ANALYSIS OF FREE AND DAMPED VIBRATION OF AN ELLIPTICAL PLATE RESTING ON ELASTIC FOUNDATION

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Abstract

In present study, we have analyzed free and damped vibration of an elliptical plate resting on elastic foundation with thickness changes linearly. The model equation is solved using energy strain method i.e., Rayleigh-Ritz Technique. The parameter under study are damping parameter, time period, aspect ratio, taper constant, logarithmic decrement, elastic foundation, also the effect of these parameter on the deflection of plates is also provided with numerical and graphical result. The relationship between various parameters as told above are also presented numerically and graphically for first two modes of vibration for thin clamped elliptical plate.

Keywords: *Vibration, Elliptical plate, Damped vibration, Elastic Foundation, aspect ratio, taper constant, logarithmic decrement.*

INTRODUCTION

Modern structures and machine designs requires stability, durability, reliability, strength and efficiency. So we need critical thinking and analysis to study these design structures with the advancement of technology and use of new developed material in the field of construction of equipment and structures. We requires various factors which could be bounded with the construction in control of different type of plate vibrations. Vibration of plates plays a vital role in the field of mechanical engineering, machine designing, nuclear reactor technology, big architectural design, naval structures etc. and various parameters effect these structures namely variable thickness, homogeneity, temperature, foundation etc. Plates of various shapes are used in construction which are studied with reference to above parameters. A review of literature provide us the insight of the studies being conducted in structural analysis. We observe that a large amount of work has been carried out for rectangular plate, circular plate, infinite plate but lesser amount of work has been taken for elliptical plates. Elliptical plates are widely used in various structures. So the need to study these elliptical plates arises. J. S. Tomar and A. K. Gupta (1984) [12] have studied vibration of orthotropic elliptic plate in which thickness is non-uniform and temperature variation using Galerkin's Method corresponding to two modes of frequencies for clamped boundary conditions. B. Singh and S. Chakraverty (1992) [8] used thickness variation which varies quadratically to study the transverse vibration of elliptic plates and considering circular plate as a special case by employing ritz method. Bani Singh and Vipin Saxena (1995) [10] considered quarter of an elliptic plate which is widely used in industry and applied linear and quadratic variation in thickness for obtaining frequencies and mode shapes. The method was used rayleigh ritz method. B. Singh and V. Saxena[1996] [9] thoroughly worked on circular plate and considered its vibration studying the effect of unidirectional quadratic variation in plate thickness. S. Chakraverty, Ragini Jindal and V. K. Agarwal (2005) [2] have determined Flexural vibration of elliptical plate with Rayleigh-Ritz method and Gram Schmidt process. S. Chakraverty, Ragini Jindal and V. K. Agarwal (2007) [3] have investigated the vibration of Orthotropic elliptic and circular plate with non-homogeneity and variable thickness. A.K. Gupta, A. Khanna [2007] [5] considered the effect of visco elasticity on rectangular plate and solved this problem with thickness variation in both direction considering it linearly. A. K. Gupta and et.al [2009] [7] extended their work and studied visco-elastic effect on rectangular plate considering bi-direction variation in thickness which varies exponentially. A. K. Gupta, L. Kumar (2009) [4] considered visco elastic elliptic plate in their study with reflecting the effect of thermal gradient non-homogeneous and variable thickness. E. Kago, J. Lellep [2013][7] worked on Free vibrations of plates on Elastic foundation. P. Arora[2016] [1] established results for tapered parallelogram plate with two dimensional thickness, non homogeneity and also considering change in temperature the vibration analysis of non-homogeneous tapered parallelogram plate with two dimensional thickness and temperature variation. Nitu Singh and Vipin Saxena (2017)[11] presented a numerical experiment on quarter of an elliptic plate in which thickness varies exponentially.

By above discussion we observe that damping effect on vibration analysis of elliptical plate is being least studied. Through this paper we shall study elliptical plate with various parameters as damping factor, elastic foundation factor, taper constant, aspect ratio, time period, deflection, logarithmic decrement and distance of elliptical plate from centre to any point. We shall carry our work to find the relation between the parameters as told above. Here, we will solved our problem of free and damped vibration of elliptical plates resting on elastic foundation by Rayleigh Ritz method taking two term deflection function.

Assumptions and governing equation of motion

We consider a elliptical plate occupy the domain defined by $E=\{(x,y), x^2/a^2+y^2/b^2=1, x,y \text{ belong to } R\}$

The equation of motion for plate under study is obtained using of reference [4] and then adding damping and elastic foundation factor, we get

$\check{D}$ $[D_1(W_{,xxxx} +2W_{,xxyy} +W_{,yyyy}) + 2D_{1,x} (W_{,xxx} +W_{,xyy}) + 2D_{1,y} (W_{,yyy} +W_{,yxx}) + D_{1,xx} (W_{,xx} +\nu W_{,yy}) + D_{1,yy} (W_{,yy} +\nu W_{,xx}) + 2(1-\nu)D_{1,xy}W_{,xy}+K_fW]T+\rho hWT_{,tt}+K_dWT_{,t} = 0$ (1)

Here, a comma followed by a suffix denotes partial differentiation with respect to that variable.

By the method of separation of variables, we get

$[D_1(W_{,xxxx} +2W_{,xxyy} +W_{,yyyy}) + 2D_{1,x} (W_{,xxx} +W_{,xyy}) + 2D_{1,y}(W_{,yyy} +W_{,yxx}) + D_{1,xx} (W_{,xx}+\nu W_{,yy}) + +D_{1,yy} (W_{,yy} +\nu W_{,xx}) + 2(1-\nu)D_{1,xy}W_{,xy}+K_fW]/\rho hW=(-T_{,tt} -T_{,t} K_d/\rho h)/\check{D}T$ (2)

The above condition only hold when LHS and RHS both equal to a constant. Let P^2 be the required constant. We get,

$[D_1(W_{,xxxx} +2W_{,xxyy} +W_{,yyyy}) + 2D_{1,x} (W_{,xxx} +W_{,xyy}) + 2D_{1,y} (W_{,yyy} +W_{,yxx}) + D_{1,xx}(W_{,xx}+\nu W_{,yy}) +D_{1,yy} (W_{,yy} +\nu W_{,xx}) + 2(1-\nu)D_{1,xy}W_{,xy} +K_fW]-\rho hP^2W=0$ (3)

And $T_{,t} (K_d/\rho h) +T_{,tt} + P^2T \check{D}= 0$ (4)

where (3) is differential equations of motion for plate and (4) is differential equation for time function for plate.

Solution with Rayleigh Ritz Method

We shall employ Rayleigh-Ritz technique to solve our model equation (3). Thus, we have

$\delta(V_{max} - T_{max})=0$ (5)

We have considered different values of W which satisfies our geometrical conditions for clamped elliptic plate. The boundary conditions are that

$W = W_{,x}= 0$ along $1-x^2/a^2-y^2/b^2=0$

$W = W_{,y} = 0$ along $1-x^2/a^2-y^2/b^2=0$

The two-term deflection function is assumed that as, $W= A_1(1-x^2/a^2-y^2/b^2)^2 +A_2(1-x^2/a^2-y^2/b^2)^3$

The expressions for strain energy V_{max} and kinetic energy T_{max} are

$$V_{max} = (1/2)\int_{-a}^{a}\int_{-b\sqrt{1-(x/a)^2}}^{b\sqrt{1-(x/a)^2}} D_1\{(W,xx)^2 + (W,yy)^2 + 2\nu W,xxW,yy + 2(1-\nu)(W,xy)^2 + K_fW^2\}dxdy$$

$$T_{max} = (1/2)\rho P^2\int_{-a}^{a}\int_{-b\sqrt{1-(x/a)^2}}^{b\sqrt{1-(x/a)^2}} hW^2dxdy$$

Assuming thickness h varies as, $h=h_0[1-\beta(x^2/a^2+y^2/b^2)]$ (6)

Introducing the non-dimensional variables as $X = x/a$, $Y = y/a$, $\bar{W}=W/a$, $\bar{h}=h/a$

Substituting the thickness from (6) and above conditions, the expressions of V_{max} and T_{max} becomes as,

$$V_{max} = Q\int_{-1}^{1}\int_{-(1/m)\sqrt{1-X^2}}^{(1/m)\sqrt{1-X^2}} \{1-\beta(X^2+Y^2m^2)\}^3\{(\overline{W}_{,XX})^2+(\overline{W}_{,YY})^2+2\nu\overline{W}_{,XX}\overline{W}_{,YY}+2(1-\nu)(\overline{W}_{,XY})^2+K_f\overline{W}^2\}dXdY$$

(7)

Where $Q = (Ea^3\overline{ho}^3)/24(1-\nu^2)$

$$T_{max} = R\int_{-a}^{a}\int_{-(1/m)\sqrt{1-X^2}}^{(1/m)\sqrt{1-X^2}} \{1-\beta(X^2+Y^2m^2)\}\overline{W}^2 dXdY \quad (8)$$

Where $R= (1/2)\rho a^5P^2\overline{ho}$

substituting the values of T_{max} & V_{max} from Eqs (7) and (8) in Eq. (5), we get

$\delta(V_1 - \lambda^2P^2T_1)=0$ (9)

here V_1 and T_1 are given by

$$V_1 = \int_{-1}^{1}\int_{-(1/m)\sqrt{1-X^2}}^{(1/m)\sqrt{1-X^2}} \{1-\beta(X^2+Y^2m^2)\}^3\{(\overline{W}_{,XX})^2+(\overline{W}_{,YY})^2+2\nu\overline{W}_{,XX}\overline{W}_{,YY}+2(1-\nu)(\overline{W}_{,XY})^2+K_f\overline{W}^2\}dXdY \quad (10)$$

$$T_1 = \int_{-a}^{a}\int_{-(1/m)\sqrt{1-X^2}}^{(1/m)\sqrt{1-X^2}} \{1-\beta(X^2+Y^2m^2)\}\overline{W}^2 dXdY \quad (11)$$

Where λ^2 (frequency parameter) is equal to $=12\rho P^2a^2(1-\nu^2)/E\overline{ho}^2$

Now substituting the value of W , we obtain the result in two unknown constants A_1 and A_2 which is determined as

$(\partial /\partial A_n)(V_1 - \lambda^2P^2T_1)=0$, n = 1, 2 (12)

On solving, we get

$b_{11}A_1 + b_{12}A_2 = 0$

And $b_{21}A_1+b_{22}A_2=0$ (13)

b_{11}, b_{12}, b_{21} and b_{22} are coefficient of A_1 and A_2 which are functions of various parameters used in our problem.

To obtain non-trivial solution, from equation (13) we get frequency equation as,

$b_{11}b_{22}-b_{12}b_{21} = 0$ (14)

which gives us a second degree equation in P^2.

Here, b_{11} = (8*pi*m)/3 + (4*pi)/m + 4*pi*m^3 + (28*pi*β^2)/(5*m) - (8*pi*β^3)/(5*m) + (28*pi*β^2*m^3)/5 - (8*pi*β^3*m^3)/5 - (231*pi*β*m)/50 - (7*pi*β)/m + (462*pi*β^2*m)/125 - 7*pi*β*m^3 - (793*pi*β^3*m)/750 + (pi*k_f)/(10*m) - (pi*β*k_f)/(20*m) + (pi*β^2*k_f)/(70*m) - (pi*β^3*k_f)/(560*m) + (λ^2p^2pi*(56*β - 336))/(3360*m)

b_{12}=2*pi*m + (3*pi)/m + 3*pi*m^3 + (12*pi*β^2)/(5*m) - (3*pi*β^3)/(5*m) + (12*pi*β^2*m^3)/5 - (3*pi*β^3*m^3)/5 - (1179*pi*β*m)/500 - (18*pi*β)/(5*m) + (393*pi*β^2*m)/250 - (18*pi*β*m^3)/5 - (197*pi*β^3*m)/500 + (pi*k_f)/(12*m) - (pi*β*k_f)/(28*m) + (pi*β^2*k_f)/(112*m) - (pi*β^3*k_f)/(1008*m) + (λ^2p^2pi*(40*β - 280))/(3360*m)

b_{21}=2*pi*m + (3*pi)/m + 3*pi*m^3 + (12*pi*β^2)/(5*m) - (3*pi*β^3)/(5*m) + (12*pi*β^2*m^3)/5 - (3*pi*β^3*m^3)/5 - (1179*pi*β*m)/500 - (18*pi*β)/(5*m) + (393*pi*β^2*m)/250 - (18*pi*β*m^3)/5 - (197*pi*β^3*m)/500 + (pi*k_f)/(12*m) - (pi*β*k_f)/(28*m) + (pi*β^2*k_f)/(112*m) - (pi*β^3*k_f)/(1008*m) + ($\lambda^2 p^2$pi*(40*β - 280))/(3360*m)

b_{22}=(12*pi*m)/5 + (18*pi)/(5*m) + (18*pi*m^3)/5 + (72*pi*β^2)/(35*m) - (9*pi*β^3)/(20*m) + (72*pi*β^2*m^3)/35 - (9*pi*β^3*m^3)/20 - (1179*pi*β*m)/500 - (18*pi*β)/(5*m) + (1179*pi*β^2*m)/875 - (18*pi*β*m^3)/5 - (591*pi*β^3*m)/2000 + (pi*k_f)/(14*m) - (3*pi*β*k_f)/(112*m) + (pi*β^2*k_f)/(168*m) - (pi*β^3*k_f)/(1680*m) + ($\lambda^2 p^2$pi*(30*β - 240))/(3360*m)

Where, * indicates ordinary multiplication, pi denotes π and ^ denotes power.

Choosing A_1=1, we obtain $A_2=-b_{11}/b_{12}$ by eq (15), with these values of A_1 and A_2 we get, deflection function as

$$W = (1-X^2-Y^2m^2)^2 +(-b_{11}/b_{12})(1-X^2-Y^2m^2)^3 \quad (15)$$

Solution for time function.

Time functions T(t) given by equation (5) and visco- elastic operator Ď is equal to

$$\check{D} \equiv \{1+(\eta/G)(d/dt)\} \quad (16)$$

Time function is obtained by using equation (18) in equation (5), thus we obtain a differential equation in T whose solution is

$$T(t) = e^{a_1 t}[C_1 \mathrm{Cos} b_1 t + C_2 \mathrm{Sin} b_1 t] \quad (17)$$

where $a_1 = -(k_d/\rho h+\eta P^2/G)/2$

$$b_1 = \sqrt{4P^2 - (K_d/\rho h + \eta P^2/G)^2}\,/\,2$$

using starting conditions in equation (17) which are as T =1 & $T_{,t}$ = 0 at t = 0, we get the value of constants

C_1 =1 and $C_2 = -a_1/b_1$

After computing the required values of constants the time function T(t) becomes as

$$T(t) = e^{a_1 t}[\mathrm{Cos} b_1 t + (-a_1/b_1)\mathrm{Sin} b_1 t] \quad (18)$$

Now by using Eqs (15) and (18), the deflection comes out as

$$w =[(1-X^2-Y^2m^2)^2 +(-b_{11}/b_{12})(1-X^2-Y^2m^2)^3][e^{a_1 t}\{\mathrm{Cos} b_1 t + (-a_1/b_1)\mathrm{Sin} b_1 t\}] \quad (19)$$

time period and logarithmic decrement for the vibration of plates are represented respectively by the standard results as, $K = 2\pi/P$ (20)

And Λ= loge(w2/w1) (21)

here w_1 is the deflection at any point at $K = K_1$ and w_2 is the deflection at same point at succeeding K_1.

Numerical work and Discussion:

For the solving the problem, the following values of parameter were used [4]:

E=7.08 $\times 10^{10}$ N/M^2 ,

G=2.682 $\times 10^{10}$ N/M^{2,} ,

η=1.4612 $\times$ 10^6 N.S./M^2

ρ=2.8 $\times$ 10^3 Kg/M^3, ν=0.345,

h_o=0.001, thickness at the centre of elliptical plate.

$Z= x^2/a^2+y^2/b^2$,distance from the centre of elliptical plate to reference point.

m=a/b (aspect Ratio)

Table1: Foundation factor K_f vs. Deflection function w for different m,

K_d =0.5, β=0.6, T=5K, Z=0.2

k_f	m=1		m=1.5	
	First mode	Second mode	First mode	Second mode
0	0.375044	0.0176436	0.355247	0.0135782
0.5	0.374752	0.0176357	0.355110	0.0135758
1	0.374461	0.0176278	0.354973	0.0135734
1.5	0.374171	0.0176199	0.354836	0.0135709
2	0.373882	0.0176119	0.354700	0.0135685
2.5	0.373594	0.0176040	0.354564	0.0135661

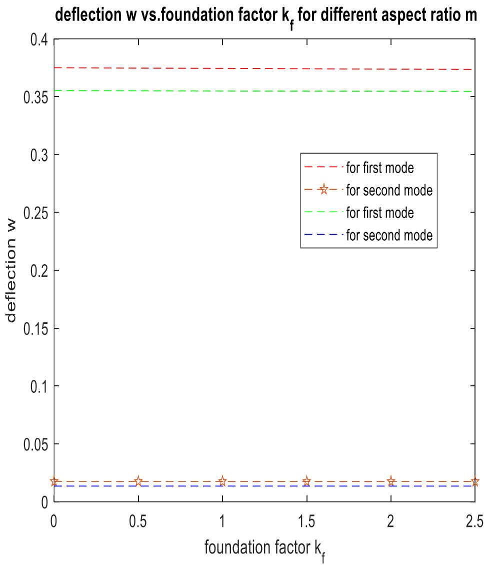

Figure 1

From table 1 and figure 1 we interpret that deflection w decreases slightly with the increasing value of elastic foundation factor k_f for both modes and for different values of m. Here different thing from figure 1 is that the value of deflection is very closed for first mode but for different value of m.

Table 2. Damping factor K_d vs. Deflection function w for different m,

K_f=1, β=0.6, T=5K, Z=0.2

k_d	m=1		m=1.5	
	First mode	Second mode	First mode	Second mode
0	0.37572	0.017640	0.35569	0.013579

0.5	0.37446	0.017627	0.35497	0.013573
1	0.37320	0.017615	0.35425	0.013567
1.5	0.37194	0.017602	0.35354	0.013561
2	0.37069	0.017589	0.35283	0.013555
2.5	0.36944	0.017576	0.35212	0.013550

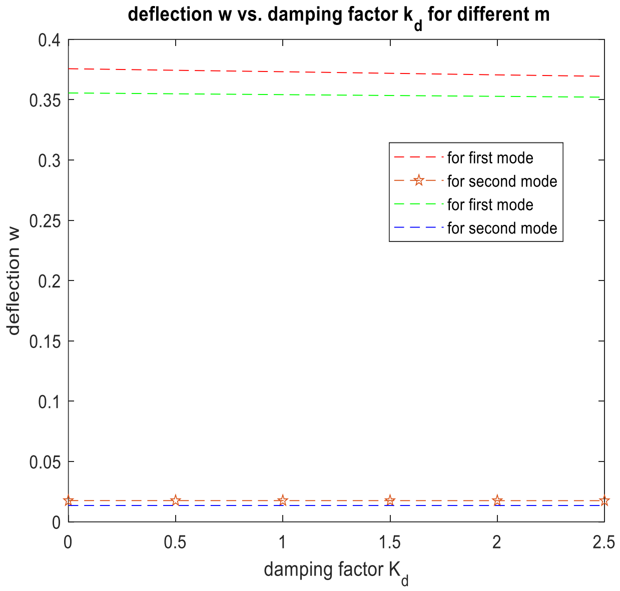

Figure 2.

From figure 2, we see that deflection w decreases slightly with the increasing value of damping factor k_d for both modes for different values of aspect ratio m. we observe also that for increasing value of m, value of deflection w decreases for both mode.

Table 3. Taper constant β vs. Deflection function w for different K_d

K_f=0.5, m=1.5, T=5K, Z=0.2

β	K_d=1		K_d=3	
	First mode	Second mode	First mode	Second mode
0	0.638351	0.011017	0.635667	0.011006
0.2	0.514659	0.012171	0.512047	0.012156
0.4	0.419594	0.012963	0.416950	0.012944
0.6	0.354393	0.013569	0.351539	0.013546
0.8	0.316689	0.014608	0.313430	0.014579

1	0.298804	0.016575	0.295096	0.016539

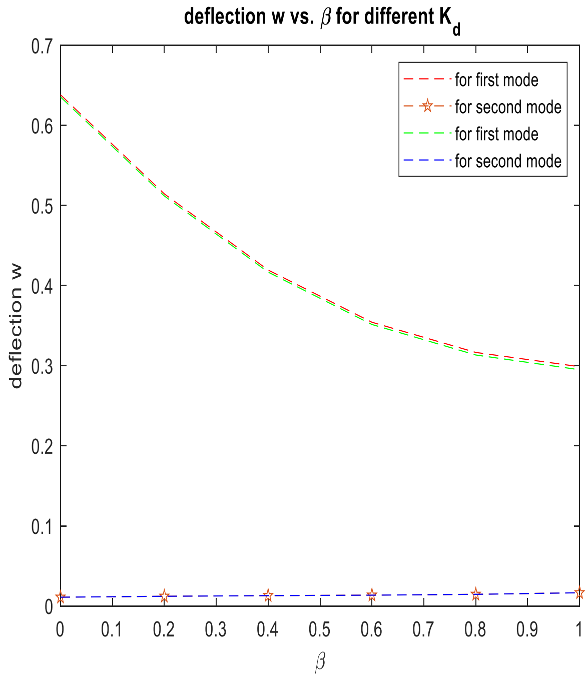

Figure 3.

Here we see that deflection w decreases with the increasing value of β for both modes for different values of K_d. we observe also that one different thing is that there is no change in deflection up to two decimal place deflection w for both mode.

Table 4. Aspect ratio m vs. Deflection function w for different K_d

K_f=0.5, β=0.6, T=5K, Z=0.2

m	K_d=0.5		K_d =3	
	First mode	Second mode	First mode	Second mode
0.5	0.3839	0.01993	0.3745	0.01975
1	0.3747	0.01763	0.3741	0.01754
1.5	0.3551	0.01357	0.3737	0.01353
2	0.3268	0.00914	0.3733	0.00912
2.5	0.2928	0.00542	0.3729	0.00541

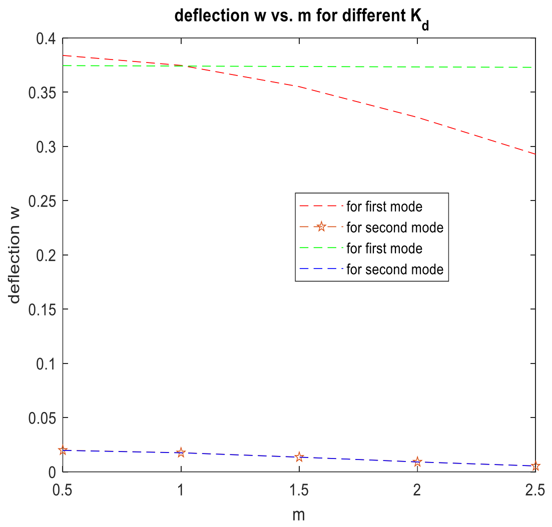

Figure 4.

Figure 4 shows that deflection w decreases with the increasing value of m for both modes for different values of K_d. here we also observe that the graph of deflection w cuts each other at one point for first mode.. there is no change in deflection up to four decimal place for second mode. For K_d=3, there is no change in deflection up to to decimal place.

Table 5. Z vs. Deflection function w for different K_d,

K_f=0.5, β=0.6, m=1.5

Z	K_d=0.5		K_d=3	
	First mode	Second mode	First mode	Second mode
0	0.47625	-0.10449	0.47204	-0.10429
0.2	0.35511	0.01357	0.35153	0.01354
0.4	0.22801	0.05288	0.22536	0.05275
0.6	0.11388	0.04361	0.11230	0.04348
0.8	0.03159	0.01592	0.03106	0.01586

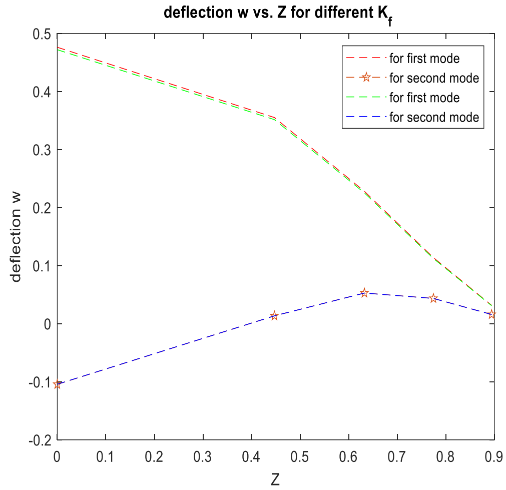

Figure 5.

From figure 5 we see that deflection w decreases for first mode but for second mode it increases up to the value of Z=0.6 and then decreases.

Table 6. Time period vs. foundation factor K_f for different β ,

m=1.5

K_f	β=0.2		β=0.4	
	First mode	Second mode	First mode	Second mode
0	0.027385	0.006489	0.032613	0.007488
0.5	0.027356	0.006488	0.032567	0.007487
1	0.027327	0.006488	0.032522	0.007487
1.5	0.027298	0.006488	0.032476	0.007486
2	0.027269	0.006487	0.032431	0.007486
2.5	0.027240	0.006487	0.032386	0.007485

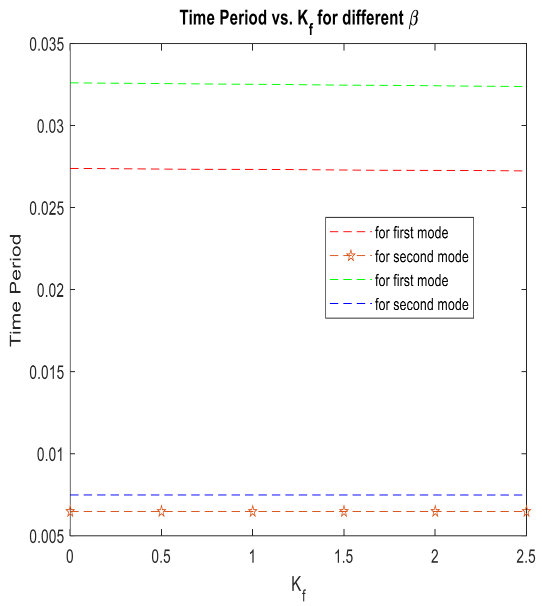

figure 6.

Figure 6 shows that time period slightly decreases with increasing value of K_f for both modes of vibration for different value of **β**

Table 7. Time period vs. m for different K_f

β=0.6,

m	K_f=0.5		K_f=2	
	First mode	Second mode	First mode	Second mode
0	0.023592	0.005602	0.023533	0.005601
0.5	0.027356	0.006488	0.027269	0.006487
1	0.032567	0.007487	0.032431	0.007486
1.5	0.039834	0.008486	0.039606	0.008484
2	0.048650	0.009293	0.048268	0.009290

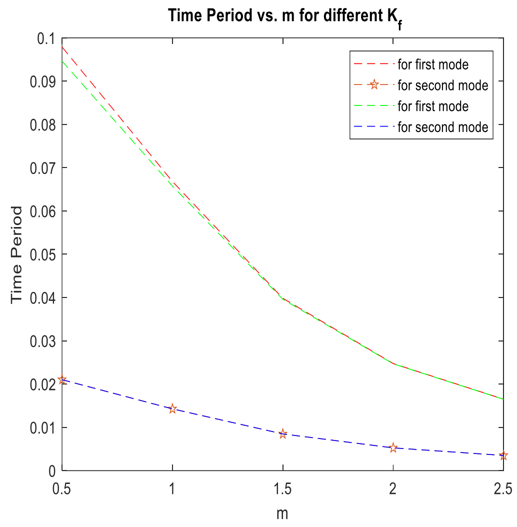

Figure 7.

Figure 7. shows that time period increases with increasing value of m for both modes vibration for different value of K_f and for first mode time period varies parabolically and for second mode it varies linearly.

Table 8. Deflection w vs. m for different Z,

$k_{f=}0$, $K_{d=}0$,**β=0.6,T=5K,Z=0.2,**

m	Z=0.2		Z=0.4	
	First mode	Second mode	First mode	Second mode
0	0.734871	0.019771	0.391985	0.069014
0.5	0.703066	0.016406	0.375021	0.057270
1	0.639822	0.011023	0.341286	0.038478
1.5	0.555023	0.006041	0.296053	0.021089
2	0.460339	0.002725	0.245548	0.009514

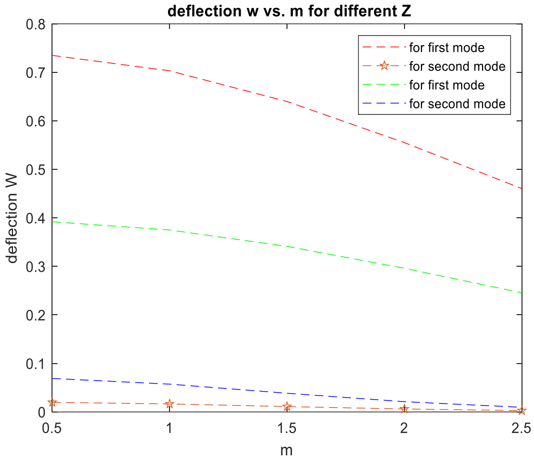

Figure 8.

In figure 8, our result for deflection matches with reference [4] for the values of elastic foundation factor and damping factor equal to zero.

Table 9. Logarithmic decrement vs. K_f for different K_d,

B=0.6, m=1.5

K_f	K_d=0.2		K_d=0.4	
	First mode	Second mode	First mode	Second mode
0	-0.027888	-0.129251	-0.028319	-0.129343
0.5	-0.027940	-0.129261	-0.028371	-0.129353
1	-0.027992	-0.129271	-0.028422	-0.129363
1.5	-0.028045	-0.129281	-0.028474	-0.129372
2	-0.028097	-0.129290	-0.028525	-0.129382
2.5	-0.028149	-0.129300	-0.028576	-0.129392

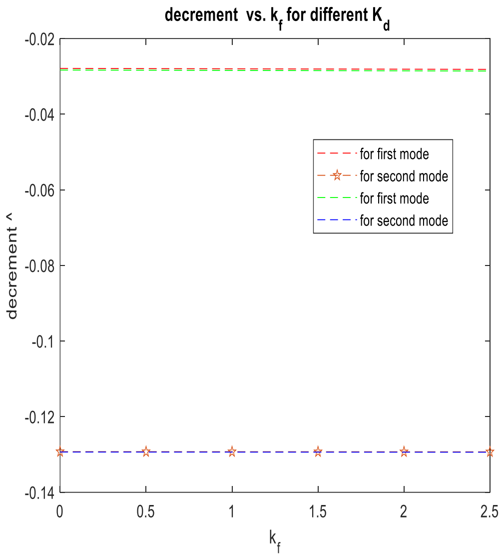

Figure 9.

From table 9 we see that there is no change in logarithmic decrement up to two decimal place for both mode of vibration for different value of Kd and logarithmic decrement varies linearly for both mode of vibration.

Conclusion:

On comparing with reference [4], it is observed that:

When we increases the value of elastic foundation factor and damping factor from 0 to 2.5 close agreement is found for the free and damped vibration of elliptical plate with resting on elastic foundation.

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List of Symbols:

a	length of the semi major axis of elliptical plate	ρ	mass density per unit volume of the plate material,
b	b length of the semi minor axis of elliptical plate,	t	time,
m	(a/b) aspect Ratio	η	Visco-elastic constant ,
h	thickness of the plate at the point (x,y),	w(x,y,t)	deflection of the plate i.e. amplitude,
x, y	co-ordinates in the plane of the plate.	T(t)	time function,
E	young's modulus,	W(x,y)	deflection function
G	shear modulus,	β	taper constant
ν	Poisson's ratio,	Λ	Logarithmic decrement,
Ď	visco-elastic operator	K	time period,
D_1	$Eh_0^3/12(1-\nu^2)$, Flexural rigidity,	$\overline{ho}$	$=h_0$ at x = y = 0
Q	$=(Ea^3\overline{ho}^3)/24(1-\nu^2)$, a constant	R	$= (1/2)\rho P^2a^5\overline{ho}$, a constant
λ^2	$=12\rho P^2a^2(1-\nu^2)/E\overline{ho}^2$, a frequency parameter	K_d	Damping factor
K_f	Elastic foundation factor		