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Group Runs Control Chart For Process Dispersion Based On Downton Statistic

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Abstract
The Group Runs (GR) control chart approach suggested in the literature of control charts is an improved version of the synthetic control chart. The use of ranked set sampling (RSS) scheme is also found to improve the performance of control chart in process monitoring. This study proposes combination of these two approaches to construct a GR control chart using RSS scheme for efficiently monitoring the changes in the process dispersion for normally distributed process. The proposed GR chart integrates the D chart based on Downton's estimator with extended form of conforming run length (CRL) chart and the chart is denoted as GR-D chart. The procedure to determine optimal design parameters of the chart is explained in detail. The average run length (ARL) performance of the proposed chart is evaluated using Monte Carlo simulations under simple random sampling (SRS) and RSS schemes. The proposed GR-D chart is compared with the synthetic-D chart under both the sampling schemes and the results show the outperformance of the GR-D control chart in terms of ARL. Further, the GR-D chart with RSS scheme performs uniformly better than GR-D chart with SRS scheme. An illustrative example is also provided to demonstrate the implementation and working of the GR-D chart.

***Keywords**: Statistical Process Control, Control Chart, Process Monitoring, Average Run Length, Group Runs.*

INTRODUCTION

The concept of control charts for monitoring process parameters was first introduced by [1] in his seminal work. Shewhart $\bar{X}$ chart is most popular for monitoring process mean of normally distributed process, whereas Shewhart R and S charts are the commonly considered charts to monitor the dispersion. These charts are based on subgroups drawn using SRS scheme. The application of Shewhart charts is straightforward and yields favorable results in pinpointing significant changes; however, they have limited capacity to detect minor or moderate shifts in process parameters. ([2]). Various methods have been suggested in the literature for enhancing the efficiency of Shewhart-type control charts. One such approach is the use of the synthetic control chart. In order to improve the performance of Shewhart-type control chart, [3] first proposed synthetic control chart to monitor process mean, which is composed of Shewhart $\bar{X}$ chart and CRL chart. They have shown that the synthetic chart outperforms the $\bar{X}$ chart. The use of synthetic charts was extended for monitoring process dispersion by [4] by developing synthetic-S chart as a combination of S chart and CRL chart. [5] developed synthetic-R chart to monitor dispersion, which consists of R chart and CRL chart. [6] examined a modification of the synthetic control chart that could be utilized to supervise not only the average but also the standard deviation. As pointed out by [6], the synthetic control charts became more popular because of the fact that many practitioners opt to delay the announcement of a process being out-of-control until there is another noncompliant occurrence. The advancement of synthetic control charts for univariate and multivariate process is recorded by many researchers including [7], [8], [9] [10], [11], [12]. [13] provided review of more than 100 research articles on univariate and multivariate synthetic-type control charts. The synthetic charts are found to outperform the corresponding Shewhart-type charts for detecting shifts in the process parameters.

There are alternative methods in literature that aim to enhance the effectiveness of Shewhart-type charts. One such approach is use of runs rules. [14] introduced 2-out-of-2 runs rule control chart as efficient alternative to Shewhart chart. This chart is superior to Shewhart chart in terms of performance. These two approaches, synthetic and runs rule charts, are better than the Shewhart-type charts in detecting shifts in the process parameters. [15] first designed GR control chart as an combination of Shewhart chart and an extended version of the CRL chart to enhance the efficacy of the Shewhart chart and synthetic chart for detecting shifts in the process mean of normally distributed process. [16] proposed a multivariate GR control chart for monitoring process variation by integrating the generalized sample variance chart and the extended version of the CRL chart. It was exhibited that the GR chart is better than the synthetic chart given by [10]. [17] extended the GR control chart for monitoring mean vector of the multivariate process by combining Hotelling's T^2 chart with modified version of the CRL chart. They reported that the proposed GR T^2 chart performs better than the synthetic T^2 chart by [9]. Some nonparametric GR charts are also proposed for keeping track of the location of the process. [16] developed a nonparametric GR control chart to detect shift in the process median as a combination of Shewhart's control chart based on signed-rank statistic and group-based CRL chart. [18] have developed a nonparametric GR chart for location of the process based on sign statistic. [19] developed GR control chart for profile monitoring.

Researchers have also attempted to examine the performance of the control charts by choosing different charting statistics. [20] constructed a control chart based on Downton's estimator, named D chart, for identifying increase in the process standard deviation. The statistic D proposed by [21] is an unbiased estimator of process standard deviation, if the process distribution is normal. [20] demonstrated that for processes with normal distribution, the D chart is equally efficient as Shewhart S chart in identifying the changes in the process dispersion. The performance of D chart was improved by [12] by developing synthetic-D chart as a combination of D chart and CRL chart. This paper attempts to improve the performance of the synthetic-D chart by using the idea of GR chart.

The regular design of Shewhart-type control chart is based on SRS scheme. The performance of the control chart is greatly affected by the sampling scheme by which the subgroups are selected. Many recent studies have developed control charts using RSS scheme. While proposing the scheme [22], noted that RSS scheme is better than regular SRS for precise estimation of the population mean. Recently, RSS scheme has got due interest in the development of control charts. The pioneering attempt of using RSS scheme for monitoring process mean was done by [23]. They found that the RSS charts are performing superior to the SRS charts. [24] constructed control charts for process average using various RSS schemes, and determined that the suggested RSS-based control charts exhibit superior performance compared to traditional SRS charts. [25] proposed sample mean control charts using various sampling methods such as RSS and its recent variations. These were designed under the assumption of a normal underlying distribution with a mean μ and variance σ^2. [26] improved the performance of synthetic chart developed by [12] by applying various RSS schemes for subgroup selection. [27] discussed the usefulness of control charts with various RSS schemes in manufacturing industries.

Attracted by the features of the GR chart as well as RSS scheme, in this paper, we have proposed GR-D to enhance the performance of the synthetic-D chart to efficiently monitor process dispersion. The performance of the proposed chart is evaluated under both SRS and RSS sampling schemes.

D CHART FOR PROCESS DISPERSION

Let $X_1, X_2, ..., X_n$ be a simple random sample of size n from $N(\mu, \sigma^2)$. Further, let $X_{(1)}, X_{(2)}, ..., X_{(n)}$ be the order statistics corresponding to this sample. For monitoring shifts in process standard deviation, [20] developed Shewhart-type control chart based on the statistic

$$D = \frac{2\sqrt{\pi}}{n(n-1)} \sum_{i=1}^{n} \left[i - \frac{1}{2}(n+1) \right] X_{(i)} \tag{1}$$

where, statistic D is proposed by [21] which is an unbiased estimator of σ. [20] have demonstrated that for processes with normal distribution, the D chart is equally effective as Shewhart S chart in identifying changes in the process standard deviation. They used probability limits of D chart, obtained using the distribution of $Z = D/\sigma$. The upper control limit for the D chart is-

$$UCL = Z_{1-\alpha}\,\overline{D} \text{ with } P(Z \geq Z_{1-\alpha}) = 1 - \alpha \tag{2}$$

where α is pre-fixed probability Type-I error, Z_α is α^{th} quantile point of the distribution of Z. The process standard deviation is supervised by a plot of D values with UCL as given by Eq.(2). $D < UCL$ will imply that the process is under control, else it is understood that process standard deviation has increased, leading to out-of-control state.

We assume that the process parameters μ and σ^2 are known, that is, $\mu = \mu_0$ and $\sigma^2 = \sigma_0^2$ are in-control mean and variance of the process respectively. Let σ_0 and $\sigma_1 = \delta\sigma_0\ (0 < \delta \neq 1)$ be the in-control and the out-of-control standard deviations respectively. The process is said to be in-control when $\delta = 1$. For detecting shift when $\delta > 1$, an upper control limit $k^+\sigma_0$ of D chart is calculated, and a signal is given if $D > k^+\sigma_0$. When $\delta < 1$, a lower limit $k^-\sigma_0$ for D chart is calculated and a signal is given if $D < k^-\sigma_0$. The average run length (ARL) can be calculated as

$$ARL_D(\delta) = \frac{1}{P(\delta)} \tag{3}$$

where $P(\delta)$ is the probability of detecting a shift δ in the process standard deviation. For $\delta > 1$,

$$P(\delta) = P(D > k^+\sigma_0 \mid \sigma = \delta\ \sigma_0)$$

$$= (Z > k^+/\delta) = 1 - F(k^+/\delta) \tag{4}$$

For $\delta < 1$,

$$P(\delta) = P(D < k^-\sigma_0 \mid \sigma = \delta\ \sigma_0)$$

$$= \Pr(k^-/\delta) = F(k^-/\delta) \tag{5}$$

where $F(.)$ denotes the cumulative distribution function.

CONFORMING RUN LENGTH (CRL) CHART

The CRL chart suggested by [28] is an attribute control chart to detect shift in the fraction non-conforming p. The random variable CRL is the number of conforming items between two successive non-conforming items inclusive of the last non-conforming item. The charting statistic CRL follows a geometric distribution with cumulative distribution function (cdf)

$$F(CRL) = 1 - (1-p)^{CRL},\ CRL = 1, 2, 3, \ldots \tag{6}$$

where p is the probability of the non-conforming unit. If the practitioner is only concerned about detecting an increase in p, the lower control limit (L) of the CRL chart is sufficient. It is as follows-

$$L = \frac{\ln(1-\alpha_{CRL})}{\ln(1-p_0)} \tag{7}$$

where $\alpha_{CRL} = 1-(1-p_0)^L = F_{p_0}(L)$ is the type I error probability for the CRL chart and p_0 is the in-control fraction non-conforming. If the CRL value of a sample is less than or equal to L, this signifies an increase in non-conforming fraction and triggers issuance of out-of-control signal.

The ARL of the CRL chart to detect the increase in the fraction non-conforming is given by

$$ARL_{CRL} = \frac{1}{F_{CRL}(L)} = \frac{1}{1-(1-p)^L} \tag{8}$$

[12] developed the synthetic D chart based on SRS scheme as a combination of the D chart of [20] and the CRL chart for improved monitoring of the process dispersion. The numerical comparison revealed that the synthetic D chart produces significant improvement as compared with R, S and D charts as well as the synthetic charts of [5] and [4] for normally distributed process data. The operational mechanism of synthetic D chart based on SRS scheme is provided in [12].

RANKED SET SAMPLING SCHEME

This section explains the operational principles of the RSS scheme. Let x be the study characteristic. To obtain an RSS sample of size n, a total of n^2 units are randomly selected from the population. A random arrangement of these n^2 units is made in n sets each of size n. Let $\{X_{i,k}, i=1,2,...,n\}$ be the k^{th} set, where $k=1,2,...,n$. The observations in every set are ranked by visual inspection on characteristic x or with the help of some other auxiliary variable. These sets are then called as ranked sets. The resulting RSS scheme is called as RSS with perfect ranking or RSS with imperfect ranking, based on whether the ranked sets have exact ranking or non-exact ranking with respect to study variable. This study considers RSS scheme with perfect ranking. Let k^{th} ordered set be $\{X_{[i,k]}, i=1,2,...,n\}$, where $X_{[i,k]}$ is the i^{th} order statistic in k^{th} set, $i=1,2,...,n$; $k=1,2,...n$. Then i^{th} order statistic is picked from i^{th}, $i=1,2,\ldots,n$ ranked set. The final chosen sample of size n is $\{X_{[1,1]}, X_{[2,2]},..., X_{[n,n]}\}$, which is referred to as a ranked set sample.

GR-D CHART FOR PROCESS DISPERSION

In this section, the design structure of the proposed GR-D chart to monitor the change (positive and negative) in the process dispersion based on SRS and RSS schemes is presented. Following the work of [15], in order to increase the detection ability of the synthetic-D chart, we integrate the D chart of [20] and extended version of the CRL chart to form the GR-D chart. The D chart has only upper control limit (UCL) and CRL chart has only the lower control limit L. Let Y_r be the r^{th} group based CRL and L be the lower limit of the CRL chart. Then GR-D chart declares the process as out-of-control if $Y_1 \le L$ or for some $r(>1)$, $Y_r \le L$ and $Y_{r+1} \le L$.

A. GR-D Chart Based on SRS Scheme

Following [15], the operation of the GR-D chart is outlined by the following steps:

1. Fix the upper control limit UCL for the D chart and lower control limit L of the CRL chart.
2. Take a group of n items (based on SRS of size n) at each inspection point j and compute the chart statistic, say D_j.
3. If $D_j \le UCL$, then the group is considered a conforming group and control flow returns back to previous step. Otherwise, it is considered as non-conforming and control flow goes to step (3).
4. Obtain the value of statistic CRL as the number of D samples between the current and previous non-conforming groups.
5. If $CRL > L$, the process is thought to be under control, and control flow moves back to step 2. If $CRL_1 \le L$ or two successive $CRL_i \le L$ and $CRL_{i+1} \le L$, for $i=2,3,...$.for the first time, the process is thought to be out-of-control, and control flow moves to the next step.

6. Signal the out-of-control state.
7. An assignable cause needs to be identified and corrective action should be taken to remove it.

Let the average number of groups (samples) required for a GR-D chart to detect a shift of magnitude δ in process dispersion be denoted by $ARL_{GR}(\delta)$. To construct the GR-D chart, the following ARL model is considered.

Minimize $ARL_{GR}(\delta)$

Subject to $ARL_{GR}(0) \geq \tau$

where τ is the minimum required value of $ARL(0)$.

Following [15], the ARL performance measure for the GR-D chart can be expressed as follows:

For increase in the process standard deviation, i.e. when $\delta > 1$,

$$ARL_{GR}(\delta) = \frac{1}{1 - F\,(k^+/\delta)} \times \frac{1}{[1 - F\,(k^+/\delta)^L\,]^2} \tag{9}$$

For decrease in the process standard deviation, i.e. when $\delta < 1$,

$$ARL_{GR}(\delta) = \frac{1}{F\,(k^-/\delta)} \times \frac{1}{\{1 - [1 - F\,(k^-/\delta)^L\,]\}^2} \tag{10}$$

where $F(.)$ is cumulative distribution function.

Optimal Design Procedure

We present the optimal design to obtain the optimal parameters (k^+, L) and (k^-, L) for the GR-D chart for increases and decreases respectively in process standard deviation that results in minimum $ARL_{GR}(\delta)$ value, subject to in-control ARL (ARL_0) which is at least equal to $\tau = 200$.

The optimal design procedure of GR-D chart using SRS scheme is as follows:

1. Specify group size n, shift size δ^* and in-control ARL as ARL_0.
2. Commence with L as 2.
3. If $\delta^* > 1$, obtain the value of k^+ by solving Eq. (9) (use $\delta = 1$) taking $ARL_{GR}(\delta)$ as ARL_0. If $\delta^* < 1$, obtain k^- by solving Eq. (10) (use $\delta = 1$) taking $ARL_{GR}(\delta)$ as ARL_0.
4. When $\delta^* > 1$, evaluate ARL for δ^* from the present value of L and k^+ using Eq. (9) (use $\delta = \delta^*$). When $\delta^* < 1$, evaluate ARL for δ^* from present value of L and k^- using Eq. (10) (use $\delta = \delta^*$).
6. If L is not equal to 2, proceed to step (7); else, increase L by one and go back to step (3).
7. If the present ARL for δ^* is greater than the preceding one, go to the step (8); otherwise, increase L by one and go back to step (3).
8. Take the preceding L and k^+ (or k^-) as the optimal design parameters for the GR-D chart.

Table I: Different Pairs of L and k^+ For $\mathbf{n=8}$, $\boldsymbol{\delta^*=1.2}$ and Specified $\mathbf{ARL_0=200}$

L_1	k^+	$ARL_{GR}(\delta)$
2	1.260601	14.658
3	1.341355	10.774
4	1.38514	9.331
5	1.414736	8.591
6	1.436912	8.164
7	1.4546	7.904
8	1.469275	7.746
9	1.481688	7.641
10	1.492551	7.587
11	1.502118	7.566
12	1.510681	7.568

In order to illustrate design of GR-D chart using SRS scheme, consider $n=8$, $\delta^*=1.2$ and in control ARL=200. Table I shows how $ARL(\delta^*)$ changes with respect to (L, k^+). The ARL value first decreases, attains a minimum of 7.566 and then starts increasing. Therefore, the optimal parameters for SRS GR-D chart, in this case are $L=11$ and $k^+=1.502118$.

Table II: ARL Comparison for Process Dispersion with Positive Shift

	Synthetic-D		**GR-D**	
	SRS	**RSS**	**SRS**	**RSS**
		$n=5$		
Shift (δ)	$L=17$ $k^+=1.843$	$L=15$ $k^+=1.7792$	$L=16$ $k^+=1.7327$	$L=15$ $k^+=1.688$
1.0	200 (2.371)	200.16 (1.059)	199.99 (1.247)	201.69 (1.251)
1.1	43.92 (0.560)	39.77 (0.230)	33.58 (0 .232)	30.82 (0.210)
1.2	15.89 (0.202)	13.89 (0.078)	11.51 (0.074)	10.28 (0.065)
1.3	8.34 (0.093)	7.09 (0.035)	6.08 (0.033)	5.38 (0.028)
1.4	5.38 (0.055)	4.61 (0.02)	4.13 (0.019)	3.69 (0.016)
1.5	3.92(0.036)	3.4 (0.013)	3.19 (0.012)	2.83 (0.010)
2.0	1.79 (0.012)	1.64 (0.005)	1.62 (0.005)	1.52 (0.004)
		$n=10$		
Shift (δ)	$L=12$ $k^+=1.519$	$L=8$ $k^+=1.3681$	$L=10$ $k^+=1.4315$	$L=7$ $k^+=1.3491$

1.0	201 (2.388)	199.75 (1.021)	200.95 (1.210)	200.04 (1.172)
1.1	28.61 (0.366)	20.2 (0.115)	21.04 (0.142)	12.39 (0.083)
1.2	8.66 (0.103)	5.49 (0.029)	6.14 (0.037)	3.32 (0.018)
1.3	4.41 (0.045)	2.75 (0.011)	3.24 (0.015)	1.90 (0.007)
1.4	2.89 (0.024)	1.86 (0.006)	2.25 (0.008)	1.43 (0.004)
1.5	2.16 (0.016)	1.48 (0.004)	1.79 (0.005)	1.22 (0.002)
2.0	1.18 (0.005)	1.04 (0.001)	1.128 (0.002)	1.00 (0.000)

B. GR-D chart Based on RSS

A RSS subgroup sample of size n is generated from the underlying process using the sampling mechanism discussed in Section 4. Downton statistic based on RSS sample $\{X_{[1,1]}, X_{[2,2]}, \ldots, X_{[n,n]}\}$ of size n is then given as

$$D_{RSS} = \frac{2\sqrt{\pi}}{n(n-1)} \sum_{i=1}^{n} \left[i - \frac{1}{2}(n+1) \right] X_{[i,i]} \qquad (11)$$

Let $Z_{RSS} = D_{RSS} / \sigma$ be the relative D_{RSS} with respect to σ. Due to the unknown distribution of Z_{RSS} statistic quantiles for are computed through a simulation approach. As RSS ensures thorough representative sample, we further extended the GR-D chart for the case, where group sampling is done using RSS. The working mechanism of GR-D chart under SRS and RSS schemes is very much similar, except the fact that scheme by which samples are taken from the process is SRS or RSS. The operation of GR-D chart under RSS scheme is same as that of GR-D chart under SRS scheme, except that the samples are selected by RSS instead of SRS.

PERFORMANCE EVALUATION

In this section, the performance of proposed GR-D chart is investigated using ARL under SRS and RSS schemes and is compared with synthetic D chart. A simulation study is conducted to evaluate the performance of the proposed GR-D chart under RSS and SRS schemes. In-control observations are generated from $N(\mu_0, \sigma_0^2)$ distribution, where $\mu_0 = 0$ and $\sigma_0^2 = 1$. For out-of-control process, the observations are generated from $N(\mu_0, \sigma_1^2)$ process, where $\sigma_1 = \delta \sigma_0$. The in-control ARL of 200 is considered. The out-of-control ARL values along with their standard errors are determined using 50000 simulations. For increases in the standard deviation, we have considered shift sizes as $\delta = 1.1, 1.2, 1.3, 1.4, 1.5, 2.0$ and for decreases in the standard deviation, size of shifts considered are $\delta = 0.9, 0.8, 0.7, 0.6, 0.5, 0.1$. The sample sizes considered are $n = 5$ and 10. The simulated ARL values of the proposed GR-D and synthetic-D charts along with their standard errors (shown in parenthesis) are presented in Table II and Table III for increases and decreases in shifts respectively.

Following are the findings from Table II and Table III.

- For any change in process dispersion in either direction, the synthetic D chart under RSS has smaller out-of-control ARLs than SRS synthetic D chart under SRS.
- In case of SRS scheme, the proposed GR-D chart has a better performance than synthetic D-chart.
- For the case of RSS scheme also, the proposed GR-D chart has a better performance than the synthetic D-chart.
- Although the Synthetic-D chart with RSS scheme underperforms than GR-D chart with SRS scheme,with the increase in n, Synthetic-D chart under RSS scheme dominates SRS GR-D chart in the ARL performance. The increase in the sample size becomes advantageous to RSS scheme to exhibit its effect.

- Although the Synthetic-D chart with RSS scheme underperforms than GR-D chart with SRS scheme,with the increase in n, Synthetic-D chart under RSS scheme dominates SRS GR-D chart in the ARL performance. The increase in the sample size n becomes advantageous to RSS scheme to exhibit its effect.

The comparison between GR-D chart under RSS and SRS schemes show that the GR-D chart under RSS scheme performs uniformly better than GR-D chart under SRS scheme for all sample sizes.

Table III: ARL Comparison for Process Dispersion with Negative Shift

	Synthetic-D		**GR-D**	
	SRS	**RSS**	**SRS**	**RSS**
		$n=5$		
Shift (δ)	$L=5$ $k^-=0.396$	$L=3$ $k^-=0.5867$	$L=17$ $k^-=0.3921$	$L=12$ $k^-=0.554$
1.0	200 (2.252)	200.74 (0.975)	200.73 (1.254)	200.40 (1.220)
0.9	96.82 (1.125)	81.72 (0.409)	78.68 (0.523)	62.65 (0.412)
0.8	44.86 (0.551)	31.13 (0.163)	31.10 (0.213)	20.00 (0.138)
0.7	19.88 (0.247)	11.68 (0.063)	13.07 (0.085)	7.27 (0.045)
0.6	8.80 (0.108)	4.5 (0.024)	6.19 (0.033)	3.36 (0.015)
0.5	3.83 (0.043)	1.96 (0.008)	3.37 (0.013)	1.90 (0.006)
0.1	1.00 (0.000)	1.00 (0.000)	1.00 (0.000)	1.00 (0.000)
		$n=10$		
Shift (δ)	$L=6$ $k^-=0.5757$	$L=5$ $k^-=0.757$	$L=6$ $k^-=0.646$	$L=2$ $k^-=0.893$
1.0	200 (2.228)	199.53 (0.992)	200.93 (1.154)	200.00 (1.045)
0.9	55.55 (0.651)	28.1 (0.154)	38.64 (0.248)	21.62 (0.128)
0.8	15.47 (0.198)	5.4 (0.029)	9.07 (0.060)	3.94 (0.024)
0.7	5.19 (0.061)	1.86 (0.007)	3.03 (0.016)	1.44 (0.006)
0.6	2.28 (0.019)	1.15 (0.002)	1.58 (0.005)	1.03 (0.001)
0.5	1.35 (0.007)	1.01 (0.000)	1.13 (0.002)	1.00 (0.000)
0.1	1.00 (0.000)	1.00 (0.000)	1.00 (0.000)	1.00 (0.000)

Illustrative Example

To enhance the understanding of proposed chart, an illustrative example is given in this section. To serve the purpose, two datasets are simulated. Out of these 40, first 10 samples are from $N(0,1)$ and the later 30 samples are from $N(0,1.2)$. Here, $N(0,1)$ is in-control setup whereas $N(0,1.2)$ refers to a shift in the dispersion. Table IV contains the values of D-statistic for these samples. The synthetic-D chart and GR-D charts are applied for this dataset to identify change in process standard deviation. The graphical display of the control charts applied to dataset 1 is shown in Figure.1.

For $n=10$, the charting parameters for synthetic D chart for positive shift in standard deviation are $L=12$ and $k^+=1.5192$ whereas the same for GR-D chart are $L=10$ and $k^+=1.4315$. From Figure 1, it can be seen that synthetic D chart gives out-of-control signal at point no.30, whereas GR-D chart gives the signal at point no. 26, which is earlier than that of the synthetic D chart.

Table IV: D-Values for Simulated Data with SRS

In-control Process		Out-of-control process with shift $\delta=1.2$					
No	**D-value**	**No.**	**D-value**	**No.**	**D-value**	**No.**	**D-value**
1	0.72583	11	0.97835	21	1.37029	31	0.95931
2	0.85104	12	0.85493	22	1.39693	32	1.40776
3	0.91398	13	0.90268	23	0.97221	33	1.70162
4	1.32501	14	1.32644	24	0.9898	34	1.71875
5	0.62823	15	1.43776	25	0.89427	35	1.06642
6	1.16342	16	0.80115	26	1.63038	36	0.98644
7	0.9279	17	1.30024	27	1.11742	37	1.58068
8	1.11559	18	1.20472	28	1.12241	38	1.30733
9	1.02387	19	0.79584	29	1.07751	39	0.92811
10	1.39351	20	1.4635	30	1.60206	40	1.24987

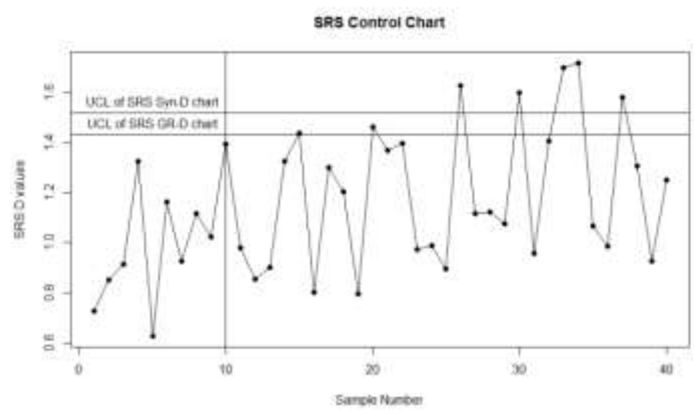

Figure 1. Synthetic D and GR-D Charts Under SRS Scheme

Dataset 2 contains 40 RSS samples of same size. Out of these 40, first 10 samples are from $N(0,1)$ and the later 30 samples are from $N(0,1.2)$. Here, $N(0,1)$ is in-control setup whereas $N(0,1.2)$ refers to a shift in the dispersion. Table V contains the values of D-statistic for these samples. The synthetic D chart and GR-D charts are applied for this dataset to identify change in process standard deviation. The graphical display of the control charts applied to dataset 2 is shown in Figure 2.

For $n=10$, the charting parameters for RSS synthetic D chart for positive shift in standard deviation are $L=8$ and $k^+=1.3681$, whereas the same for GR-D chart are $L=7$ and $k^+=1.3491$. From Figure 2, it can be seen that synthetic D chart gives out-of-control signal at point no.17, whereas GR-D chart gives the signal at point no. 15, which is earlier than that of synthetic D chart.

Table V: D-Values for Simulated Data with RSS

In-control Process		Out of control process with shift $\delta=1.2$					
No.	**D-value**	**No.**	**D-value**	**No.**	**D-value**	**No.**	**D-value**
1	0.74711	11	1.25095	21	1.03814	31	1.24992
2	1.27355	12	1.2723	22	1.38267	32	1.21955
3	1.10721	13	1.54455	23	1.24793	33	1.44044
4	1.25319	14	1.36456	24	1.062	34	0.87794
5	1.08601	15	1.35025	25	0.99952	35	1.47924
6	1.02107	16	1.08913	26	1.23756	36	1.36146
7	1.16927	17	1.54455	27	1.56702	37	1.60746
8	1.0569	18	1.18551	28	1.16449	38	0.81373
9	1.21497	19	1.34813	29	1.34919	39	1.37829
10	0.91067	20	1.38548	30	1.29238	40	1.29895

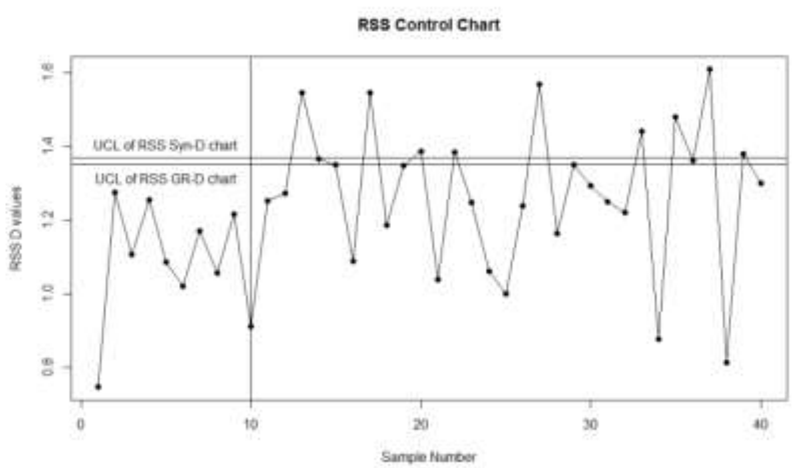

Figure 2. Synthetic D and GR-D Charts Under RSS Scheme

CONCLUSIONS

This study presents a GR-D control chart utilizing Downton's estimator to better monitor the changes in the process dispersion of a process with normal distribution. The proposed GR-D chart is a combination of the D chart and extended version of CRL chart. The performance of the proposed chart is evaluated using Monte Carlo simulations with SRS and RSS sampling schemes and is compared with the synthetic-D chart with RSS and SRS schemes. The results of comparison showed the superiority of the GR-D chart over synthetic-D chart under both SRS and RSS schemes. Further, comparison of GR-D chart with SRS and RSS schemes show that GR-D chart based on RSS scheme exhibits better performance.

REFERENCES

[1] Shewhart, W. A. (1931). Economic control of quality of manufactured product, Reprinted by the American Society for Quality Control in 1980.

[2] Montgomery, D. C. (2009). Introduction to Statistical Quality Control, 6th Ed. New York,Wiley.

[3] Wu, Z. and Spedding, T. A. (2000). A synthetic control chart for detecting small shifts in the process mean, Journal of Quality Technology, 32 (1), 32-38.

[4] Huang, H. J. and Chen, F. L. (2005). A synthetic control chart for monitoring process dispersion with sample standard deviation, Computers and Industrial Engineering, 49 (2), 221-240.

[5] Chen, F. L. and Huang, H. J. (2005). A synthetic control chart for monitoring process dispersion with sample range, The International Journal of Advanced Manufacturing Technology, 26, 842-851.

[6] Costa, A. F. B. and Rahim, M. A. (2006). A synthetic control chart for monitoring the process mean and variance, Journal of Quality in Maintenance Engineering, 12 (1), 81-88.

[7] Davis, R. B., and Woodall, W. H. (2002). Evaluating and improving the synthetic control chart, Journal of Quality Technology, 34 (2), 200-208.

[8] Ghute, V. B. and Shirke, D. T. (2007). Joint monitoring of multivariate process using synthetic control charts, International Journal of Statistics and Management System, 2, (1-2), 129-141.

[9] Ghute, V. B., and Shirke, D. T. (2008a). A multivariate synthetic control chart for monitoring process mean vector, Communications in Statistics-Theory and Methods, 37 (13), 2136-2148.

[10] Ghute, V. B. and Shirke, D. T. (2008b). A multivariate synthetic control chart for process dispersion, Quality Technology and Quantitative Management, 5 (3), 271-288.

[11] Aparisi, F. and de Luna, M. A. (2009). The design and performance of the multivariate synthetic- control chart, Communications in Statistics-Theory and Methods, 38 (2), 173-192.

[12] Rajmanya, S. V. and Ghute, V. B. (2014). A synthetic control chart for monitoring process variability, Quality and Reliability Engineering International, 30 (8), 1301-1309.

[13] Rakitzis, A. C., Chakraborti, S., Shongwe, S. C., Graham, M. A., and Khoo, M. B. C. (2019). An overview of synthetic-type control charts: Techniques and methodology, Quality and Reliability Engineering International, 35 (7), 2081-2096.

[14] Klein, M. (2000). Two alternatives to the Shewhart X control chart, Journal of Quality Technology, 32 (4), 427-431.

[15] Gadre, M.P. and Rattihalli, R.N. (2004). A group runs control chart for detecting shifts in the process mean, Economic Quality Control, 19 (1), 29-43.

[16] Gadre, M. P.and Kakade, V. C. (2014). A nonparametric group runs control chart to detect shift in the process median, IAPQR Transactions, 39 (1), 29-53.

[17] Gadre, M. P.and Kakade, V. C. (2016). Some group runs based multivariate control charts for monitoring the process mean vector, Open Journal of Statistics, 6 (6), 1098-1109.

[18] Khilare, S. K., and Shirke, D. T. (2023). A nonparametric group runs control chart for location using sign statistic, Thailand Statistician, 21 (1), 165-179.

[19] Ghadge, O.D. and Ghute, V. B. (2023). Group runs and modified group runs control charts for monitoring linear regression profiles. Reliability: Theory & Applications, 18 (4 (76)), 513-524.

[20] Abbasi, S. A. and Miller, A. (2011). D chart: An efficient alternative to monitor process dispersion, In Proceedings of the World Congress on Engineering and Computer Science, 2, 19-21.

[21] Downton, F. (1966). Linear estimates with polynomial coefficients, Biometrika, 53, (1/2), 129-141.

[22] McIntyre, G. A. (1952). A method for unbiased selective sampling, using ranked sets, Australian Journal of Agricultural Research, 3(4), 385-390.

[23] Salazar, R. D., and Sinha, A. K. (1997). Control chart X based on ranked set sampling, ComunicacionTecica, 97 (9).

[24] Muttlak, H. and Al-Sabah, W. (2003). Statistical quality control based on ranked set sampling, Journal of Applied Statistics, 30 (9), 1055-1078.

[25] Al-Rawwash, M., Al-Nasser, A. D. and Al-Omari, A. I. (2014). Quality control charts using different ranked set sampling schemes, Mathematical Methods in Science and Mechanics, 173-77.

[26] Haq, A., Brown, J. and Moltchanova, E. (2015). New synthetic control charts for monitoring process mean and process dispersion, Quality and Reliability Engineering International, 31 (8), 1305-1325.

[27] Mehmood, R., Riaz, M. and Does, R. J. (2014). Quality quandaries: on the application of different ranked set sampling schemes, Quality Engineering, 26 (3), 370-378.

[28] Bourke, P. D. (1991). Detecting a shift in fraction nonconforming using run-length control charts with 100% inspection, Journal of Quality Technology, 23 (3), 225-238.