Fractional Order Modeling And Control For Non-Linear System (Inverted Pendulum)

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Abstract

This piece of literature presents a fractional order model (FOM) of an inverted pendulum system (IPS) based on simulated and actual data. We first perform the estimation and optimization operations using the simulated data sets extracted from a reference simulated IPS prototype. This is related to FOM validation. First, we developed, simulated, and manipulated the mathematical model of IPS using the MATLAB/Simulink environment. We utilized the simulated data sets derived from the virtualized IPS prototype. We test the proposed FOM and compare it with a traditional controller to see if it can effectively describe the behaviour of the IPS. The comparative results demonstrate a rather high degree of consistency between the simulated IPS prototype and the FOM's output. Using the Euler-Langerange equation to dynamically simulate the inverted pendulum system, the primary goal of the fractional order controller is to manage the angle with the position of the non-linear system's cart. When utilizing the FOM controller, experimental results demonstrate significantly enhanced performance, providing compelling proof of the FOM's dependability in control system design.

Keywords: *Inverted pendulum, Euler-Langerange equation, Fractional order model.*

I Introduction

An accurate dynamical model has a great importance for IPS analysis and control design. So far, the focus of the researchers had been primarily on describing IPS with integer order differential equations and using a multitude of analytical and numerical solutions to optimize the formulation and analysis procedure through system identification. System identification is the application of mathematical tools and algorithms in order to build system dynamical models from measured data [1,2]. Identifying a dynamical model for IPS, at the unstable upright-position, is a challenging task because of its unstable nature [3–5]. The identification of the pendulum was carried out under the application of PI-PD real time control in [6]. In [7] a feed forward neural network with different number of neurons in the hidden layer was used for the identification of IPS. The kernel dynamics of most real systems are actually fractional [8]. IOM can describe the features of many systems which have less fractionalities but it will not be highly accurate [9,10]. The main reason for using the IOM was the absence of solution methods for fractional differential equations [11]. However, the recent evolution in hardware implementation has brought a renewed wave in the modelling and analysis of new class of fractional-order systems [12,13]. Fractional calculus allows alleviating the limitations of conventional differential equations where only integer operator powers are used [14]. This gives rise to system-models that take into account phenomena such as self-similarity and system state history dependence. Contemporary, industrial control systems are of considerable complexity [15], therefore such systems are likely to exhibit such phenomena [16]. Hence, interest for fraction system has been raised in the area of identification. Many authors pointed out that, fractional-order calculus is most suitable for the description of memory and genetic properties of various materials and processes, which are neglected in the classical integer-order models [17]. Fractional differential equations can be used to describe many systems in many interdisciplinary fields, for example [18–21].

In this paper, a FOM is proposed to capture the inverted pendulum dynamics accurately. Further, a FOM is optimized, based on the FOM, to effectively control the IPS. First, the mathematical model of Inverted pendulum model system has been derived, simulated and controlled using the MATLAB/Simulink environment. An identified using the simulated data sets which were obtained from the simulated IPS prototype. The proposed FOM is tested and compared with and to check its competitiveness to describe the behavior of the IPS.

II. Modeling of Inverted Pendulum and Controllers

The inverted pendulum is a highly nonlinear andopen-loop unstable system. This means that standard lineartechniques cannot model the nonlinear dynamics of the system. Before the inverted pendulum model can be developed in simulink, the system dynamical equations are derived using 'Lagrange Equations'. [9] The Lagrangian equations are one of many methods of determining the system equations. Using this method it is possible to derive dynamical system equations for a complicated mechanical system such as the inverted pendulum.

Where,

M = Mass of the cart

m = Mass of the pole

l= Length of the pole

f=control force

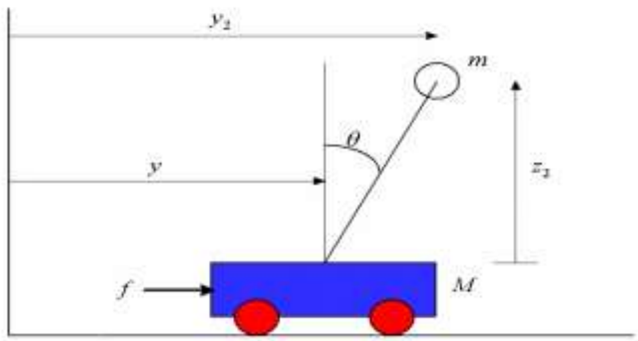

Fig.1 is a free-bodied diagram of the pendulum system.

The Lagrange equations use the kinetic and potential energy in the system to determine the dynamical equations of the cart-pole system. The kinetic energy of the system is the sum of the kinetic energies of each mass.

Kinetic energy of cart,

$$K_1 = \frac{1}{2}M\dot{x}^2$$

Kinetic energy of pole,

$$K_2 = \frac{1}{2}M\dot{x}_2^2 + \frac{1}{2}m\dot{p}^2$$

Where,

$$x_2 = l\sin\theta + x$$

$$=> \dot{x}_2 = l\cos\theta\dot{\theta} + \dot{x}$$

and,

$$p = l\cos\theta$$

$$=> \dot{p} = -l\sin\theta\dot{\theta}$$

$$=> \dot{p}^2 = \dot{\theta}^2 l^2 \sin^2$$

Total the kinetic energy of the system can thus be formulated as

$$K = K_1 + K_2$$

$$K = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m(\dot{x}^2 + l^2\theta^2\cos^2\theta + 2\dot{x}l\dot{\theta}\cos\theta + l^2\dot{\theta}^2\sin^2\theta$$

Potential energy,

$$P = mgl\cos\theta$$

In classical mechanics, the natural form of the Lagrangian is defined as the kinetic energy, K, of the system minus its potential energy, P.

L = K-P

To obtain a closed-form dynamic model of the pendulum, the energy expressions are used to formulate the Lagrangian L=K-P. Let the generalized forces corresponding to the generalized displacements$\bar{q} = \{x, \theta\}$ be $F = \{Fx\ ,0\}$. Using Lagrangian's equation..

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = F_j; \qquad j= 1,2$$

the equation of motion is obtained as below,

$$(M + m)\ddot{x} + ml\cos\theta\ddot{\theta} - ml\dot{\theta}\sin\theta = F$$

$$ml\cos\theta\ddot{x} - ml\sin\theta\dot{\theta}\dot{x} - mgl\sin\theta + ml^2\ddot{\theta} = 0$$

As θ is very small, for linearization, we take

$$\sin\theta \approx \theta$$

$$\text{And} \qquad \cos\theta \approx 1$$

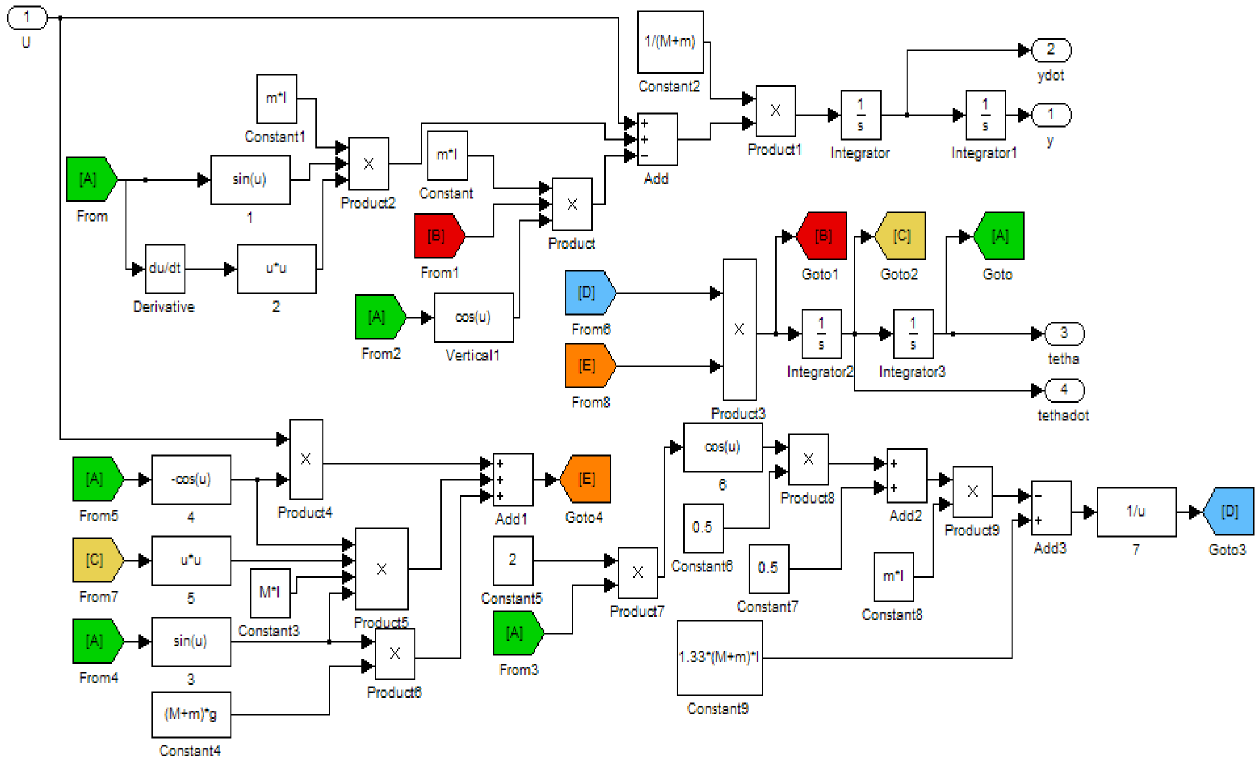

Fig.2: Simulink model of nonlinear pendulum system

III. DESIGN OF CONTROL LAW FOR NON-LINEAR IP MODEL

Now, here the control law for nonlinear inverted pendulum is shown. to provide the training data for neural network controller we use direct adaptive control technique referred in[10]. First four equations are entered into main equation

The main equation given below calculates the required force, U to keep the pendulum stable.

$$h_1 = \frac{3}{4l} g \sin\theta$$

$$h_2 = \frac{3}{4l} \cos\theta$$

$$f_1 = m\left(l \sin\theta \dot{\theta}^2 - \frac{3}{8} g \sin 2\theta\right) - f\dot{x}$$

$$f_2 = M + m(1 - \frac{3}{4}\cos^2\theta)$$

$$u = \frac{f_2}{h_2}\left[h_1 + k_1\theta + k_2\dot{\theta} + c_1 x + c_2\dot{x}\right] - f_1$$

For the simulations M, m, l, g are set to the values of the pendulum model. The following numeric values are used: M = 1.2 Kg, m = 0.109 Kg, l = 0.25m, g = 9.81 m/s, k_1=25, k_2=10,c_1=1, c_2= 2.6 [4].

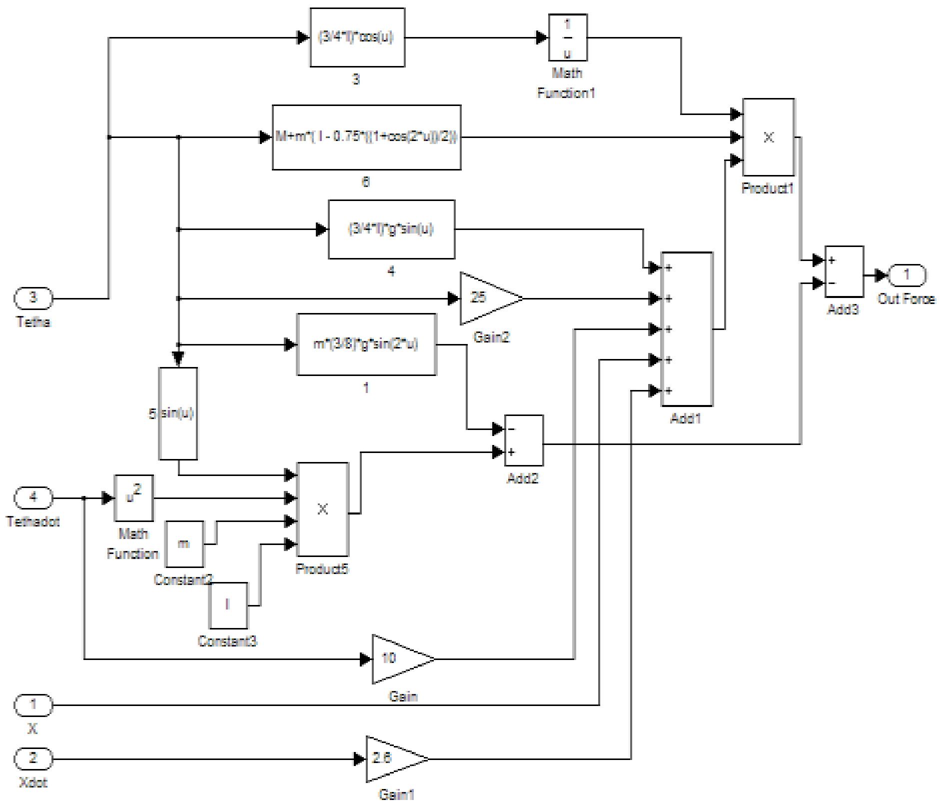

Fig.3: Simulink model of nonlinear control law

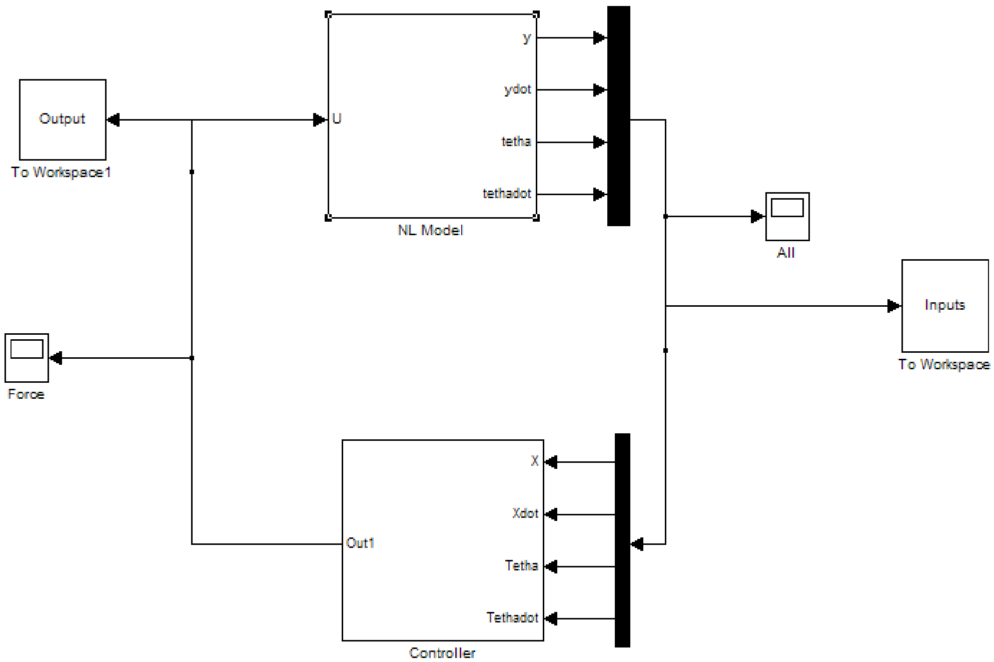

Fig.4: Simulink blocks of non-linear model of IP and controller

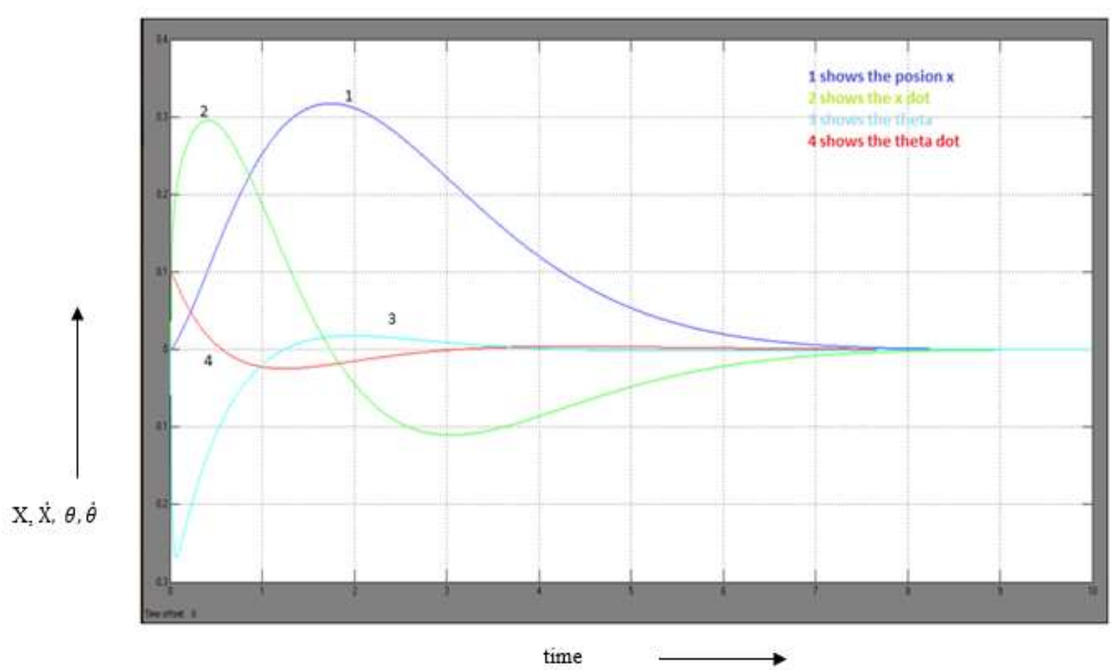

Fig. 5: simulation model result of non-linear model of inverted pendulum and conventional controller

IV Controller design of Fractional order model

In general, thebasicproblemwhich refers to control the angle and position of cart of the non-linear inverted pendulum system from the initial point to the final point.

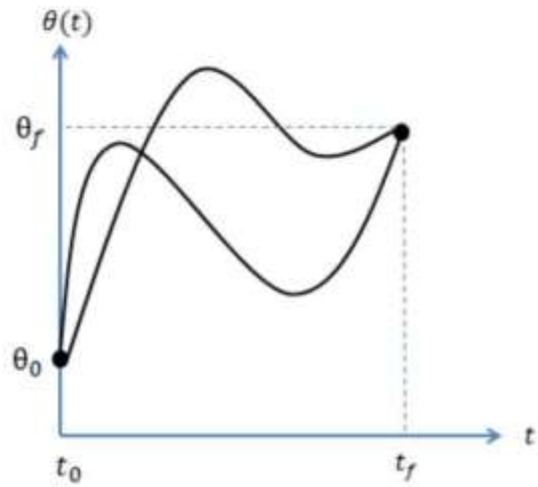

Fig.6 Initial, Via and Goal Points with Respect to Time

Initially four restrictions are required to produce a polynomial of fractional order. The manipulator's initial position and its intended position are both restrictions.

$$\delta(0) = \delta_0$$

and

$$\delta(t_f) = \delta_f$$

Imposed another two constrained which can be defined as

$$\dot{\delta}(0) = \dot{\delta}_0$$

and

$$\dot{\delta}(t_f) = \dot{\delta}_f$$

The proposed four equations describing this general fractional order polynomial are:

$$\delta_0 = a_0$$

$$\delta_f = a_0 + a_1 t_f + a_2 {t_f}^2 + a_3 {t_f}^{3.5}$$

$$\dot{\delta}_0 = a_1$$

$$\dot{\delta}_f = a_1 + 2a_2 t + 3.5 a_3 {t_f}^{2.5}$$

The coefficient value is as follows when four constraints are applied to (19) and (21).

$$a_2 = \frac{3}{1.5t_f^2}(\delta_f - \delta_0) - \frac{\dot{\delta}_f}{1.5t_f} - \frac{2.5\dot{\delta}_0}{1.5t_f}$$

$$a_3 = -\frac{4}{3.5t_f^{3.5}}(\delta_f - \delta_0) + \frac{2.33}{3.5t_f^{2.5}}(\dot{\delta}_f - \dot{\delta}_0)$$

With the help of the information above, the first simulation model for trajectory generation in Cartesian space using partially fractional-order polynomials has been created.

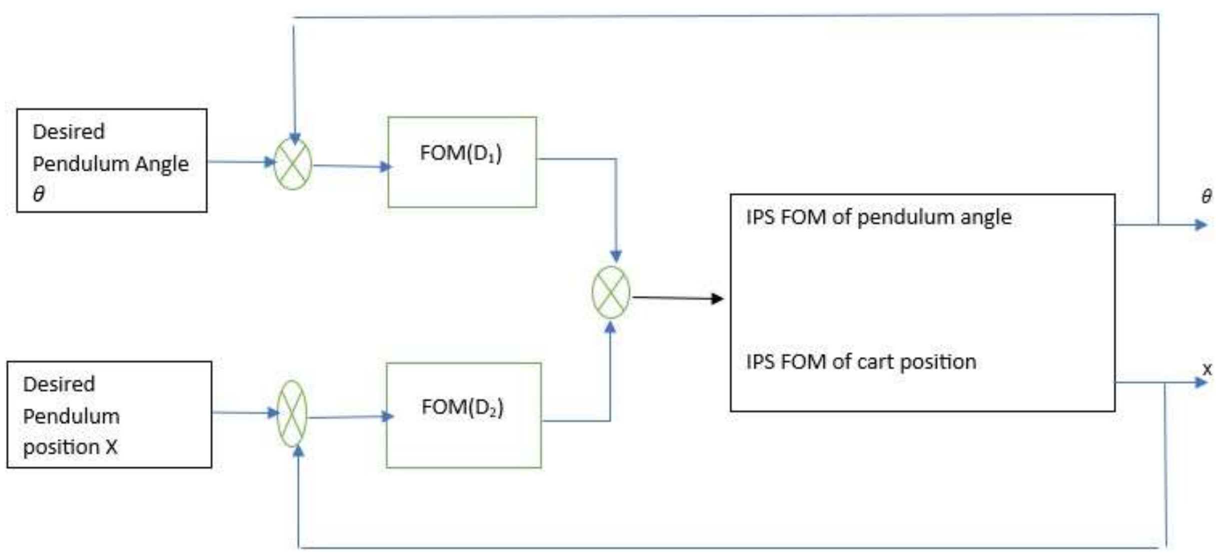

Fig.7Fractional order closed loop system.

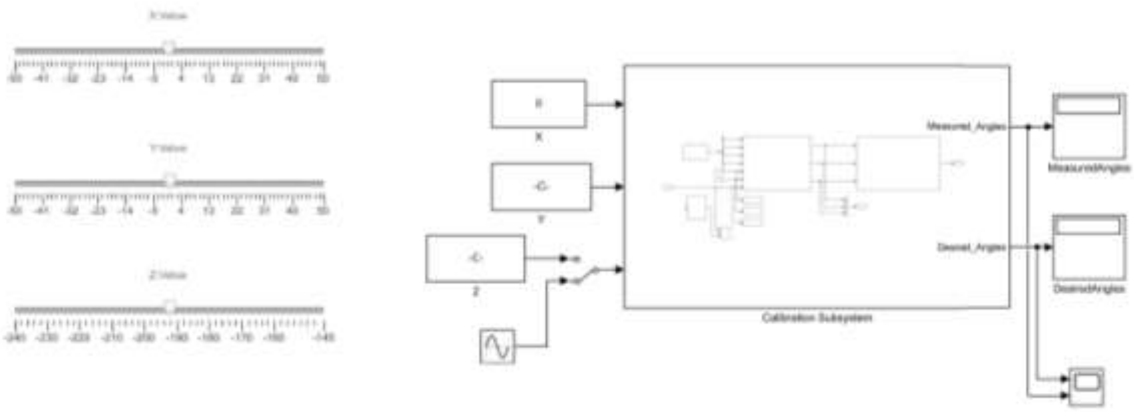

Fig. 8 Simulation model of fractional order model with changing values.

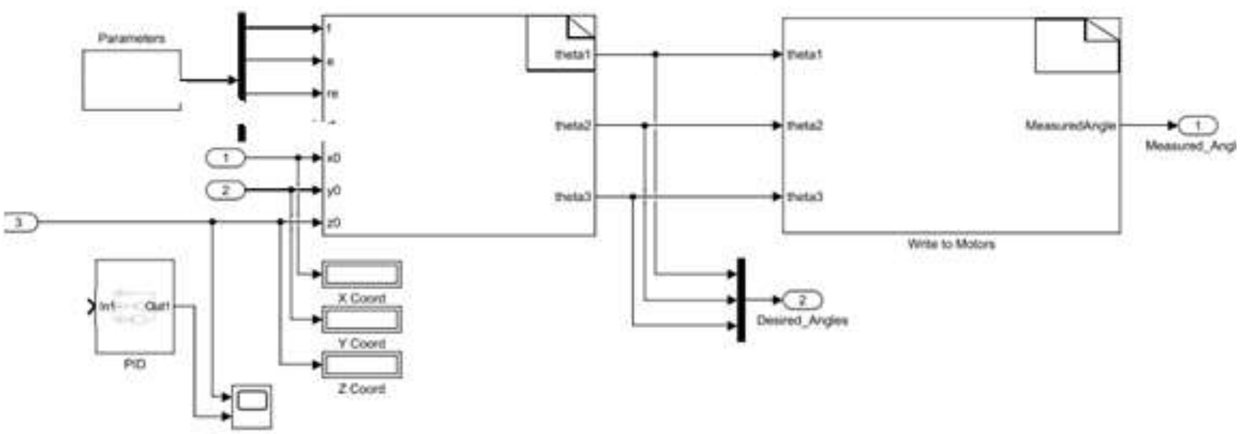

Fig.9 Simulation model of IPS with fractional order model.

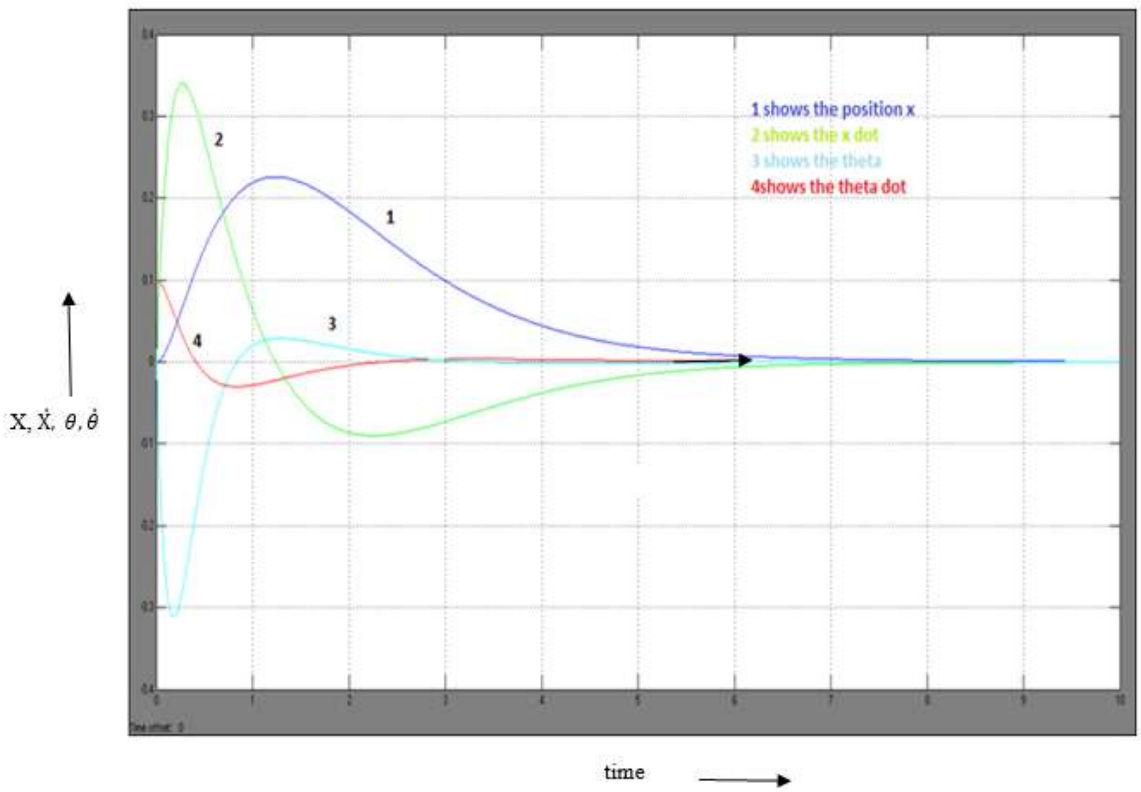

Fig. 10 Simulation model result of derivative angle and position of non-linear model of inverted pendulum and Fractional order controller

CONCLUSION

We have discovered and applied a novel kind of fractional order calculus, known as FOM, to an inverted pendulum system. We derive the structure of the FOM and employ a novel optimization technique to simultaneously estimate and optimize many parameters of the proposed FOM. A simulation study validates the effectiveness of the suggested FOM and demonstrates the viability of the proposed model. The simulation results demonstrate that the proposed model is much better than the integer model. It was proven through the experiments that the FOM controller works better than traditional controllers when it comes to speed response, steady-state error, pendulum fluctuation, and the cost of control effort. The experimental results further demonstrate the effectiveness of the suggested FOM controller in handling uncertainty regarding the mass of the pendulum. As a result, incorporating the FOM into controller design is feasible and encouraging.

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