}{P_m}\right) - Q\pi^2\right\}\right] + Q\pi^2 \left(\frac{\in bP_r}{P_m}\right)^3}{\left[\left\{\left(\frac{\in bP_r}{P_m}\right)^2 + \in^2 \beta_e \pi^2 b\right\}\left\{\frac{1}{K_1} + Fb^2 + b\right\} + Q\pi^2 \left(\frac{\in bP_r}{P_m}\right)\right]^2} \tag{7.3}$$

It is always negative if $Q < \frac{\in b^4 P_r}{P_m \pi^2}$.

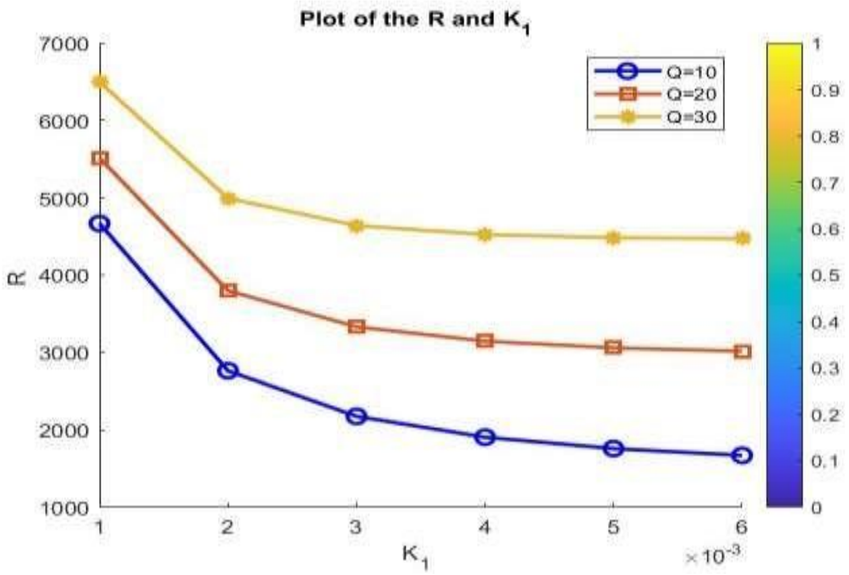


Figure 7.1: Plot between the Rayleigh number R and medium permeability K_1.

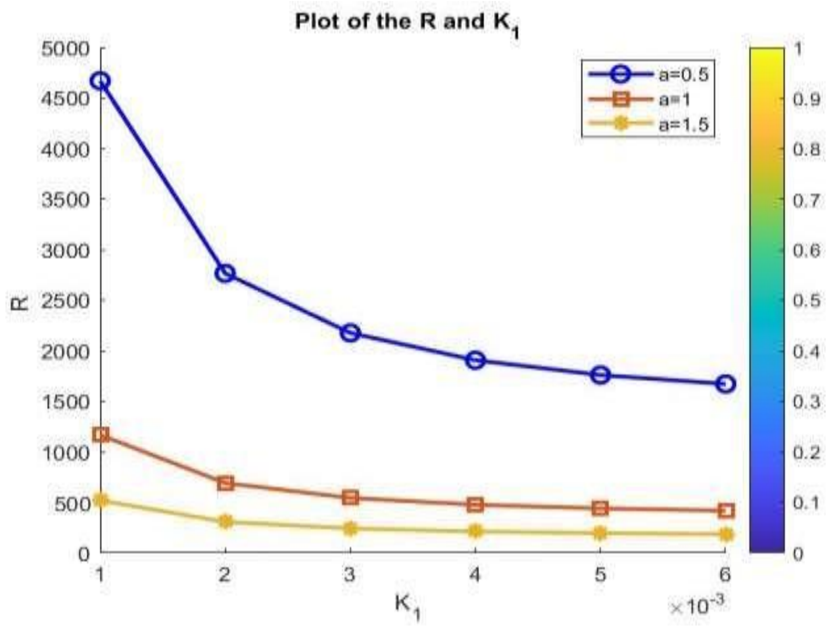


Figure 7.2: Plot of the Rayleigh number R and medium permeability K_1.

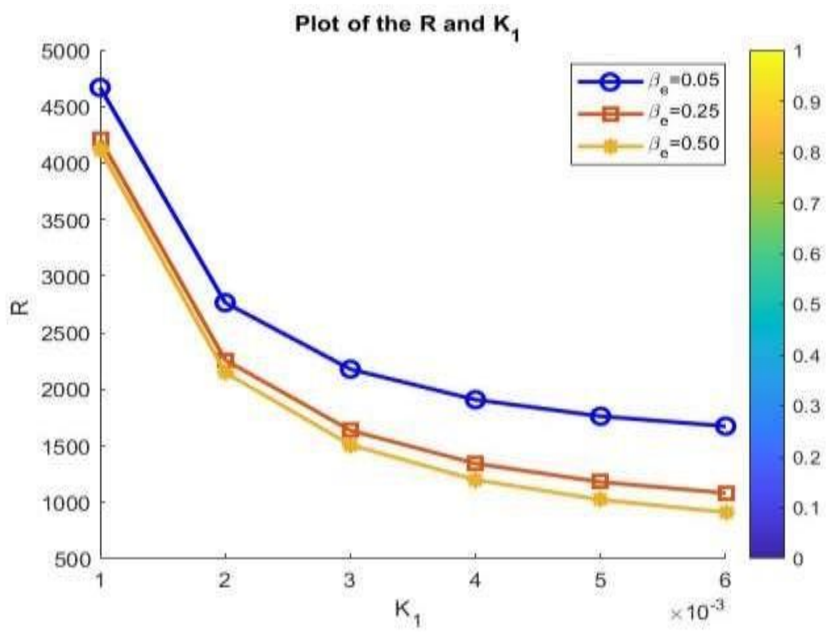


Figure 7.3: Graph between the Rayleigh number R and medium permeability K_1.

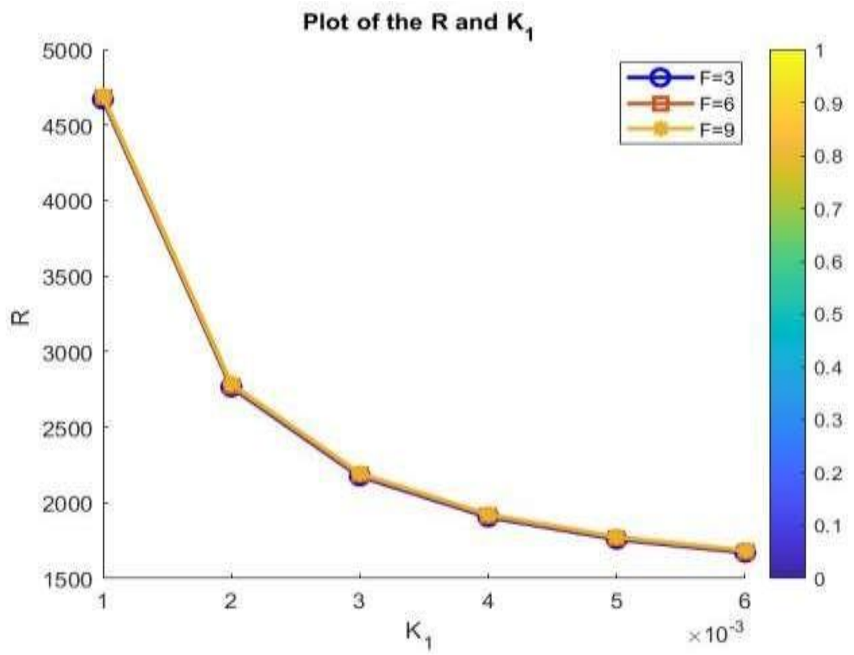


Figure 7.4: Plot of the Rayleigh number R and medium permeability K_1.

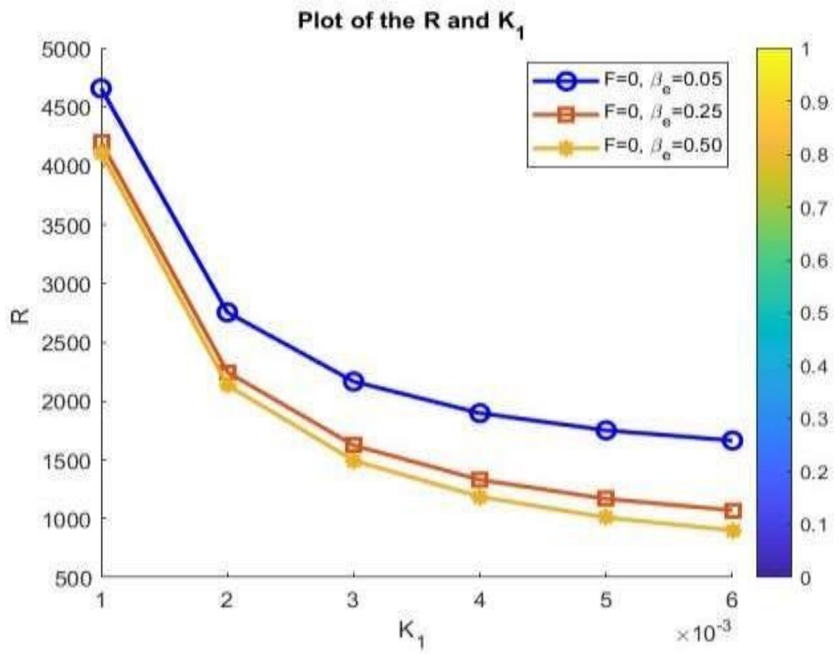


Figure 7.5: Graph between the Rayleigh number R and medium permeability K_1.

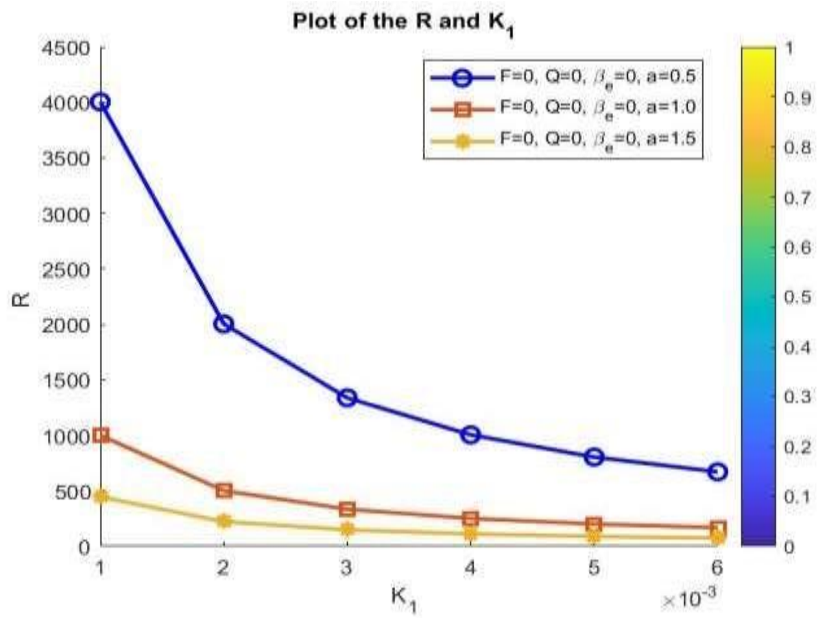


Figure 7.6: Graph of the Rayleigh number R and medium permeability K_1.

Figure 7.1, 7.2, 7.3 and 7.4 illustrations the graphs of Rayleigh number R with permeability K_1 i.e. as the permeability increase then Rayleigh number decrease with magnetic field $Q=(10,20,30)$, wave number $a=(0.5,1.0,1.5)$, Hall current parameter $\beta_e=(0.05,0.25,0.50)$ and couple-stress parameter $F=(3,6,9)$ when $\pi=3.14$, $\in=0.5$, $P_m=4$ and $P_r=2$. Now, neglecting the couple stress in Figure 7.5 and the Hall current, magnetic field and couple-stress parameters in Figure 7.6, then the Rayleigh number decreases as the permeability of the medium increases, which is satisfied the previous results [6], [5], and [16-18].

It is clear that, the effect of medium permeability is destabilizing. In the absence Hall current, magnetic Field and couple-parameter, the effect of medium permeability is always destabilizing.

$$\frac{dR}{dQ}=\frac{b^2}{a^2}\frac{\left[\pi^2\left\{\frac{1}{K_1}+Fb^2+b\right\}\left(\frac{\in bP_r}{P_m}\right)^3+Q\pi^4\left(\frac{\in bP_r}{P_m}\right)^2\right]+\left(Q\pi^6\in^2\beta_e b\right)\left\{\frac{1}{K_1}+Fb^2+b\right\}+\left[\in^2\beta_e\pi^4 b\left(\frac{\in bP_r}{P_m}\right)\left\{\frac{1}{K_1}+Fb^2+b\right\}^2\left\{2-b^2\right\}\right]}{\left[\left\{\left(\frac{\in bP_r}{P_m}\right)^2+\in^2\beta_e\pi^2 b\right\}\left\{\frac{1}{K_1}+Fb^2+b\right\}+Q\pi^2\left(\frac{\in bP_r}{P_m}\right)\right]^2} \tag{7.4}$$

It is always positive if $b<\sqrt{2}$.

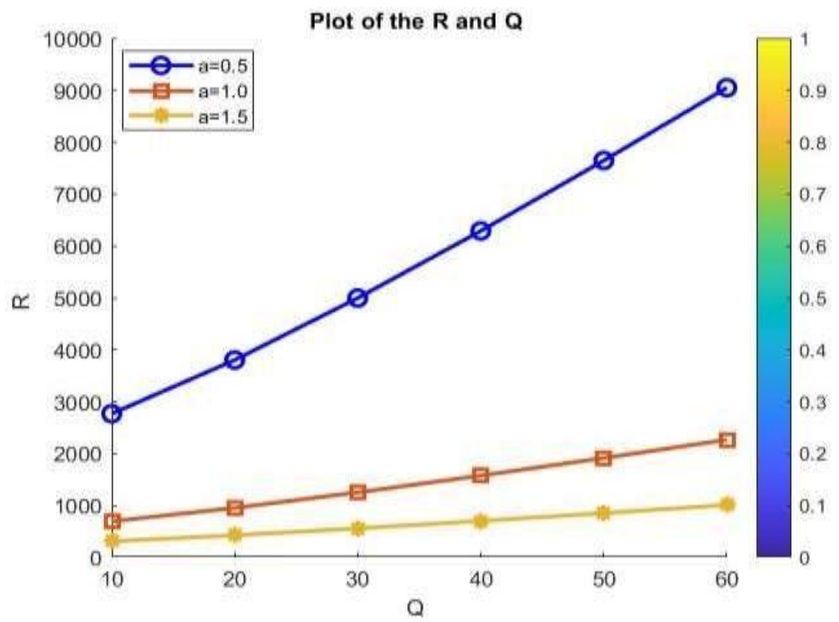


Figure 7.7: Plot between the Rayleigh number R and Magnetic field parameter Q.

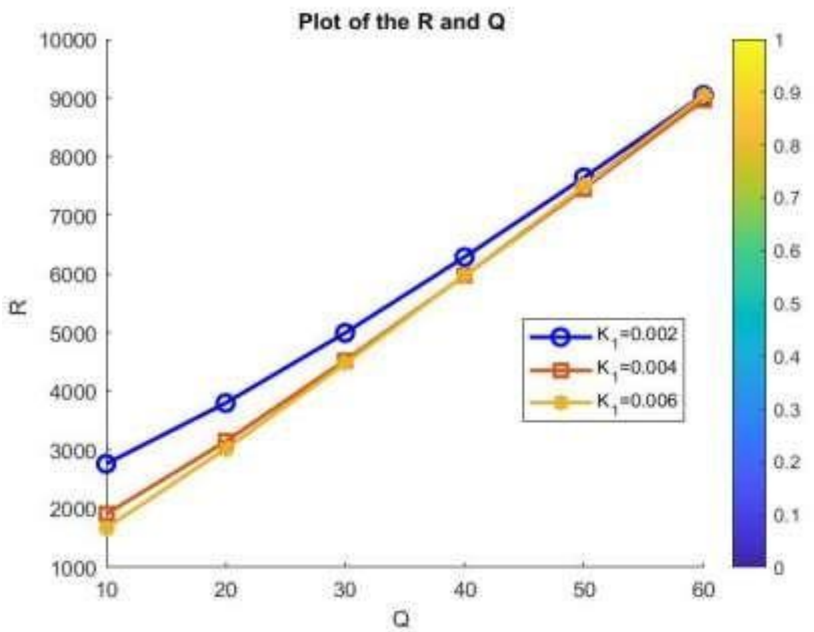


Figure 7.8: Graph of the Rayleigh number R and Magnetic field parameter Q.

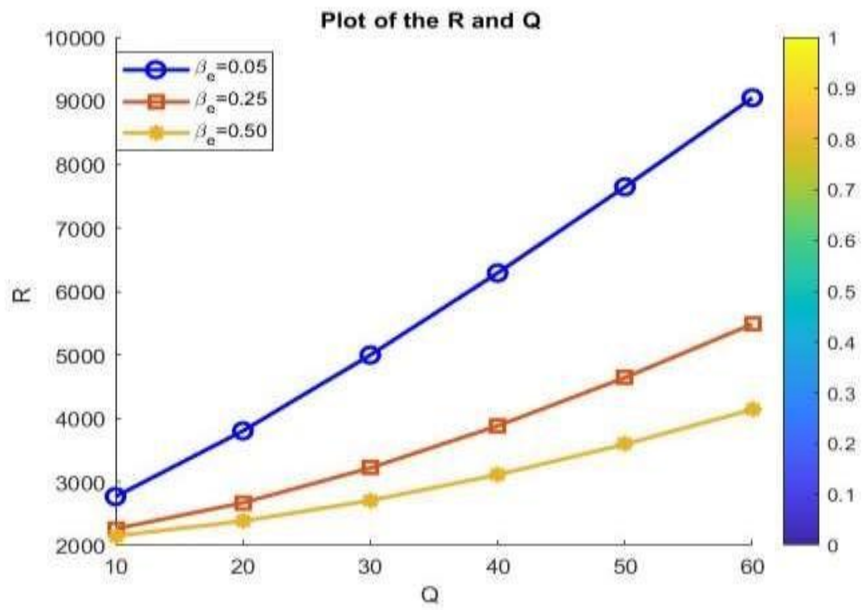


Figure 7.9: Plot between the Rayleigh number R and medium Magnetic field parameter Q.

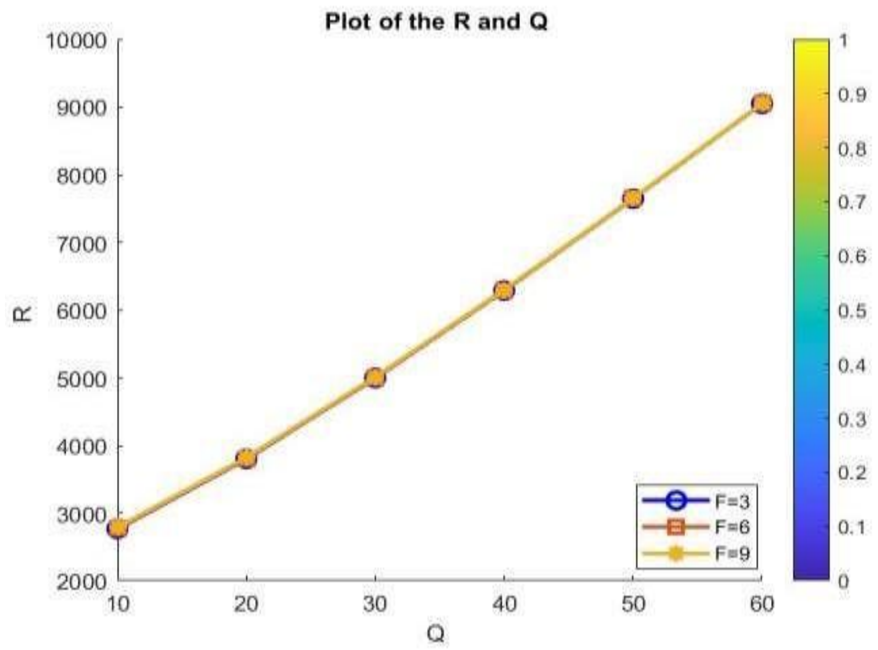


Figure 7.10: Plot of the Rayleigh number R and Magnetic field parameter Q.

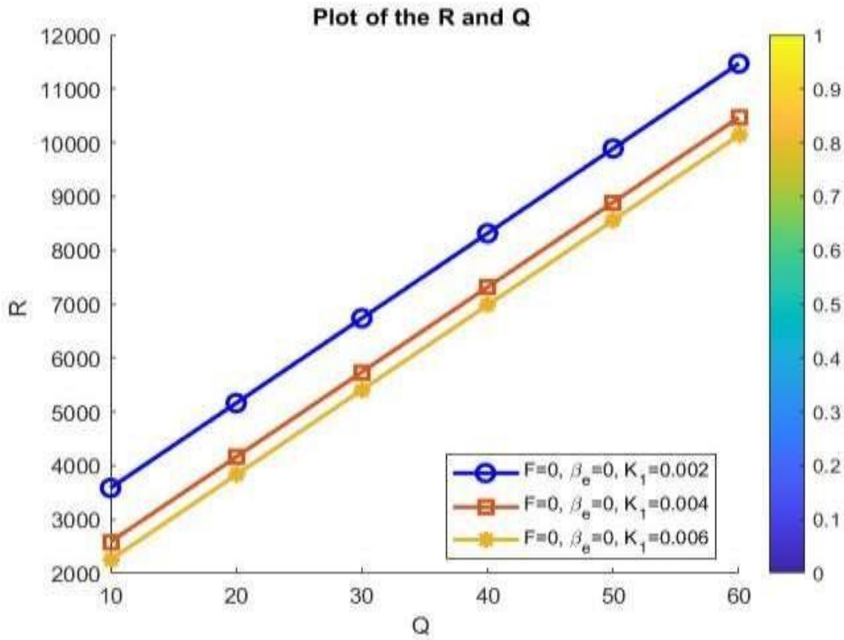


Figure 7.11: Graph between the Rayleigh number R and Magnetic field parameter Q.

Figure 7.7, 7.8, 7.9 and 7.10 shows the graphs of Rayleigh number R with magnetic field i.e. as the magnetic field parameter increase then Rayleigh number increase with the wave number $a=(0.5,1.0,1.5)$, Hall current parameter $\beta_e=(0.05,0.25,0.50)$, medium permeability $K_1=(0.002,0.004,0.006)$, and couple-stress parameter $F=(3,6,9)$ when $\pi=3.14$, $\in=0.5$, $P_m=4$ and $P_r=2$. Now, neglecting the Hall current parameter and couple-parameter in Figure 7.11, then the Rayleigh number increase as the magnetic field parameter increase, which is satisfied the previous result [18].

Thus, the effect of magnetic field is stabilizing. In the absence Hall current parameter and couple-parameter, the effect of magnetic field is always stabilizing.

$$\frac{dR}{d\beta_e}=-\frac{b^2}{a^2}\frac{\left[\in^2\pi^2 b\left\{\frac{1}{K_1}+Fb^2+b\right\}^3\left(1-b^2\right)+Q^2\pi^6\in^2 b\left\{\frac{1}{K_1}+Fb^2+b\right\}\right]+\left[Q^2\pi^4\in^2 b\left(\frac{\in bP_r}{P_m}\right)\left\{\frac{1}{K_1}+Fb^2+b\right\}^2\left(2-b^2\right)\right]}{\left[\left\{\left(\frac{\in bP_r}{P_m}\right)^2+\in^2\beta_e\pi^2 b\right\}\left\{\frac{1}{K_1}+Fb^2+b\right\}+Q\pi^2\left(\frac{\in bP_r}{P_m}\right)\right]^2} \quad (7.5)$$

It is always negative $b<\sqrt{2}$.

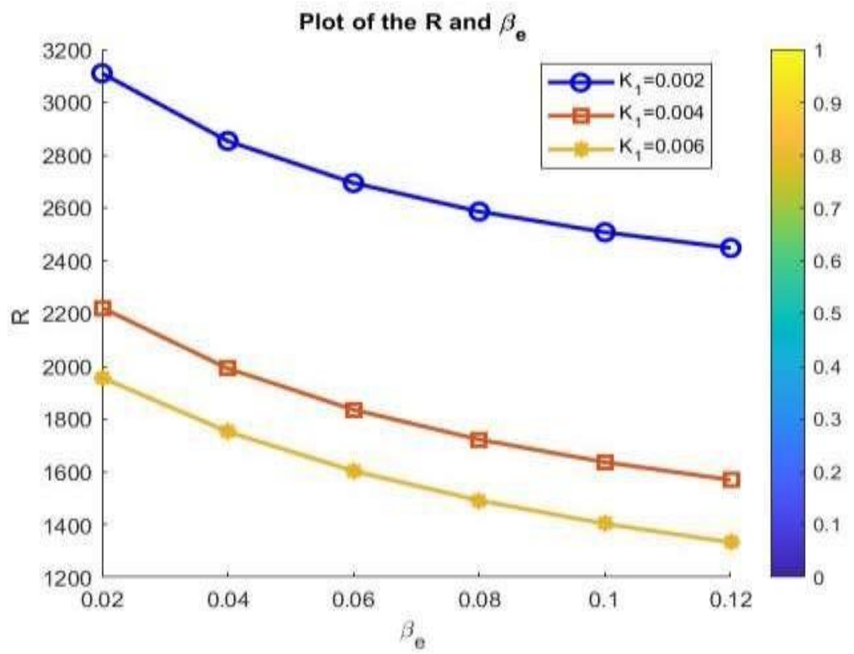


Figure 7.12: Plot between the Rayleigh number R and medium Hall current parameter β_e .

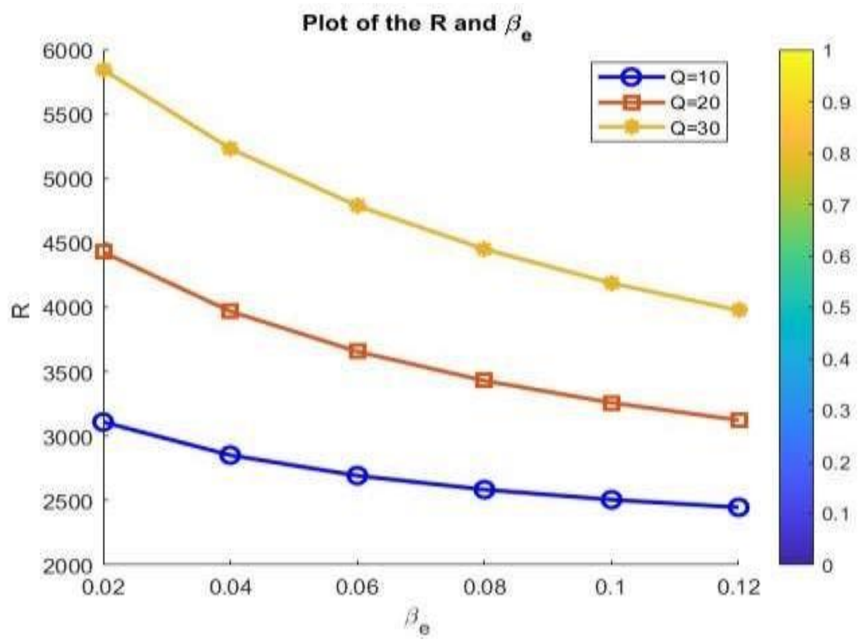


Figure 7.13: Graph of the Rayleigh number R and Hall current parameter β_e.

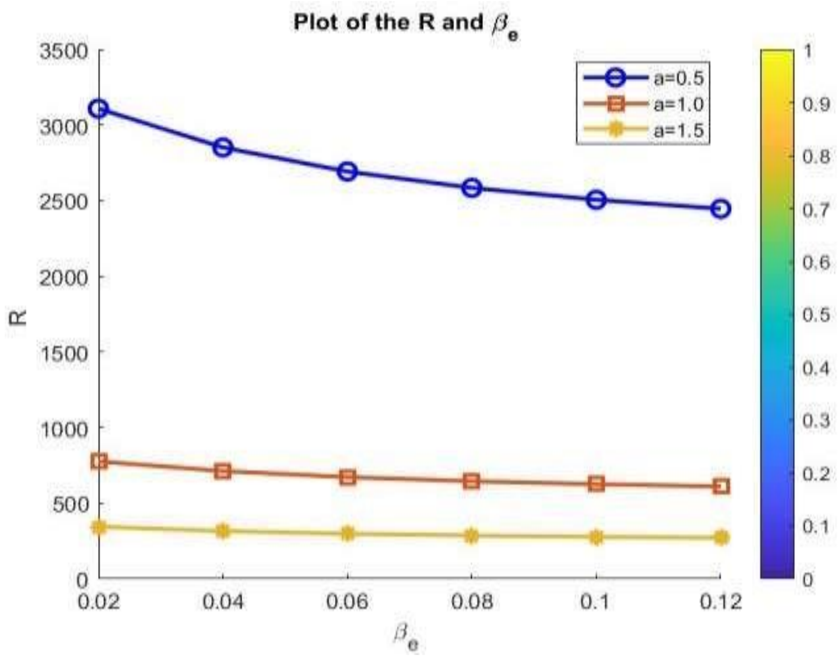


Figure 7.14: Plot between the Rayleigh number R and Hall current parameter β_e .

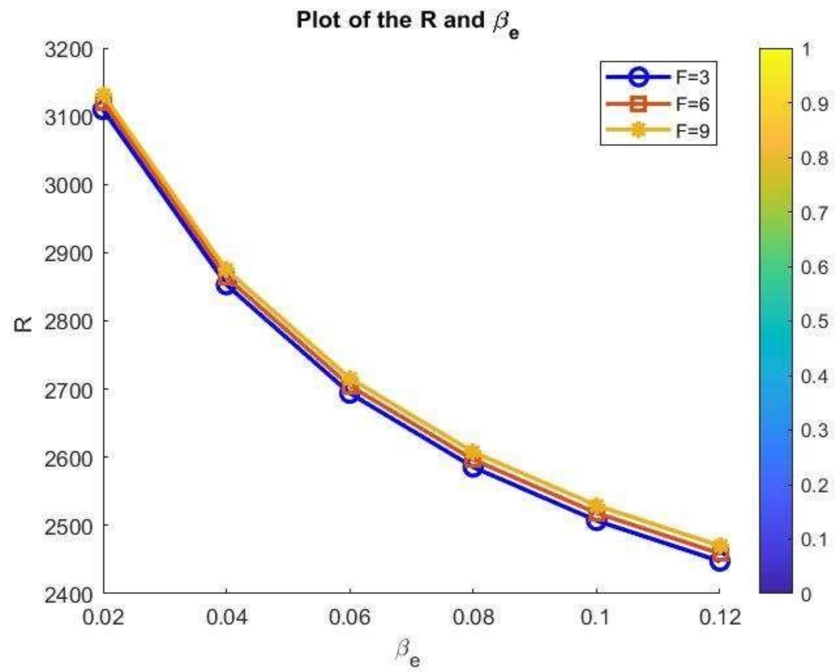


Figure 7.15: Graph between the Rayleigh number R and Hall current parameter β_e .

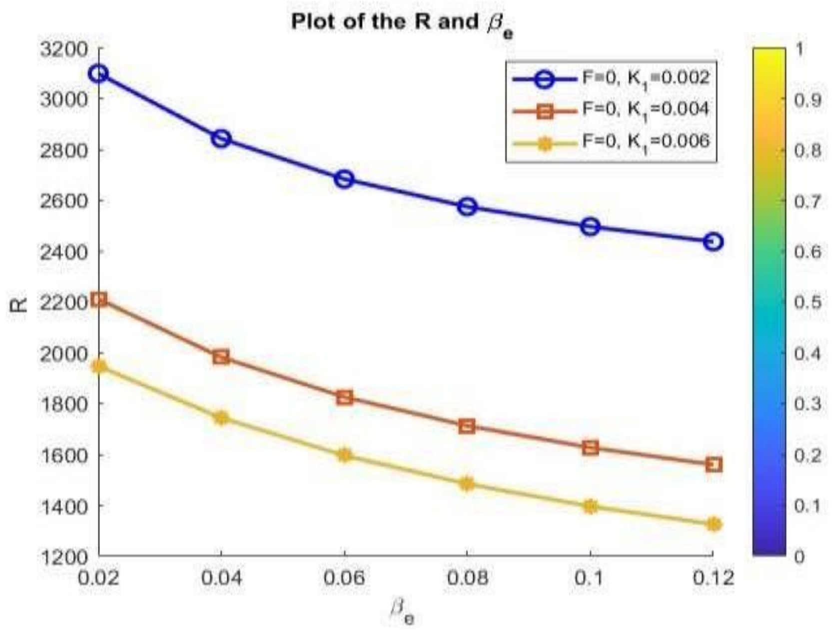


Figure 7.16: Plot of the Rayleigh number R and Hall current parameter β_e.

Figure 7.12, 7.13, 7.14 and 7.15 plots between the Rayleigh number R versus Hall current parameter β_e i.e. as the Hall current parameter increase then Rayleigh number decrease with the medium permeability $K_1 = (0.002, 0.004, 0.006)$, magnetic field $Q = (10, 20, 30)$, wave number $a = (0.5, 1.0, 1.5)$ and couple-stress parameter $F = (3, 6, 9)$ when $\pi = 3.14$, $\in = 0.5$, $P_m = 4$ and $P_r = 2$. Now, neglecting the couple-stress in Figure 7.16, then the Rayleigh number decrease as the Hall current parameter increase, which is satisfied the previous results [5], [7], and [8].

It is clear that, the effect of Hall current is destabilizing. In the absence couple-parameter, the effect of Hall current is always destabilizing.

$$\frac{dR}{dF} = \frac{b^2}{a^2} \frac{\begin{gathered} \in^2 \beta_e \pi^2 b^5 \left[\left\{ \left(\frac{\in bP_r}{P_m} \right)^2 + \in^2 \beta_e \pi^2 b \right\} \left\{ \frac{1}{K_1} + Fb^2 + b \right\} + 2Q\pi^2 \left(\frac{\in bP_r}{P_m} \right) \right] \\ \left\{ \frac{1}{K_1} + Fb^2 + b \right\} + b^2 \left[\left\{ \left(\frac{\in bP_r}{P_m} \right)^2 + \in^2 \beta_e \pi^2 b \right\} \left\{ \frac{1}{K_1} + Fb^2 + b \right\} + 2Q\pi^2 \left(\frac{\in bP_r}{P_m} \right) \right] \\ \left(\frac{\in bP_r}{P_m} \right)^2 \left\{ \frac{1}{K_1} + Fb^2 + b \right\} + Q^2 \pi^4 b^2 \left[\left\{ \left(\frac{\in bP_r}{P_m} \right)^2 + \in^2 \beta_e \pi^2 b \right\} \right] \end{gathered}}{\left[\left\{ \left(\frac{\in bP_r}{P_m} \right)^2 + \in^2 \beta_e \pi^2 b \right\} \left\{ \frac{1}{K_1} + Fb^2 + b \right\} + Q\pi^2 \left(\frac{\in bP_r}{P_m} \right) \right]^2} \tag{7.6}$$

It is always positive.

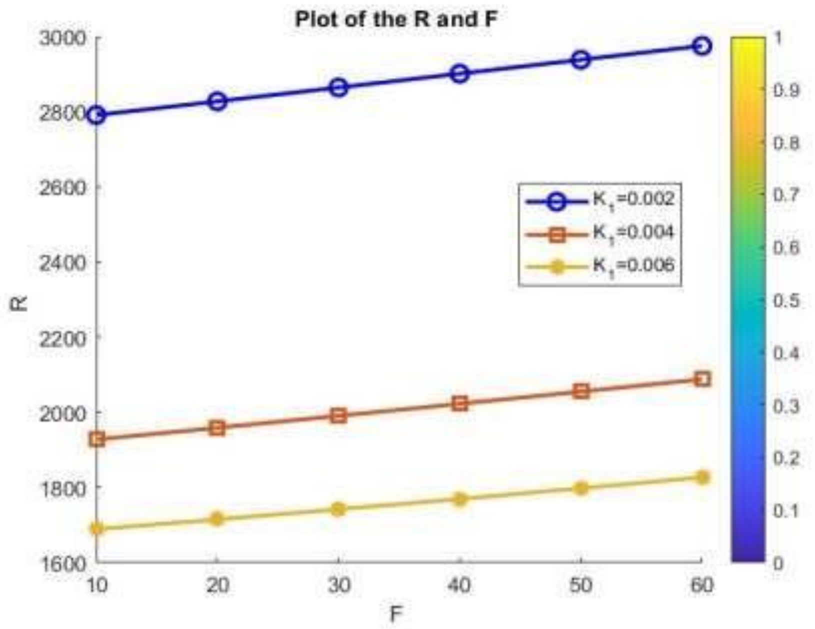


Figure 7.17: Plot between the Rayleigh number R and couple-stress parameter F.

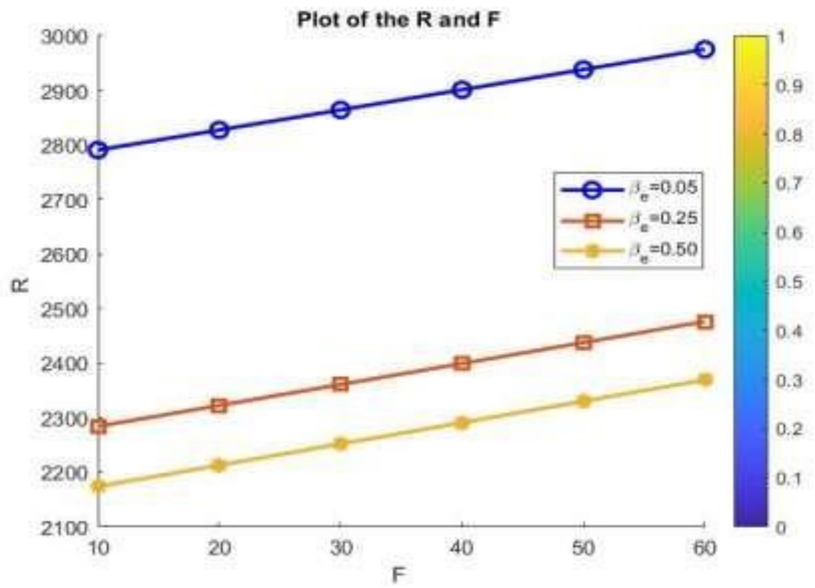


Figure 7.18: Plot of the Rayleigh number R and couple-stress parameter F.

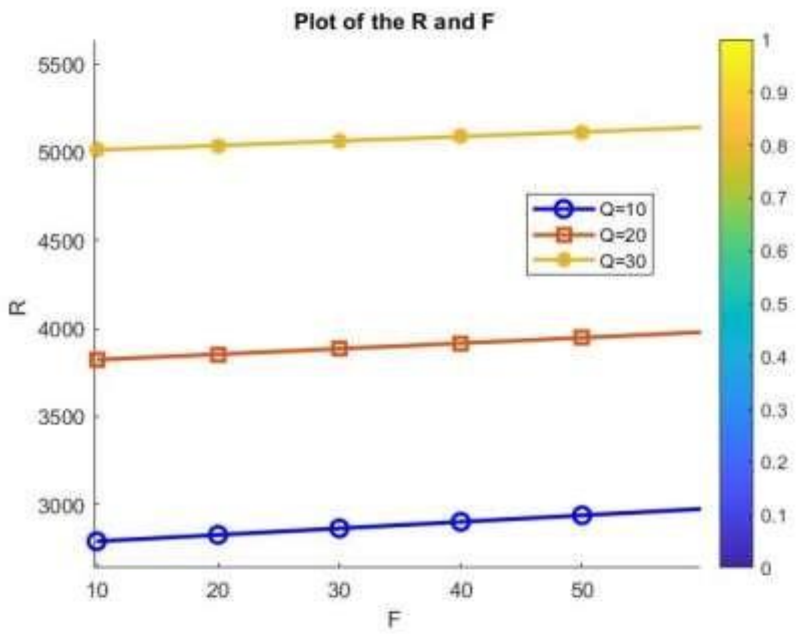


Figure 7.19: Graph of the Rayleigh number R and couple-stress parameter F.

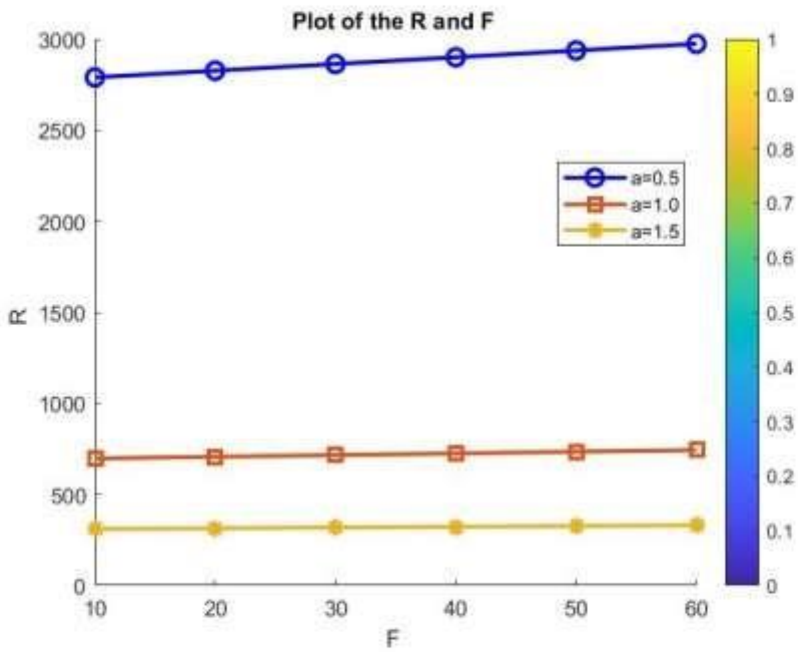


Figure 7.20: Plot between the Rayleigh number R versus couple-stress parameter F.

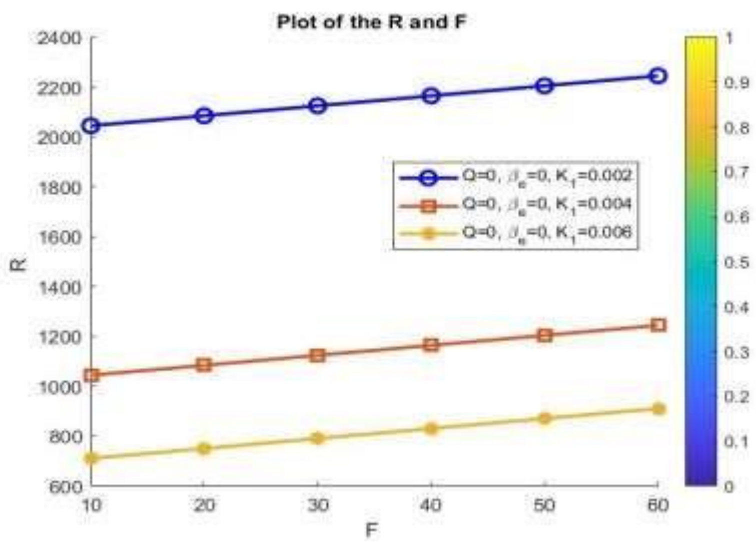


Figure 7.21: Graph between the Rayleigh number R and couple-stress parameter F.

Figure 7.17, 7.18, 7.19 and 7.20 illustrations the graphs of Rayleigh number R with couple-stress parameter i.e. as the couple-stress parameter increase then Rayleigh number increase with wave number $a=(0.5,1.0,1.5)$, Hall current parameter $\beta_e=(0.05,0.25,0.50)$, medium permeability $K_1=(0.002,0.004,0.006)$, and couple-stress parameter $F=(3,6,9)$ when $\pi=3.14$, $\in=0.5$, $P_m=4$ and $P_r=2$. Now, neglecting the Hall current parameter and couple-stress parameter in Figure 7.11, then the Rayleigh number increase as the couple-stress parameter increase, which is satisfied the previous results [13], [16], [17], and [18].

Thus, the effect of magnetic field is stabilizing. In the absence Hall current parameter and magnetic field parameter, the effect of couple-stress is always stabilizing.

8. Conclusions

According to the above numerical and analytical discussion, we found that the effect of medium permeability has destabilizing. In the absence of Hall current, magnetic field, and couple-stress as the effect of medium permeability has always destabilizing. Hall current also has destabilizing effect, magnetic field and couple-stress have stabilizing effect. Among them the most important result that the effect of Hall current destabilizes on the system. We compare these results with previous work and agree with each other.

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