AN OPTIMAL REPLENISHMENT POLICY FOR NON-INSTANTANEOUS DETERIORATING ITEMS WITH FINITE PRODUCTION RATE, STOCK DEPENDENT DEMAND AND PARTIAL BACKLOGGING

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ABSTRACT:
In this paper an optimal replenishment policy for non-instantaneous deteriorating items with finite production rate is developed. Shortages are allowed and are partially backlogged. The backlogging rate varies inversely as the waiting time for the next replenishment. Moreover, the demand is considered to be stock dependent demand. The EOQ models developed in this paper for perishable products consider continuous deterioration of the items after some period. Linear and non-linear deterioration cost function is considered and its effect on total cost function is highlighted in this paper. The objective of this work is to minimize the total inventory cost and to find the optimal length of replenishment and the optimal order quantity. Computational algorithms for this model are designed to find the optimal order quantity and the optimal cycle time. The results have been elucidated with numerical examples. Numerical illustrations and managerial insights are obtained to demonstrate the application and the performance of the proposed theory.

KEYWORDS: *Stock dependent demand, Non-instantaneous deteriorating items, Finite production, Deterioration cost function, Opportunity cost, Partial backlogging.*

INTRODUCTION:

Generally, it has been assumed that items start deteriorating as soon as they arrive in the stock. Deterioration is defined as decay, damage, obsolescence, evaporation, spoilage, loss of utility, or loss of marginal value of a commodity which decreases the original quality of the product. Many researchers such as Ghare and Schrader (1963), Philip (1974), Goyal and Giri (2001), Li and Mao (2009), Geetha and Udayakumar (2015) and Mahata (2015) assume that the deterioration of the items in inventory starts from the instant of their arrival. But in real life there are various products, like fresh fruits, vegetables, milk, meat, medicine, volatile liquids, and blood banks etc., that have a shelf-life and start deteriorating after a time lag. This underlines the fact that for some initial period of time, there is no deterioration in items. This phenomenon is termed as non instantaneous deterioration and the items are termed as non-instantaneous deteriorating items. Wu et al. (2006) defined the term ‘‘non-instantaneous’’ for such deteriorating items. He gave an optimal replenishment policy for non instantaneous deteriorating items with stock-dependent demand and partial backlogging. Large quantity of goods displayed in market attract the customers to buy more. If the stock is insufficient, customer may prefer some other brand, as a result it will fetch loss to the supplier.

This paper aims to develop a finite production inventory model for non instantaneous deteriorating items with stock dependent demand and partial backlogging. The degree of deteriorating of product utility is treated as a deterioration cost. Here the production rate is assumed to be finite. In this model shortages are allowed which are partially backlogged. Here we have considered two types of deterioration cost function

(i) Linear deterioration cost function and
(ii) Non linear deterioration cost function

A linear deterioration cost function

$$P(t) = \begin{cases} \varphi(t - t_d), & t \geq t_d \\ 0 & \text{Otherwise} \end{cases}$$

Which gives the cost of keeping one unit of product in stock until age t, where t_d be the time period at which deterioration of product starts and φ is constant. There will be no deterioration cost incurred upon the products upto time period (0, t_d).

An non-linear deterioration cost function is also taken as

$$P(t) = \begin{cases} \alpha(e^{\beta(t-t_d)} - 1), & t \geq t_d \\ 0 & \text{Otherwise} \end{cases}$$

Which also gives the cost of keeping one unit of product in stock until age t, where t_d be that time period at which deterioration of product starts and α and β are constants.

Assumptions and notations:

The following assumptions and notations have been used in the entire paper.

Assumptions

1. A single product is considered over a prescribed period of T unit of time
2. The replenishment occurs at an finite rate.
3. The demand rate function D(t) is deterministic and is functional given by $D(t) = \begin{cases} a + bI(t), & I(t) > 0 \\ a \quad , & I(t) \leq 0 \end{cases}$
4. Shortages are allowed and are subject to partial backlogging, the backlogged rate is defined to be $\frac{1}{1+\delta(T-t)}$ when inventory is negative. The backlogging parameter δ is a positive constant and $t_1 \leq t < T$.
5. Lead time is zero.

NOTATIONS

The following notations are used in this full paper

P	Production rate (finite)
Q	Number of items produced per production run
D(t)	Demand rate
h	Inventory holding cost per unit per unit time
K	Setup cost per unit per unit time
t_d	Time at which deterioration of product start
t_Ω	Time at which production resumed
q	Maximum inventory level at time t_Ω
T	Length of replenishment cycle, which will not exceed product lifetime
s	Shortage cost per unit per unit time
P(t)	Deterioration cost function
π	Opportunity cost per unit per unit time
δ	Backlogging parameter
t_1	The optimum time at which the inventory reaches zero and inventory starts to accumulate
Q*	Optimum value of Q (Units)
T*	Optimum value of T (Days)
TC(t)	Total cost per unit time

MODEL FORMULATION

EOQ Model for finite production with shortages

The rate of production P units per period is finite. From the figure it is clear that initially the stock is zero and the production starts with a finite rate P(>D) units per unit time while the demand is D(t) units per unit time. If "Q" be the number of items produced per production run and the production continue for a period t_Ω. Therefore the inventory increases with a rate (P-D) units per unit per unit time till it reaches its maximum at time t_Ω. Due to combined effects of demand and deterioration in the interval $[t_\Omega, t_1]$, the level of inventory gradually decreases. At t_1 the inventory level reaches zero.

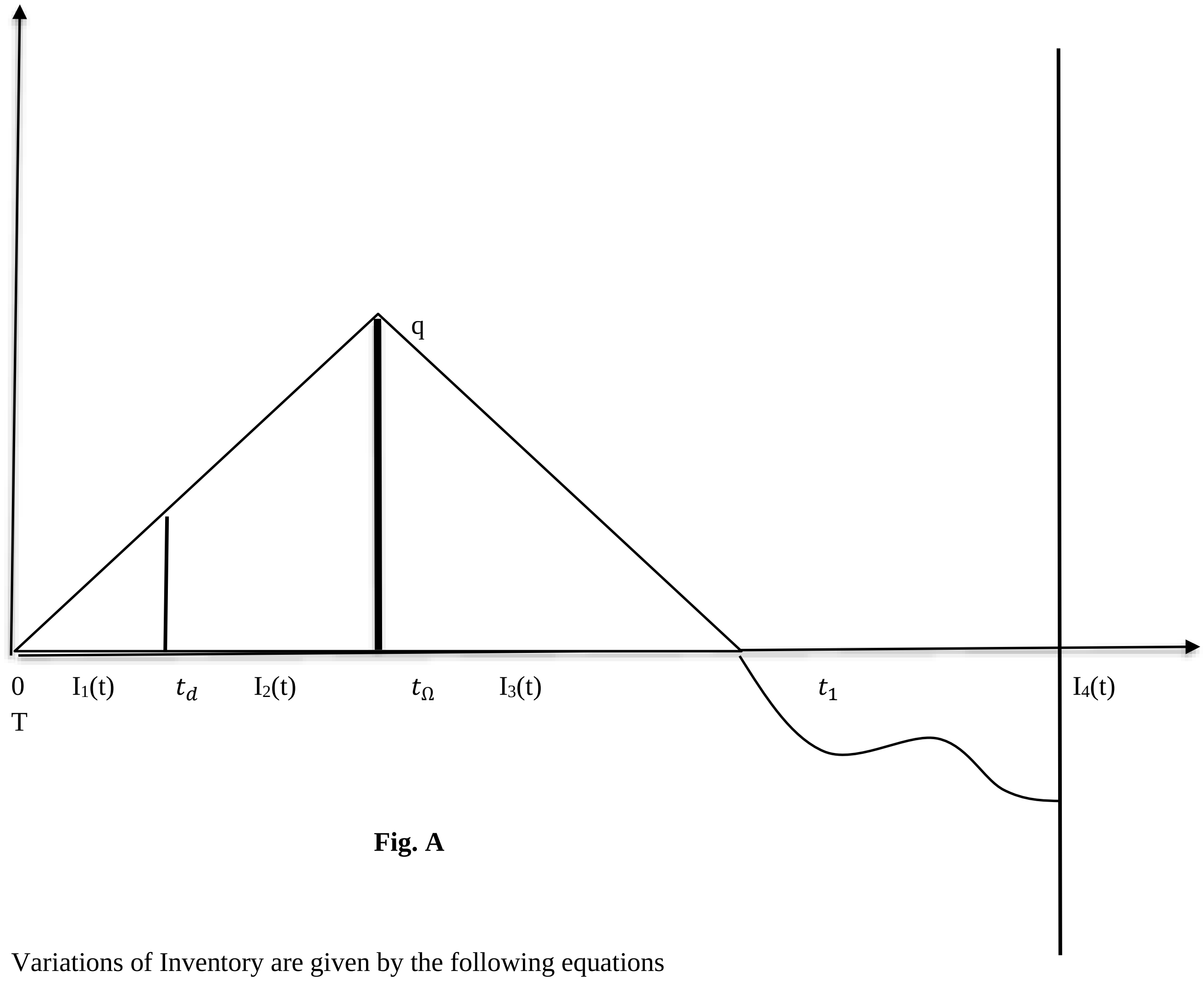


Fig. A

Variations of Inventory are given by the following equations

$$\frac{dI_1(t)}{dt} = P - a - b\, I_1(t) \qquad 0 \le t \le t_d$$

$$\frac{dI_2(t)}{dt} + \theta\, I_2(t) = P - a - bI_2(t) \qquad t_d < t \le t_\Omega$$

$\frac{dI_3(t)}{dt} + \theta I_3(t) \;\; = -a - bI_3(t) \qquad t_\Omega < t \le t_1$

$\frac{dI_4(t)}{dt} = \frac{-a}{1+\delta(T-t)} \qquad t_1 < t \le T$

Using boundary conditions

$I_1(0) = 0$, $I_2(t_\Omega) = q$, $I_3(t_1) = 0$, $I_4(T) = 0$

$I_1(t) = (P\text{-}a)\frac{(e^{-bt}+1)}{b}$

$I_2(t) = \frac{P-a}{b+\theta} + \frac{e^{(b+\theta)(t_\Omega - t)}(qb+q\theta+a-b)}{(b+\theta)}$

$I_3(t) = \frac{a}{(b+\theta)}\left[\, e^{(b+\theta)(t_\Omega - t)} - 1\right]$

$I_4(t) = \frac{a(log(1+\delta(T-t)) - log(1+\delta(T-t_1)))}{\delta}$

From the continuity of $I_1(t_d) = I_2(t_d)$ gives the inventory level at t_Ω

$q = \frac{1}{be^{-(b+\theta)x}\,(b+\theta)}\;(e^{-(b+\theta)z-bx}\,((b+\theta)(a-P) + (P-a)(be^{-(b+\theta)x}\;+\theta e^{-(b+\theta)z})$

The maximum backlogging quantity is given by

$I_b = \frac{-a(log(1+\delta(T-t_1)))}{\delta}$

The number of items produced per production run Q is given by

$Q = q + I_b$

The total cost per cycle consists of setup cost, inventory holding cost,

deterioration cost, shortage cost and opportunity cost.

Setup cost

The setup cost of inventory for the period is given by

$S = \frac{K}{T}$

Holding cost

The holding cost per cycle is given by

$H = h\int_0^{t_d} I_1(t)dt + h\int_{t_d}^{t_\Omega} I_2\,(t)dt + h\int_{t_\Omega}^{t_1} I_3\,(t)dt$

$= \frac{h(P-a)}{b^2}\left(e^{-bt_d}+bt_d-1\right)-\frac{h}{(b+\theta)^2}\left[(P-a)(b+\theta)(t_d-t_\Omega)+(qb+q\theta-P+a)\left(e^{(b+\theta)(t_1-t_\Omega)}-t_d e^{(b+\theta)(t_1-t_d)}\right)\right]-\frac{ha}{(b+\theta)^2}[(b+\theta)(t_1-t_\Omega)+1-e^{(b+\theta)(t_1-t_\Omega)})$

Shortage cost

Now the cost of shortage for the period (t_1,T) is given by

$SC = s\int_{t_1}^{T} -I_4(t)dt$

$= -sa[\frac{\left(log\left(1+\delta(T-t_1)\right)-\delta(T-t_1)\right)}{\delta^2}]$

Opportunity cost:

The opportunity cost due to sales lost during the replenishment cycle for the period (t_1,T) is given by $OC = \pi\int_{t_1}^{T}[a-\frac{a}{1+\delta(T-t)}]\,dt$

$= \pi a\{(T-t_1)+\frac{log\left(1+\delta(T-t_1)\right)}{\delta}\}$

Case I: Linear deterioration cost function with shortage:

Deterioration cost:

A linear deterioration cost function $P(t) = \varphi(t-t_d),\ t \geq t_d$, which gives the cost of keeping one unit of product in stock until age t, where t_d be the time period at which deterioration of product starts and φ is constant**.**

If linear deterioration cost function is used, then the cost due to deterioration of products during the period (t_d, t_1) is given by

$DC1 = \int_{t_d}^{t_\Omega}[\varphi(t-t_d)\theta I_2(t)dt + \int_{t_\Omega}^{t_1}[\varphi(t-t_d)\theta I_3(t)dt$

$= -\frac{1}{2}\frac{1}{(b+\theta)^3}\left[\varphi\theta({t_d}^2+{t_\Omega}^2)(a-P)(b^2+\theta^2)+2(P-a)\left(-\varphi\theta^2{t_d}^2b+e^{(b+\theta)(t_1-t_d)}-e^{(b+\theta)(t_1-t_\Omega)}+2t_dt_\Omega b\theta-2{t_\Omega}^2b\theta\right)+2e^{(b+\theta)(t_1-t_\Omega)}(2(t_\Omega-t_d)(qb\theta+qb^2+q\theta^2+pb-p\theta+ab+a\theta)+q(b+\theta))-2qe^{(b+\theta)(t_1-t_d)}(b+\theta)+\{\emptyset\theta a(b+\theta)^2{t_\Omega}^2+2e^{(b+\theta)(t_1-t_\Omega)}(b+\theta)(t_\Omega+2t_d)+2t_dt_\Omega(b+\theta)^2+(t_1(b+\theta)+1)^2+1\right\}$

The total cost per unit time is

$TC_1(t) = \frac{S+H+DC1+SC+OC}{T}$

$= \frac{1}{T}[K + \frac{h(P-a)}{b^2}\left(e^{-bt_d}+bt_d-1\right)-\frac{h}{(b+\theta)^2}\left[(P-a)(b+\theta)(t_d-t_\Omega)+(qb+q\theta-p+a)\left(e^{(b+\theta)(t_1-t_\Omega)}-t_d e^{(b+\theta)(t_1-t_d)}\right)\right]-\frac{ha}{(b+\theta)^2}[(b+\theta)(t_1-t_\Omega)+1-e^{(b+\theta)(t_1-t_\Omega)})]$

$-\frac{1}{2}\frac{1}{(b+\theta)^3}$ $[\,\varphi\theta({t_d}^2+{t_\Omega}^2)(a-P)(b^2+\theta^2)+2(P-a)\big(-\varphi\theta^2{t_d}^2b+e^{(b+\theta)(t_1-t_d)}-$
$e^{(b+\theta)(t_1-t_\Omega)}+2t_dt_\Omega b\theta-2{t_\Omega}^2b\theta\,\big)+2e^{(b+\theta)(t_1-t_\Omega)}(2(t_\Omega-t_d)(qb\theta+qb^2+{t_\Omega}^2+$
$2e^{(b+\theta)(t_1-t_\Omega)}(b+\theta)(t_\Omega+2t_d)+2t_dt_\Omega(b+\theta)^2+\big(t_1(b+\theta)+1q\theta^2+pb-p\theta+ab+a\theta\big)+$
$q(b+\theta))-\ 2qe^{(b+\theta)(t_1-t_d)}(b+\theta)+\{\emptyset\theta a(b+\theta)^2)\big)^2+1\}]$-sa$\left[\frac{\left(log\left(1+\delta(T-t_1)\right)-\delta(T-t_1)\right)}{\delta^2}\right]+$
$\pi a(T-t_1+\frac{log\left(1+\delta(T-t_1)\right)}{\delta})$ **…(1)**

The necessary conditions for the total annual cost $TC_1(t_1,T)$ is minimum with respect to t_1 and T are $\frac{\partial TC_1(t_1,T)}{\partial t_1}=0$ and $\frac{\partial TC_1\,(t_1,T)}{\partial T}=0$ **…(2)**

Such that they have to satisfy the following conditions

$\frac{\partial TC_1\,(t_1,T)}{\partial t_1^2}>0$, $\frac{\partial TC_1\,(t_1,T)}{\partial T^2}>0$ and

$\{(\frac{\partial TC_1(t_1,T)}{\partial t_1^2})\,(\frac{\partial TC_1\,(t_1,T)}{\partial T^2})-\ \frac{\partial^2 TC_1\,(t_1,T)}{\partial t_1\partial T}\}>0$ at $t_1=t_1^*$ and T = T* **…(3)**

Algorithm – 1 [case(1)]

Step 1: Start

Step 2: Evaluate $\frac{\partial TC_1(t_1,T)}{\partial t_1}$ and $\frac{\partial TC_1\,(t_1,T)}{\partial T}$

Step 3: Solve the simultaneous equations $\frac{\partial TC_1(t_1,T)}{\partial t_1}=0$ and $\frac{\partial TC_1\,(t_1,T)}{\partial T}=0$ and and fine the vales of $t_1\ and\ T$ by fixing $t_d and\ t_\Omega$ and initializing the values of P,k,h,a,b,q,c, $\varphi,\theta,\delta,\pi,\alpha,\beta$

Step 4: Choosing one set of solution from 3.

Step 5: If the values in equation (3) are greater than zero, then this set of solution is optimal and go to step 6. Otherwise go to step 4 with next set of solution obtained from step 3.

Step 6: Evaluate $TC_1\ (t_1^*,T^*)$

Step 7: End

Case II: Non linear deterioration cost function with shortage:

A non linear deterioration cost function , P(t) = $\alpha(e^{\beta(t-t_d)}-1),\quad t\geq t_d$

which gives the cost of keeping of one unit of product in stock until age t, where t_d be the time period at which deterioration of product start and $\alpha\ and\ \beta$ are constant

Deterioration cost:

If non linear deterioration cost function is used, then the cost due to deterioration of products delivered during the period (t_d, t_1) is given by

DC2 = $\int_{t_d}^{t_\Omega} \alpha(e^{\beta(t-t_d)} - 1)\theta I_2(t)dt + \int_{t_\Omega}^{t_1} \alpha(e^{\beta(t-t_d)} - 1)\theta I_3(t)dt$

$= \frac{\alpha\theta e^{-t_d(\beta+b+\theta)}}{(b+\theta)^2\,\beta(b+\theta-\beta)}[e^{t_d(\beta+b+\theta)}(P\theta\beta(t_d-t_\Omega)(\theta-\beta)+2pb\beta\theta(t_d-t_\Omega)+t_d\beta^2(a\theta-Pb)-2Pb\theta+\beta\theta(p-a)-ab\beta+(a-p)(\theta^2+b^2)-2ab\beta\theta(t_d-t_\Omega)+Pb\beta+Pb^2t_d\beta+2ab\theta+ab\beta^2(t_\Omega+t_d)+Pzb\beta^2-at_\Omega\beta^2\theta-pb^2t_\Omega p)+e^{t_d(b+\theta)+\beta t_\Omega}(2Pb\theta+\alpha\beta\theta-a\theta^2-ab\theta+Pb+p\theta^2-ab^2+ab\beta-P\beta\theta)+e^{t_d(\beta+b+\theta)+(b+\theta)(t_1-t_\Omega)}\big(\beta^2(P-\alpha)+\beta(\alpha\theta+ab)+P\beta(b-\theta)+q\beta\theta(\theta-\beta)\big)+e^{t_d(b+\theta)+\beta t_\Omega+(b+\theta)(t_1+t_\Omega)}\big(b\beta(p-a)-q\beta(b^2+2\beta\theta)-q(\beta\theta^2+b^2)\big)+e^{t_d\beta+(b+\theta)t_1}(\beta^2(\alpha-P+bq+q\theta)-\alpha\beta\theta+P\beta\theta)]+\frac{1}{\beta(-b^2\beta+b^3+3b^2\theta-2b\theta\beta+3b\theta^2-\theta^2\beta+\theta^3)}[\frac{a\theta\alpha}{e^{-\beta t_d}}(e^{\beta t_d}(\beta(b+\theta)-\beta^2+2b\theta\beta(t_1-t_\Omega)-t_\Omega\beta(\theta^2+b^2)+\theta\beta^2(t_\Omega-t_1)+bt_1\beta(b-\beta)+\theta^2\beta)-e^{\beta y}(b+\theta)^2+e^{\beta t_\Omega}((b+\theta)\big((b+\theta)-\beta\big)+e^{(b+\theta)(t_1-t_\Omega)+\beta t_d}\big(\beta^2-\beta(b+\theta)\big)+e^{(b+\theta)(t_1-t_\Omega)+\beta t_\Omega}\beta(\theta+b)]$

The total cost per unit time is

TC_2(t) $= \frac{S+H+DC2+SC+OC}{T}$

$=\frac{1}{T}$[K+ $\frac{h(P-a)}{b^2}\left(e^{-bt_d}+bt_d-1\right)-\frac{h}{(b+\theta)^2}\left[(P-a)(b+\theta)(t_d-t_\Omega)+(qb+q\theta-P+a)\left(e^{(b+\theta)(t_1-t_\Omega)}-t_de^{(b+\theta)(t_1-t_d)}\right)\right]-\frac{ha}{(b+\theta)^2}[(b+\theta)(t_1-t_\Omega)+1-e^{(b+\theta)(t_1-t_\Omega)})]$

$+\frac{\alpha\theta e^{-t_d(\beta+b+\theta)}}{(b+\theta)^2\,\beta(b+\theta-\beta)}[e^{t_d(\beta+b+\theta)}(P\theta\beta(t_d-t_\Omega)(\theta-\beta)+2Pb\beta\theta(t_d-t_\Omega)+t_d\beta^2(a\theta-pb)-2Pb\theta+\beta\theta(p-a)-ab\beta+(a-P)(\theta^2+b^2)-2ab\beta\theta(t_d-t_\Omega)+Pb\beta+Pb^2t_d\beta+2ab\theta+ab\beta^2(t_\Omega+t_d)+Pzb\beta^2-at_\Omega\beta^2\theta-Pb^2t_\Omega p)+e^{t_d(b+\theta)+\beta t_\Omega}(2Pb\theta+\alpha\beta\theta-a\theta^2-ab\theta+Pb+P\theta^2-ab^2+ab\beta-p\beta\theta)+e^{t_d(\beta+b+\theta)+(b+\theta)(t_1-t_\Omega)}\big(\beta^2(P-\alpha)+\beta(\alpha\theta+ab)+P\beta(b-\theta)+q\beta\theta(\theta-\beta)\big)+e^{t_d(b+\theta)+\beta t_\Omega+(b+\theta)(t_1+t_\Omega)}\big(b\beta(p-a)-q\beta(b^2+2\beta\theta)-q(\beta\theta^2+b^2)\big)+e^{t_d\beta+(b+\theta)t_1}(\beta^2(\alpha-P+bq+q\theta)-\alpha\beta\theta+P\beta\theta)]+\frac{1}{\beta(-b^2\beta+b^3+3b^2\theta-2b\theta\beta+3b\theta^2-\theta^2\beta+\theta^3)}[\frac{a\theta\alpha}{e^{-\beta t_d}}(e^{\beta t_d}(\beta(b+\theta)-\beta^2+2b\theta\beta(t_1-t_\Omega)-t_\Omega\beta(\theta^2+b^2)+\theta\beta^2(t_\Omega-t_1)+bt_1\beta(b-\beta)+\theta^2\beta)-e^{\beta y}(b+\theta)^2+e^{\beta t_\Omega}((b+\theta)\big((b+\theta)-\beta\big)+e^{(b+\theta)(t_1-t_\Omega)+\beta t_d}\big(\beta^2-\beta(b+\theta)\big)+e^{(b+\theta)(t_1-t_\Omega)+\beta t_\Omega}\beta(\theta+b)]-sa\left[\frac{\left(log(1+\delta(T-t_1))-\delta(T-t_1)\right)}{\delta^2}\right]+\pi a[(T-t_1)+\frac{log(1+\delta(T-t_1))}{\delta}]$ **...(4)**

The necessary conditions for the total annual cost $TC_2(t_1,T)$ is convex with respect to t_1 and T are $\frac{\partial TC_2\,(t_1,T)}{\partial t_1}=0$ and $\frac{\partial TC_2\,(t_1,T)}{\partial T}=0$ **…(5)**

Such that they have to satisfy the following conditions

$\frac{\partial TC_2(t_1,T)}{\partial t_1^2} > 0$, $\frac{\partial TC_2\,(t_1,T)}{\partial T^2} > 0$ and

$\{(\frac{\partial TC_2\,(t_1,T)}{\partial t_1^2})\,(\frac{\partial TC_2\,(t_1,T)}{\partial T^2}) - \frac{\partial^2 TC_2(t_1,T)}{\partial t_1 \partial T}\} > 0$ at $t_1 = t_1^*$ and T = T* **…(6)**

Algorithm – 2 [case(2)]

Step 1: Start

Step 2: Evaluate $\frac{\partial TC_2(t_1,T)}{\partial t_1}$ and $\frac{\partial TC_2\,(t_1,T)}{\partial T}$

Step 3: Solve the simultaneous equations $\frac{\partial TC_2(t_1,T)}{\partial t_1} = 0$ and $\frac{\partial TC_2(t_1,T)}{\partial T} = 0$ and and find the vales of t_1 *and* T by fixing t_1 and initializing the values of p,k,h,a,b,q,c,$t_d, t_\Omega, \varphi, \theta, \delta, \pi, \alpha, \beta$

Step 4: Chosing one set of solution from 6.

Step 5: If the values in equation (6) are greater than zero, then this set of solution is optimal and go to step 6. Otherwise go to step 4 with next set of solution obtained from step 3.

Step 6: Evaluate $TC_2\,(t_1^*, T^*)$

Step 7: End

Numerical example for case I - Linear deterioration cost function:

Example :1 Consider an inventory system with the following data: P = 40 units/day, K = 50($/unit)per day, a = 20 units/day, b = 0.1 units/day, h = 0.03($/unit) per day, t_d = 1 day, t_Ω = 10 days, s = 1($/unit), $\emptyset = 3.14$, $\delta = 0.3, \theta = 0.01, \pi = 0.075$.

Therefore, applying algorithm 1 of Case 1, we get the optimal solutions, t_1 = 26.67 days, T = 41.8 days, the corresponding total cost $TC1$ = 67.88 and optimum ordering quantity Q = 137.79 units.

Numerical example for case II - Non linear deterioration cost function:

Example :2 Consider an inventory system with the following data: P = 40 units/day, K = 50($/unit)per day, a = 20 units/day, b = 0.1 units/day, h = 0.03($/unit) per day, t_d = 1 day, t_Ω = 0.027 days, s = 1($/unit), $\emptyset = 3.14$, $\delta = 0.3, \theta = 0.01, \pi = 0.075$.

Therefore, applying algorithm 2 of Case II , we get the optimal solutions, t_1 = 18.97 days, T = 31.69 days, the corresponding total cost $TC2$ = 102845.741.

Comparing the results for case I and II in the numerical examples, total cost for non linear deterioration cost function is very much higher than linear deterioration cost function.

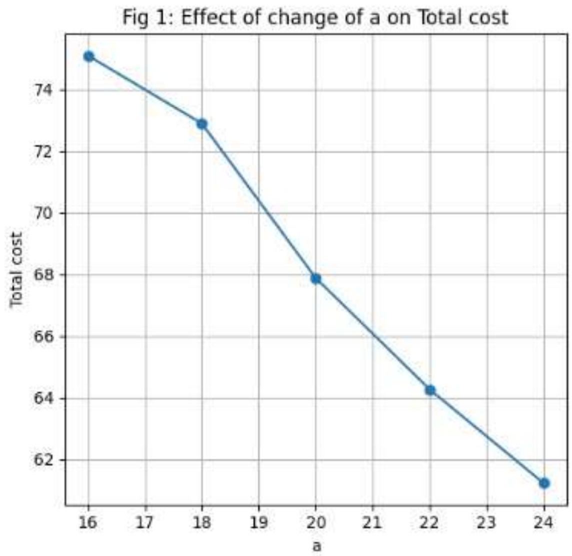
Therefore the following table , sensitivity analysis and managerial implications carried out only for the linear deterioration cost function.

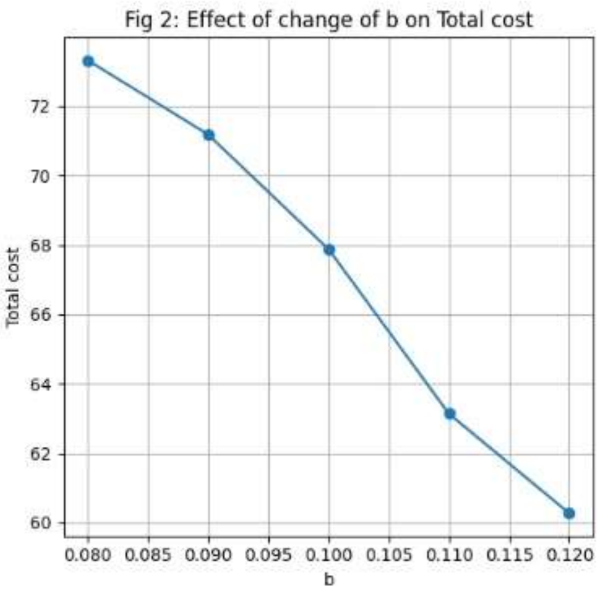
TABLE : Effect of change in various parameters of the inventory

Changing parameter	% of changing parameter	Change in values	t_1	T	Q	T C
A	-20%	16	38.67	51.67	119.9	75.09
	-10%	18	31.07	48.35	126.34	72.89
	0%	20	26.67	41.8	137.79	67.88
	+10%	22	22.64	36.79	149.78	64.27
	+20%	24	19.35	31.14	165.43	61.23
B	-20%	0.08	31.64	55.23	129.35	73.32
	-10%	0.09	28.12	48.78	132.34	71.18
	0%	0.10	26.67	41.8	137.79	67.88
	+10%	0.11	20.34	32.69	150.23	63.14
	+20%	0.12	17.65	26.14	163.07	60.27
P	-20%	32	23.34	36.67	113.32	53.45
	-10%	36	24.46	38.43	114.76	58.23
	0%	40	26.67	41.8	137.79	67.88
	+10%	44	36.88	55.46	163.41	143.14
	+20%	48	41.45	68.19	186.67	193.67
S	-20%	0.8	24.38	39.67	136.15	64.09
	-10%	0.9	25.14	40.12	136.98	65.23
	0%	1	26.67	41.8	137.79	67.88
	+10%	1.1	27.78	42.32	138.42	69.12
	+20%	1.2	28.12	43.45	139.02	70.47
θ	-20%	.008	33.67	46.63	124.65	52.45
	-10%	.009	29.07	43.45	129.32	56.23
	0%	.01	26.67	41.8	137.79	67.88
	+10%	.011	23.64	38.23	140.78	124.14
	+20%	.012	29.35	35.41	144.32	143.67
δ	-20%	.024	24.38	39.67	154.43	53.45
	-10%	.027	25.14	40.12	149.35	58.23
	0%	.03	26.67	41.8	137.79	67.88
	+10%	.033	27.78	42.32	131.79	73.14
	+20%	.036	28.12	43.45	127.23	82.67

Changing parameter	**Change of values(in days)**	**t_1**	**T**	**Q**	**T C**
t_d	1	26.67	41.8	137.79	67.88
	2	29.13	48.34	136.65	82.13
	3	31.78	56.79	135.88	98.54
	4	38.32	74.34	134.96	105.96

	5	46.15	95.02	134.01	126.98





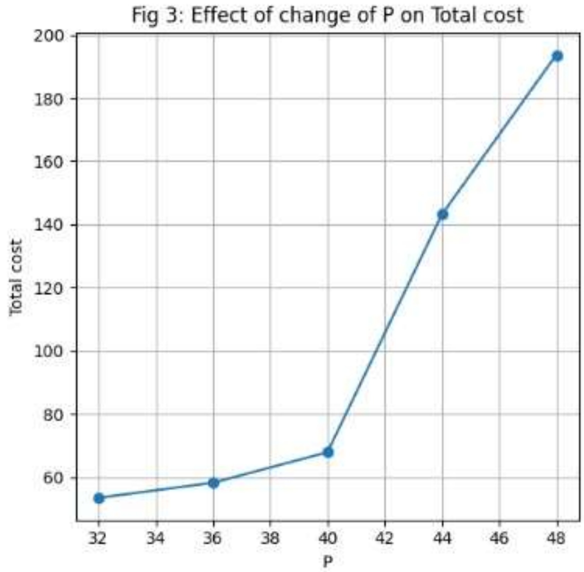


Fig 3: Effect of change of P on Total cost
Total cost
200
180
160
140
120
100
80
60
32
34
36
38
40
42
44
46
48
P

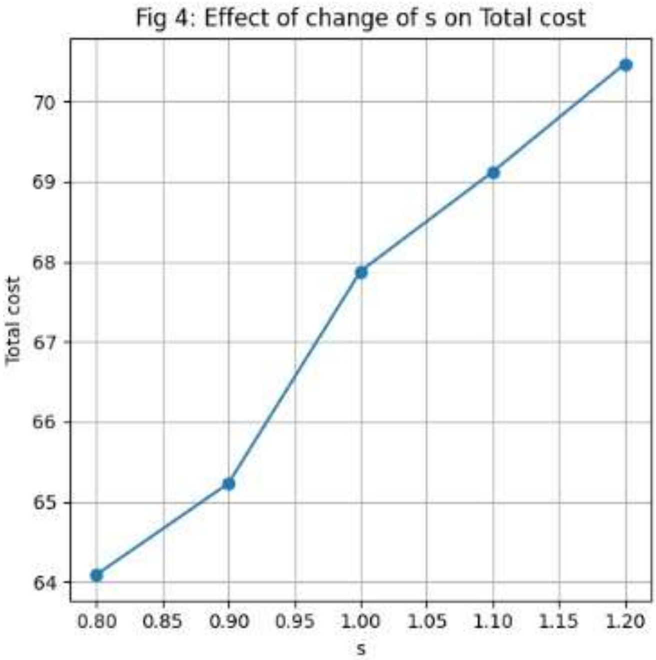


Fig 4: Effect of change of s on Total cost
Total cost
70
69
68
67
66
65
64
0.80
0.85
0.90
0.95
1.00
1.05
1.10
1.15
1.20
s

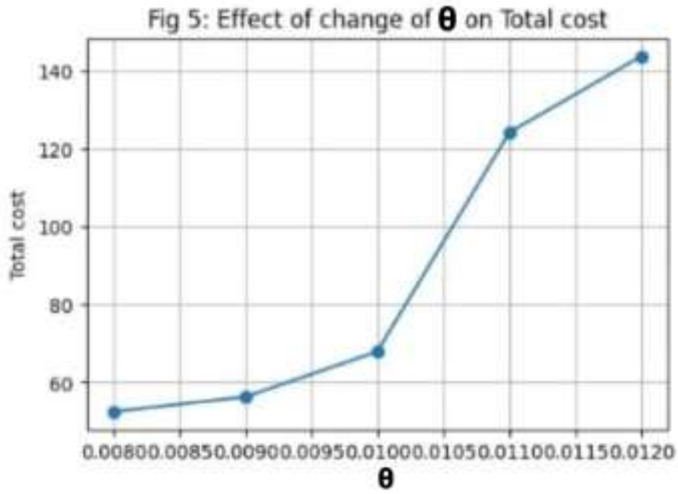


Fig 5: Effect of change of θ on Total cost
Total cost
140
120
100
80
60
0.00800.00850.00900.00950.01000.01050.01100.01150.0120
θ

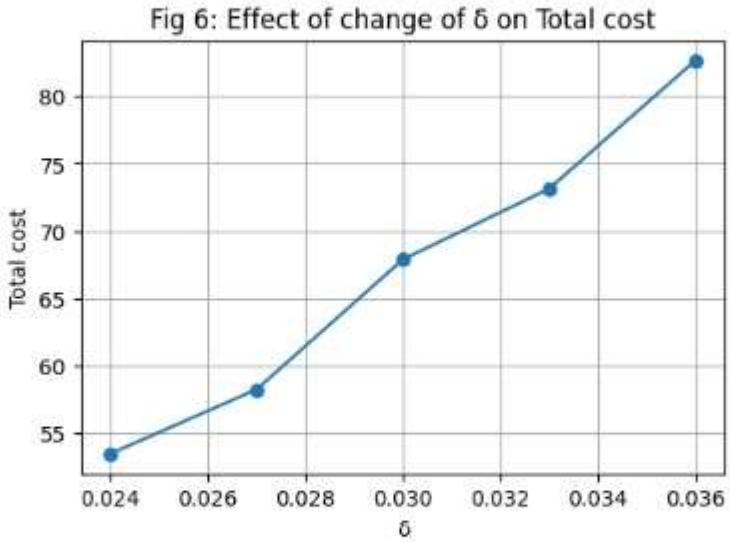


Fig 6: Effect of change of δ on Total cost
Total cost
80
75
70
65
60
55
0.024
0.026
0.028
0.030
0.032
0.034
0.036
δ

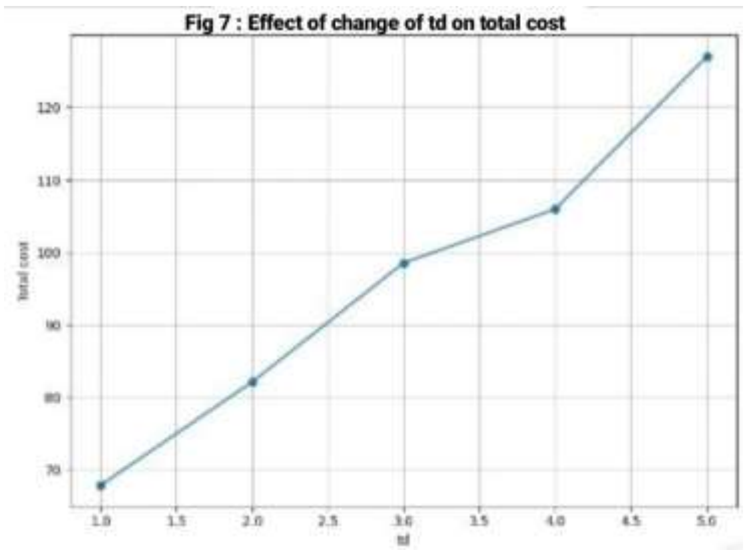


Fig 7 : Effect of change of td on total cost
Total cost
120
110
100
90
80
70
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
td

MANAGERIAL IMPLICATIONS:

We now study the effects of changes in the values of the parameters a, b, P, s, δ, θ and t_d on the optimal replenishment policy of Example 1.We change one parameter at a time, keeping the other parameters unchanged. The results are summarized in the above table. Based on the numerical results, we obtain the following managerial implications:

1. When the parameter a is increasing, the total optimal cost (TC) is highly decreasing. The time t_1 at which the inventory level becomes zero and the cycle length (T) are decreasing. But the order quantity (Q) is increasing.
2. When the parameter b is increasing, the total optimal cost (TC) is highly decreasing. The time t_1 at which the inventory level becomes zero and the cycle length (T) are decreasing. But the order quantity (Q) is increasing.
3. When the parameter P is increasing, the total optimal cost (TC) is increasing. The time t_1 at which the inventory level becomes zero and the cycle length (T) are increasing. But the order quantity (Q) is increasing.
4. When the parameter s is increasing, the total optimal cost (TC) is highly increasing. The time t_1 at which the inventory level becomes zero and the cycle length (T) are increasing. But the order quantity (Q) is increasing.
5. When the deterioration rate θ is increasing, the total optimal cost (TC) is increasing. The time t_1 at which the inventory level becomes zero and the cycle length (T) are decreasing. But the order quantity (Q) is increasing. That is, increasing of the deterioration rate α will increase the total cost.
6. If the backlogging parameter δ increases, the total optimal cost (TC), the time at which the inventory level becomes zero and the cycle length (T) are increasing. But the order quantity (Q) is decreasing. That is, in order to minimize the total cost, backlogging parameter have to be decreased.
7. When deteriorating time t_d (in days) is increasing, the total optimal cost (TC) is increasing. The time t_1 at which the inventory level becomes zero and the cycle length (T) are increasing. But the order quantity (Q) is decreasing.

CONCLUSION:

In this paper, a production inventory model for non-instantaneous deteriorating items with stock-dependent demand is developed. Production rate is postulated to be finite. In this paper the idea of continuous deterioration of utility for an individual product and a measure for the utility deterioration as a linear and non linear deterioration cost function with shortages are allowed and can be partially backlogging. The aim of this paper is to obtain the optimal solution of cycle length, time intervals and order quantity simultaneously with the objective of minimizing the total cost. Numerical example and managerial implications are given to illustrate the application and the performance of the proposed methodology. When the stock dependent demand parameters a and b are increased the total cost is decreasing drastically. When the backlogging parameter δ increases, the total optimal cost is increasing. This is due to exhibiting the items to promote and lure the customers (stock dependent demand). To conclude, sensitivity of the optimal solution with respect to changes different parameter values is carried out and some managerial implications are obtained of the proposed model.

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