# ANALYTICAL SOLUTION FOR DAMPED VIBRATIONS ANALYSIS OF CLAMPED SIMPLY SUPPORTED RIGHT TRIANGULAR PLATE

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**Abstract**

*A systematic solution for damped vibrations investigation of clamped SS right triangular plate has been studied here using separation of variable method; governing equation is solved for different boundary condition for right triangular plate. Rayleigh Ritz technique with two term deflection function is applied to obtain the desired proximate frequency equation using Gram Schmidt orthogonalization. Frequency parameter, time period, and value of deflection function at different points for first two modes of vibrations are obtained for the variation in parameters viz. taper constant, aspect ratio and damping parameter.*

***Key words**: Vibration, Rayleigh-Ritz technique, orthogonal functions Edge conditions.*

## INTRODUCTION

Different shapes and configuration of plates have been studied by Leissa [1] in his report considering flexural vibration. Large amount of research work on various plate shapes like rectangular, circular, elliptical etc. had been studied by various researcher but minimal work on vibration of plates having triangular orientation have been investigated .Triangular plates with different configuration frequently dealt across in lot of industries and structural analysis thus further research is required in this domain. The idea of free vibration in any structure or plates or system is hypothetical since damping is always present in any form. Also literature survey exhibit that few problems had been studied considering the damping parameter. By considering damping effect in the study of vibrations of plates, the research would be of great help in the field of Seismology, Earthquake study, engineering design including dams and bridges. Main hindrance in analysis of triangular plate is the labour involved in formulating the displacement function in implicit form to describe triangular domain.

As the computer technology advancement has taken place various methods had been used by researcher to studied triangular domain. F.E.M is particular example of the such method handed down by [2-4].

Gorman [5, 6] projected an examining approach which established positioning of construction structure for completely free vibration of three cornered. He also studied the right triangular plate for vibration analysis with different boundary conditions. Grid work method was applied by Christensen [7] to analyze the vibration of $45^0$ right triangular cantilevered plates. Bhatt [8] answered the question on the triangular plates using Rayleigh –Ritz approach and calculated the various parameters for vibrations of polynomial plates. Liew et al.[9] proposed orthogonal plate function to study the free vibration of rectangular plates concerning the manipulation of number shapes only natural frequencies were addressed in the research .Lam et al [10] in their research studied and analyzed natural frequencies of isotropic and orthotropic triangular plates. The present problem expands the initial work by different researchers to obtain a analytical solution of damped vibration of isotropic right triangular plate with different combination of edge conditions. For starting 2 modes the deflection function, frequencies and time period have been obtained using 2D orthogonal plate functions taking into consideration Rayleigh Ritz method considering parameters viz. taper constant, aspect ratio, damping parameters .

| System of symbols | |
|---|---|
| a ,b the side of plate | t= time |
| $\tilde{c}_i$ coefficient of $\tilde{T}(t)$ | $\tilde{w}(x, y, t)$= amplitude |
| x , y Cartesian coordinate | $\tilde{W}(x,y)$ =deflection function |
| h= plate thickness at the point (x, y) | $\tilde{T}(t)$= time function |
| f(X′, Y′ )= generating function | $\beta^*$= taper constant in x- direction |
| E =Young's modulus | $\tilde{K}$ =time period |
| υ= Poisson ratio | P =Angular frequency |
| $\tilde{d}$ = flexural rigidity $Eh^3/12(1-v^2)$ | $\tilde{T}_{max}$= maximum kinetic energy |
| ρ =density of the plate | $\tilde{V}_{max}$ =maximum strain energy |

**Mathematical model for the problem**

We obtain mathematical model for equation of isotropic right triangular plate with variable thickness by introducing K as the damping parameter and assume that the damping forces are proportional to velocity, then model equation given by Leissa [1] is transformed and equation of motion is given as

$$\tilde{d}\left(\frac{\partial^4\tilde{w}}{\partial x^4}+2\frac{\partial^4\tilde{w}}{\partial x^2\partial y^2}+\frac{\partial^4\tilde{w}}{\partial y^4}\right)+2\frac{\partial\tilde{d}}{\partial x}\left(\frac{\partial^3\tilde{w}}{\partial x^3}+\frac{\partial^3\tilde{w}}{\partial x\partial y^2}\right)+2\frac{\partial\tilde{d}}{\partial y}\left(\frac{\partial^3\tilde{w}}{\partial y^3}+\frac{\partial^3\tilde{w}}{\partial y\partial x^2}\right)+\frac{\partial^2\tilde{d}}{\partial x^2}\left(\frac{\partial^2\tilde{w}}{\partial x^2}+\upsilon\frac{\partial^2\tilde{w}}{\partial y^2}\right)+\frac{\partial^2\tilde{d}}{\partial y^2}\left(\frac{\partial^2\tilde{w}}{\partial y^2}+\upsilon\frac{\partial^2\tilde{w}}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2\tilde{d}}{\partial x\partial y}\frac{\partial^2\tilde{w}}{\partial x\partial y}+K\frac{\partial\tilde{w}}{\partial t}+\rho h\frac{\partial^2\tilde{w}}{\partial t^2}=0 \quad (1)$$

The following function is assumed to be the solution of equation (1) using method of separation of variables

$$\tilde{w}(x, y, t)=\tilde{W}(x, y)\ \tilde{T}(t) \quad (2)$$

Putting the value from (2) in (1) we get the following transformed equations i.e. (3) and (4)

$$\frac{\partial^2\tilde{T}}{\partial t^2}+\frac{K}{\rho h}\frac{\partial\tilde{T}}{\partial t}+P^2\tilde{T}=0 \quad (3)$$

and

$$\tilde{d}\left(\frac{\partial^4\tilde{w}}{\partial x^4}+2\frac{\partial^4\tilde{w}}{\partial x^2\partial y^2}+\frac{\partial^4\tilde{w}}{\partial y^4}\right)+2\frac{\partial\tilde{d}}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\partial y^2}\right)+2\frac{\partial\tilde{d}}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial y\partial x^2}\right)+\frac{\partial^2\tilde{d}}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\upsilon\frac{\partial^2 W}{\partial y^2}\right)+\frac{\partial^2\tilde{d}}{\partial y^2}\left(\frac{\partial^2\tilde{w}}{\partial y^2}+\upsilon\frac{\partial^2\tilde{w}}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2\tilde{d}}{\partial x\partial y}\frac{\partial^2\tilde{w}}{\partial x\partial y}-\rho hP^2\tilde{w}=0 \quad (4)$$

Thus Equation (3) and (4) are the required model equation for the above said plate of free and damped vibration having variable thickness respectively. We shall now separately solve above two equation.

**Time function of variable of plates**

The time function is given by equation (3) and its solution is as

$$\widetilde{T}(t) = e^{\alpha t}\{\widetilde{c_1}\cos\beta t + \widetilde{c_2}\sin\beta t\} \quad (5)$$

Where $\alpha = -\frac{K}{2\rho h}$ and $\beta = P\sqrt{1 - \frac{\alpha^2}{P^2}} = P_0$, here P is natural frequency, $P_0$ is angular frequency and $\widetilde{c_1}$ , $\widetilde{c_2}$

are arbitrary but fixed which are calculated from the primary restriction of the plate.

For the present study we have as $\widetilde{T}=1$ and $\dot{\widetilde{T}}=0$ at $t=0$ (6)

Using equation (6) in equation (5) ,one obtains $\widetilde{c_1}=1$ and $\widetilde{c_2}= -\frac{\alpha}{\beta}$ (7)

And

$$\widetilde{T}(t) = e^{\alpha t}\{\cos\beta t - \frac{\alpha}{\beta} \sin\beta t\} \quad (8)$$

Plate amplitude ($\widetilde{w}$) is this given as below for damped transverse vibration of triangular plate

$$\widetilde{w}(x,y,t) = \widetilde{W}(x,y)[\, e^{\alpha t}\left\{\cos\beta t - \frac{\alpha}{\beta}\sin\beta t\right\}] \quad (9)$$

Substituting the above value from (9) in (4) and equating the coefficient sine and cosine term we establish the equation for deflection function as

$$\tilde{d}\left(\frac{\partial^4\widetilde{W}}{\partial x^4} + 2\frac{\partial^4\widetilde{W}}{\partial x^2\,\partial y^2} + \frac{\partial^4\widetilde{W}}{\partial y^4}\right) + 2\frac{\partial\,\tilde{d}}{\partial x}\left(\frac{\partial^3\widetilde{W}}{\partial x^3} + \frac{\partial^3\widetilde{W}}{\partial x\,\partial y^2}\right) + 2\frac{\partial\,\tilde{d}}{\partial y}\left(\frac{\partial^3\widetilde{W}}{\partial y^3} + \frac{\partial^3\widetilde{W}}{\partial y\,\partial x^2}\right) + \frac{\partial^2\,\tilde{d}}{\partial x^2}\left(\frac{\partial^2\widetilde{W}}{\partial x^2} + \upsilon\frac{\partial^2\widetilde{W}}{\partial y^2}\right) + \frac{\partial^2\,\tilde{d}}{\partial y^2}\left(\frac{\partial^2\widetilde{W}}{\partial y^2} + \upsilon\frac{\partial^2\widetilde{W}}{\partial x^2}\right) + 2(1-\upsilon)\frac{\partial^2\,\tilde{d}}{\partial x\,\partial y}\frac{\partial^2\widetilde{W}}{\partial x\,\partial y} - \frac{k^2\widetilde{W}}{4\rho h} - \rho h P^2\widetilde{W} = 0 \quad (10)$$

Considering P constant and thickness variation along x axis

$$\tilde{d} = \frac{E{h_0}^3}{12(1-\upsilon^2)}\left(1 + \beta^*\frac{x}{a}\right)^3.\ ,\ \tilde{d} = \tilde{d}_0\left(1 + \beta^*\frac{x}{a}\right)^3 \text{ where } \tilde{d}_0 = \frac{E{h_0}^3}{12(1-\upsilon^2)} \quad (11)$$

Substituting (11) in (10) and we put forward unit less variables $X'=x/a$ and $Y'=y/b$ and simplifying one gets

$$a^4(1 + \beta^*X')^4\left(\frac{\partial^4\widetilde{W}}{\partial {X'}^4} + 2\frac{\partial^4\widetilde{W}}{\partial {X'}^2\,\partial {Y'}^2} + \frac{\partial^4\widetilde{W}}{\partial {Y'}^4}\right) + 2a^4(1 + \beta^*X')\frac{\partial}{\partial X'}(1 + \beta^*X')^3\left(\frac{\partial^3\widetilde{W}}{\partial {X'}^3} + \frac{\partial^3\widetilde{W}}{\partial X'\,\partial {Y'}^2}\right) + 2a^4(1 + \beta^*X')\frac{\partial(1+\beta^*X')^3}{\partial Y'}\left(\frac{\partial^3\widetilde{W}}{\partial {Y'}^3} + \frac{\partial^3\widetilde{W}}{\partial Y'\,\partial {X'}^2}\right) + a^4(1 + \beta^*X')\frac{\partial^2(1+\beta^*X')^3}{\partial {X'}^2}\left(\frac{\partial^2\widetilde{W}}{\partial {X'}^2} + \upsilon\frac{\partial^2\widetilde{W}}{\partial {Y'}^2}\right) + a^4(1 + \beta^*X')\frac{\partial^2(1+\beta^*X')^3}{\partial {Y'}^2}\left(\frac{\partial^2\widetilde{W}}{\partial {Y'}^2} + \upsilon\frac{\partial^2\widetilde{W}}{\partial {X'}^2}\right) + 2(1-\upsilon)a^4(1 + \beta^*X')\frac{\partial^2(1+\beta^*X')^3}{\partial X'\,\partial Y'}\frac{\partial^2\widetilde{W}}{\partial X'\,\partial Y'} - \frac{k^2a^4\widetilde{W}}{4\rho h_0\,\tilde{d}0} - \frac{\rho h_0 a^4P^2}{\tilde{d}0}(1 + \beta^*X')^2\widetilde{W} = 0 \quad (12)$$

Replacing $\frac{4\rho h_0\,\tilde{d}0}{a^4}$ by $\frac{{K_0}^2}{\mu^2}$ and $\frac{\rho h_0 a^4P^2}{\tilde{d}0}$ by $\Lambda^2$

We shall now find the deflection function $\widetilde{W}$ using orthogonal plate function.

**Orthogonal plate function**

To obtain deflection function for triangular plate problem in our study we assume it as

$\widetilde{W}(X', Y') = A_1\Phi_1 + A_2\Phi_2$ where $\Phi_1$ and $\Phi_2$ are orthogonal (13).

plate function $\Phi_1(X', Y')$ is so chosen for triangular plate in our study such that it at least satisfy the edges preconditions of the triangular plate and a better concentrate approximation is achieved if

$\Phi_1(X', Y')$ comply with the edge preconditions.

For present problem the required function in equation (13) is $\Phi_i(X',Y') = \prod_{k=1}^{3} \Theta_k(X',Y')$ (14)

Here $\Theta_k$ are the edge functions. The function $\Theta_k(X',Y')$ for distinct support conditions are summarized below:

(i) For simply supported edge $\Theta(X', Y') = \begin{cases} X' - c & \text{, at edge } X' = c \\ Y' - d & \text{, at edge } Y' = d \\ Y' - tX' - e \text{ ,} & \text{at edge } Y' = tX' + e \end{cases}$ (15)

(ii) For clamped edge $\Theta(X', Y') = \begin{cases} (X' - c)^2 & \text{, at edge } X' = c \\ (Y' - d)^2 & \text{, at edge } Y' = d \\ (Y' - tX' - e)^2 & \text{, at edge } Y' = tX' + e \end{cases}$ (16)

For $\Phi_2(X', Y') = f_2(X', Y')\Phi_1(X', Y') - a_{2,1}\Phi_1(X', Y')$ (17)

$\iint \Phi_i(X',Y')\Phi_j(X',Y')dX'dY' = \begin{cases} 0 & \text{, if } i \neq j \\ 1 & \text{, if } i = j \end{cases}$ (By gram Schmidt orthogonalization definition) (18)

$$a_{m,i} = \frac{\iint f_m(X',Y')\Phi_1(X',Y')\Phi_i(X',Y')dX'dY'}{\iint \Phi_i(X',Y')\Phi_i(X',Y')dX'dY'} \qquad (19)$$

Where $f_m(X', Y')$ is the generating function and $f_m(X', Y')$ is calculated by deciding the parameters

$$r = \lceil \sqrt{m-1} \rceil \qquad (20)$$

$$t = (m-1) - r^2 \qquad (21)$$

If t is even, then $s = t/2$; $0 \leq s \leq r$ (21)

$$f_m(X', Y') = X'^r Y'^s \qquad (22)$$

If t is odd, then $s = (t-1)/2$; $0 \leq s \leq r - 1$ (23)

$$f_m(X', Y') = X'^r Y'^s \qquad (24)$$

Where $\lceil \, \rceil$ is equation (20) denotes the greatest integer function.

| Boundary condition | Generating functions | Plate functions |
|---|---|---|
| CSS | $f_1(X', Y') = 1$ | $\Phi_1 = Y'^2(X'-1)(Y'-X')$ |
| | $f_2(X', Y') = X'$ | $\Phi_2 = X' \Phi_1(X', Y') - a_{2,1} \Phi_1(X', Y')$ |

**Method of analysis**

The plate geometry for thin triangular plate is given in fig. (a) and approximate solution is derived using Rayleigh's principle, which states that

$$\widetilde{V}_{max} = \widetilde{T}_{max} \quad (25)$$

Here $\widetilde{V}_{max}$ is max. Strain energy and $\widetilde{T}_{max}$ is max. Kinetic energy.

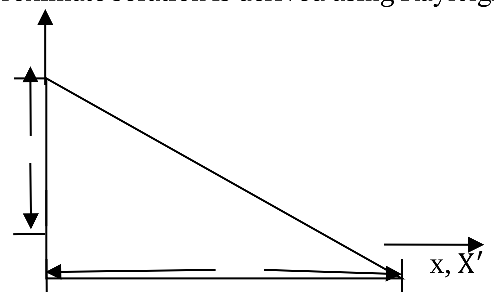



fig.(a) Geometry of right triangular plate

**Equation of motion**

The expression for K.E. (T) and S.E. (V) are

$$\widetilde{T}_{max} = \frac{1}{2} ab \Lambda^2 \iint [((1+\beta^* X')^2 + (\frac{K}{K_0})^2] \widetilde{W}^2((X',, Y') dX' dY' \quad (26)$$

$$\widetilde{V}_{max} = \frac{1}{2} ab \iint \{(1+\beta^* X')^4 \left(\frac{\partial^2 \widetilde{W}}{\partial X'^2}\right)^2 + 2\upsilon \alpha'^2 (1+\beta^* X')^4 \left(\frac{\partial^2 \widetilde{W}}{\partial X'^2} \frac{\partial^2 \widetilde{W}}{\partial Y'^2}\right) + \alpha'^4 (1+\beta^* X')^4 \left(\frac{\partial^2 \widetilde{W}}{\partial Y'^2}\right)^2 + 2(1-\upsilon)(1+\beta^* X')^4 \alpha'^2 \left(\frac{\partial^2 \widetilde{W}}{\partial X' \partial Y'}\right)^2 \} dX' dY' \quad (27)$$

**Solution and frequency equation**

$$\frac{\partial}{\partial A_i} (\widetilde{V}max - \widetilde{T}max) \quad (28)$$

Which leads to the governing Eigen value equation $\sum [K_{ij} - \Lambda^2 M_{ij}] c_i = 0$ (29)

$$K_{ij} = P_{ij} + \alpha'^4 Q_{ij} + \alpha'^2 \upsilon (R_{ij} + S_{ij}) + 2(1-\upsilon)\alpha'^2 T_{ij}$$

$M_{ij} = \iint [(1+\beta^* X')^2 + (\frac{K}{K_0})^2] \Phi_i(X', Y') \Phi_j(X', Y') dX' dY'$, $P_{ij} = \iint (1+\beta^* X')^4 \frac{\partial^2 \Phi_i(X',Y')}{\partial X' \partial X'} \frac{\partial^2 \Phi_j(X',Y')}{\partial X' \partial X'} dX' dY'$,

$Q_{ij} = \iint (1+\beta^* X')^4 \frac{\partial^2 \Phi_i(X',Y')}{\partial Y' \partial Y'} \frac{\partial^2 \Phi_j(X',Y')}{\partial Y' \partial Y'} dX' dY'$, $R_{ij} = \iint (1+\beta^* X')^4 \frac{\partial^2 \Phi_i(X',Y')}{\partial Y' \partial Y'} \frac{\partial^2 \Phi_j(X',Y')}{\partial X' \partial X'} dX' dY'$,

$S_{ij} = \iint (1+\beta^* X')^4 \frac{\partial^2 \Phi_i(X',Y')}{\partial X' \partial X'} \frac{\partial^2 \Phi_j(X',Y')}{\partial Y' \partial Y'} dX' dY'$, $T_{ij} = \iint (1+\beta^* X')^4 \frac{\partial^2 \Phi_i(X',Y')}{\partial X' \partial Y'} \frac{\partial^2 \Phi_j(X',Y')}{\partial X' \partial Y'} dX' dY'$

$F_{ij} = K_{ij} - \Lambda^2 M_{ij}$, i,j=1,2 On simplifying (29) one gets $F_{i1} A_1 + F_{i2} A_2 = 0$, i=1,2 (30)

Here $F_{i1}$, $F_{i2}$ (i=1,2) contains various parameter used and frequency parameter.

For non zero solution, we must have from equation (30)

$$\begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix} = 0 \quad (31)$$

By equation (31) we obtain a quadratic equation in $\Lambda^2$ and two values of $\Lambda$ are calculated .After deciding the values of $A_1$ and $A_2$ we obtain $\widetilde{W}$ from equation (30).

Taking $A_1$=1,we get $A_2=-\frac{b_{11}}{b_{12}}$ and so $\widetilde{W}$ is $\widetilde{W}= \Phi_1 +(-\frac{b_{11}}{b_{12}})\Phi_2$ (32)

Thus deflection $\widetilde{w}$ may be expressed, by using equation (32) and (8) in equation (2), to give

$$\widetilde{w}(x,y,t)= (\Phi_1 + (-\frac{b_{11}}{b_{12}})\Phi_2)[\, e^{\alpha t}\left\{\cos\beta t - \frac{\alpha}{\beta}\sin\beta t\right\}] \quad (33)$$

Time period for triangular plate vibration is $\widetilde{K}=\frac{2\Pi}{P}$ (34)

**Numerical evaluations**

Starting 2 modes of vibration for CSS right triangular plate is obtained by calculating frequencies ,time period and deflection function for variation in taper constant ,damping parameter and aspect ratio at different points. For 'Duralium' the following values of material parameter are used which have been documented in reference [11]

E (Young's modulus )=$7.08\times10^{10}$n/m$^2$ , ρ (density of the plate)=$2.80\times10^3$kg/m$^3$, υ(Poisson ratio)=0.3

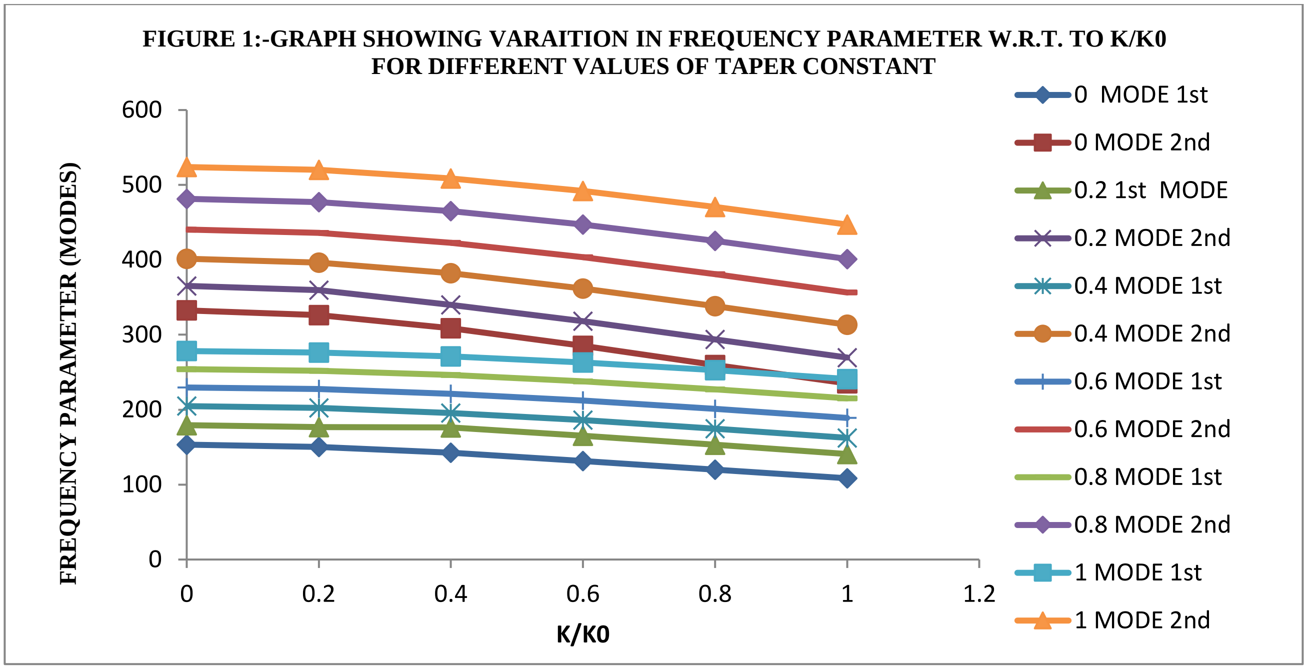
The diameter of the plate's middle section has been measured at h=0.01m.

**Numerical results and Discussion**

Table 1 constitutes 1st and 2nd frequency modes which are evaluated for the clamped simply supported right triangular boundary condition for different values of damping parameter $K/K_0$ and taper parameter β* for Poisson ratio υ=0.3, thickness of plate h=0.01 and aspect ratio a/b=3/2. Projecting table1 through graph fig.1 shows the behavior of frequency parameter ʎ with the increasing value of taper parameter (β*) for any fixed but arbitrary value of damping parameter viz. $K/K_0$ (0.0,0.2,0.4,0.6,0.8,1).

**TABLE 1:- FREQUENCY PARAMETER ʎ (a/b=3/2)**

| β* | 0.0 | | 0.2 | | 0.4 | | 0.6 | | 0.8 | | 1.0 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| K/K0 | MODE1st | MODE2nd | MODE1st | MODE2nd | MODE1st | MODE2nd | MODE1st | MODE2nd | MODE1st | MODE 2nd | MODE1st | MODE 2nd |
| 0.0 | 153.355 | 332.513 | 179.329 | 365.227 | 204.743 | 401.517 | 229.630 | 440.481 | 254.092 | 481.442 | 278.231 | 523.903 |
| 0.2 | 150.376 | 326.056 | 176.694 | 359.566 | 202.373 | 396.455 | 227.475 | 435.891 | 252.113 | 477.235 | 276.400 | 520.015 |
| 0.4 | 142.386 | 308.731 | 176.246 | 339.730 | 195.729 | 382.355 | 221.357 | 422.949 | 246.440 | 465.261 | 271.116 | 508.866 |
| 0.6 | 131.499 | 285.128 | 165.435 | 318.193 | 185.975 | 361.885 | 212.167 | 403.752 | 237.779 | 447.199 | 262.937 | 491.821 |
| 0.8 | 119.749 | 259.649 | 153.173 | 293.943 | 174.479 | 338.085 | 201.027 | 380.837 | 227.046 | 425.170 | 252.628 | 470.662 |
| 1.0 | 108.438 | 235.123 | 140.797 | 269.633 | 162.416 | 313.448 | 188.981 | 356.446 | 215.160 | 401.173 | 240.985 | 447.162 |




Frequency parameter ʎ shows increments in its value with increase in value of taper parameter for the both modes of vibration w.r.t distinct value of damping parameter. Also table 1 and figure 1, provide us the inference of damping parameter $K/K_0$ on frequency parameter for two modes of vibration, if we take any fixed but arbitrary value of taper parameter β* (0.0,0.2,0.4,0.6,0.8,1.0) that there is decrement in value of ʎ with the increase of damping parameter $K/K_0$ and this decrement is linear in nature.

Table 2 and Figure 2 explain the importance of frequency parameter for the first two modes of

Vibration at different a/b ratios in the following three cases:

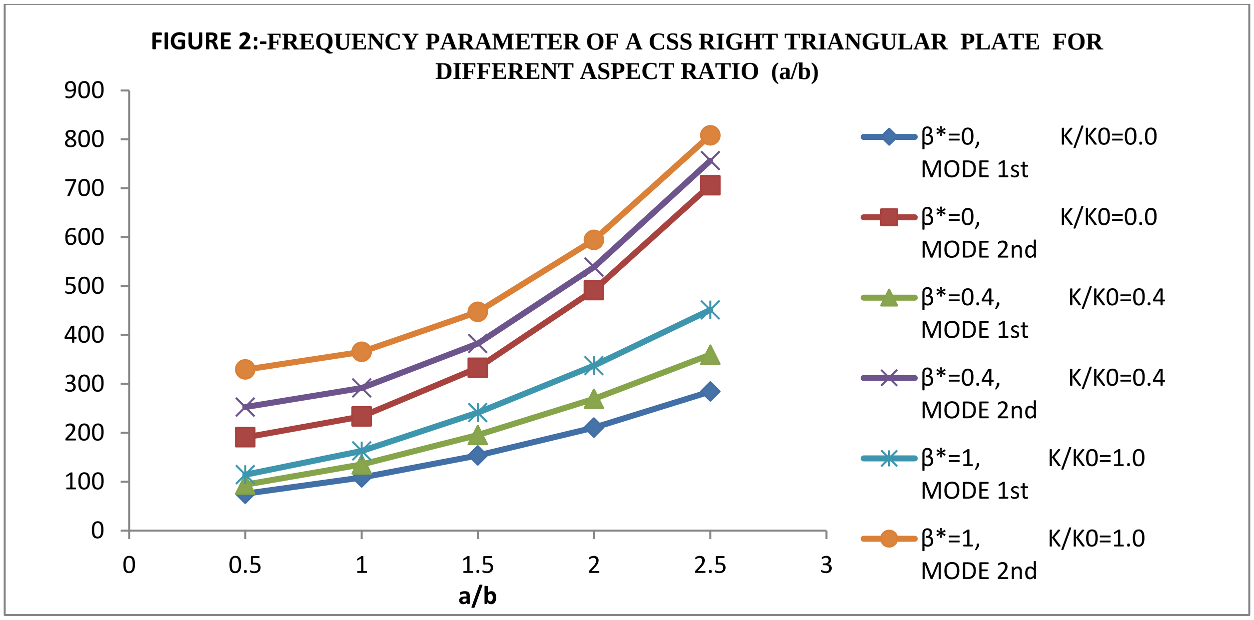
(i) $\beta^*=0, \frac{K}{K_0}=0$,(ii) $\beta^*=0.4, \frac{K}{K_0}=0.4$ ,(iii) $\beta^*=1, \frac{K}{K_0}=1$

we observe that as value of a/b increases for different β*(0.0,0.2,0.4,0.6,1.0) these is parabolic increment in frequency parameter ʎ.

**TABLE 2. FREQUENCY PARAMETER OFA CSS RIGHT TRIANGULAR PLATE FOR DIFFERENT ASPECT RATIO (a/b)**

| a/b | β*=0, $K/K_0$=0.0 | | β*=0.4, $K/K_0$=0.4 | | β*=1, $K/K_0$=1.0 | |
|---|---|---|---|---|---|---|
| | MODE 1st | MODE 2nd | MODE 1st | MODE 2nd | MODE 1st | MODE2nd |
| ½ | 75.4334 | 190.5859 | 93.9244 | 252.6519 | 114.2934 | 329.2248 |
| 1 | 108.2912 | 233.6728 | 135.1036 | 291.3157 | 163.0184 | 365.1958 |
| 3/2 | 153.3558 | 332.5131 | 195.7294 | 382.3558 | 240.9854 | 447.1621 |
| 2 | 210.4233 | 491.7641 | 269.1040 | 538.6306 | 337.2802 | 594.0084 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 5/2 | 284.2428 | 706.2499 | 359.7026 | 756.3021 | 451.2405 | 808.2765 |




From table 3 and figure 3, for aspect ratio a/b=3/2 the time period $\widetilde{K}$ have been computed for CSS right triangular plate for two modes of vibrations with different edge restrictions for variation in different value of taper constant β* and damping parameter $K/K_0$ . It can be seen from table 3 that as taper constant increases for any fixed but arbitrary value of damping parameter $K/K_0$(0.0,0.2,0.4,0.6,1.0),the time period decreases.Table3 and figure 3 also provide the result that if we take any fixed value of taper parameter β* and value of damping parameter $K/K_0$ increases frequency parameter increases and the increment is parabolic for 1$^{st}$ mode and linear for 2$^{nd}$ mode .

**TABLE3:- TIME PERIOD ($\widetilde{K}\times10^{-5}$) OF A CSS RIGHT TRIANGULAR PLATE FOR DIFFERENT VALUES OF TAPER CONSTANT (β*), DAMPING PARAMETER ($K/K_0$) AND A CONSTANT ASPECT RATIO (a/b=3/2)**

| β* | 0.0 | | 0.2 | | 0.4 | | 0.6 | | 0.8 | | 1.0 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $K/K_0$ | MODE 1$^{st}$ | MODE 2$^{nd}$ | MODE 1$^{st}$ | MODE 2$^{nd}$ | MODE 1$^{st}$ | MODE 2$^{nd}$ | MODE 1$^{st}$ | MODE 2$^{nd}$ | MODE 1$^{st}$ | MODE 2$^{nd}$ | MODE 1$^{st}$ | MODE 2$^{nd}$ |
| 0.0 | 269.248 | 124.178 | 230.251 | 113.055 | 201.670 | 102.837 | 179.814 | 93.740 | 162.503 | 85.764 | 148.404 | 78.813 |
| 0.2 | 275.305 | 126.636 | 233.749 | 114.835 | 204.032 | 104.149 | 181.517 | 94.727 | 163.779 | 86.520 | 149.387 | 79.403 |
| 0.4 | 289.990 | 133.743 | 234.278 | 121.539 | 210.958 | 107.990 | 186.534 | 97.626 | 167.548 | 88.747 | 152.299 | 81.142 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.6 | 314.0 | 144.815 | 249.588 | 129.766 | 222.022 | 114.099 | 194.614 | 102.267 | 173.651 | 92.332 | 157.036 | 83.954 |
| 0.8 | 344.809 | 159.025 | 269.569 | 140.472 | 236.651 | 122.131 | 205.399 | 108.421 | 181.860 | 97.115 | 163.445 | 87.729 |
| 1.0 | 380.775 | 175.613 | 293.263 | 153.137 | 254.228 | 131.730 | 218.491 | 115.840 | 191.907 | 102.925 | 171.341 | 92.339 |

**FIGURE 3:-TIME PERIOD ($\tilde{K}$ ×10$^{-5}$) OF A CSS RIGHT TRIANGULAR PLATE FOR DIFFERENT VALUES OF β* , K/K$_0$ AND A CONSTANT ASPECT RATIO (a/b=3/2)**

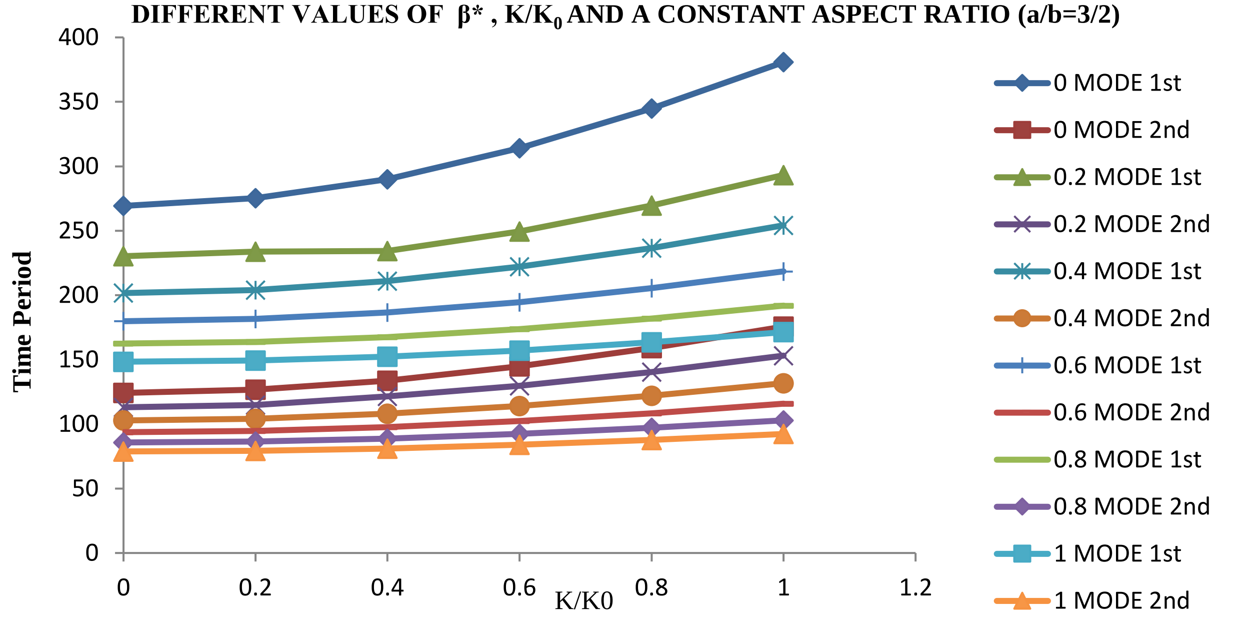



Table 4 and figure4 depicts value of time period $\tilde{K}$ for first two modes of vibration for different values of aspect ratio a/b for the following three cases: (i) β* =0, $\frac{K}{K_0}$ =0,(ii) β*=0.4, $\frac{K}{K_0}$=0.4,(iii) β* =1, $\frac{K}{K_0}$=1

It is important to observe that when aspect ratio increases, time period reduces in the preceding three situations for both modes of vibration. This reduction is again parabolic in nature.

**TABLE4:- TIME PERIOD ($\tilde{K}$×10$^{-5}$) OF A CSS RIGHT TRIANGULAR PLATE FOR DIFFERENT ASPECT RATIO (a/b)**

| a/b | β*=0 K/K$_0$=0 | | β*=0.4 K/K$_0$=0.4 | | β*=1 K/K$_0$=1 | |
|---|---|---|---|---|---|---|
| | MODE 1$^{st}$ | MODE 2$^{nd}$ | MODE 1$^{st}$ | MODE 2$^{nd}$ | MODE 1$^{st}$ | MODE 2$^{nd}$ |
| ½ | 547.3813 | 216.6521 | 439.6177 | 163.4297 | 361.2705 | 125.4183 |
| 1 | 381.2944 | 176.7036 | 305.6235 | 141.7148 | 253.2894 | 113.0649 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 3/2 | 269.2486 | 124.1780 | 210.9587 | 107.9906 | 171.3416 | 92.3397 |
| 2 | 196.2274 | 83.9647 | 153.4382 | 76.6589 | 122.4229 | 69.5122 |
| 5/2 | 145.2660 | 58.4649 | 114.7916 | 54.5956 | 91.5051 | 51.0850 |

**FIGURE4:-TIME PERIOD VARIATION IN C-SS IN RIGHT TRIANGULAR PLATES W.R.T TO DIFFERENT VALUES OF a/b**

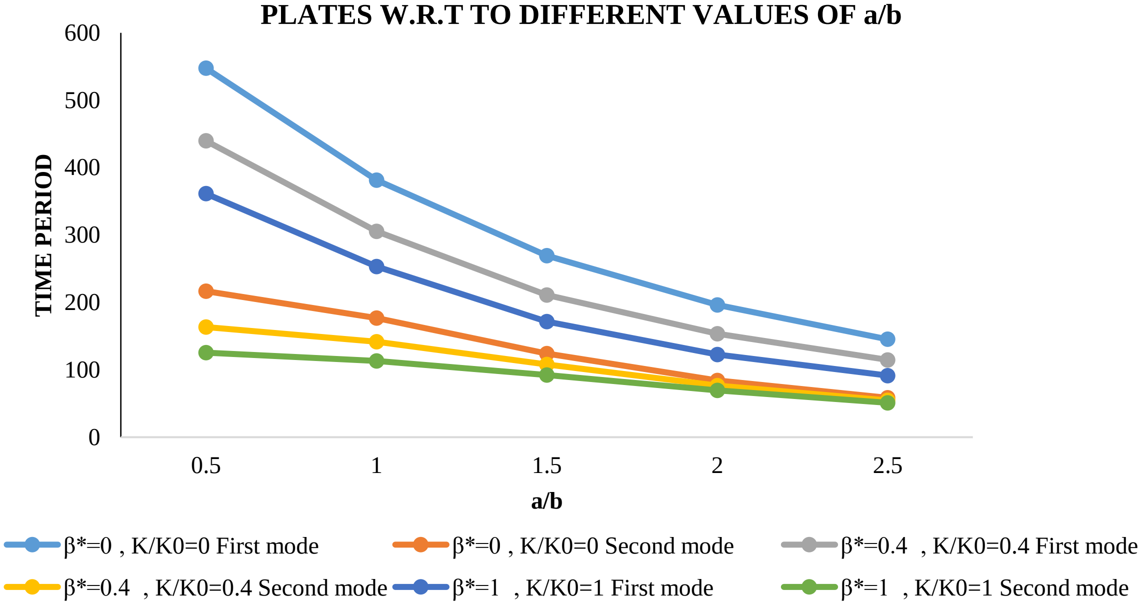



Table 5 and table 6 contains the deflection function values (i.e. the amplitude of the vibration modes )for the first two vibration modes with aspect ratios a/b = 3/2, respectively, for distinct values of $X'$ and $Y'$.The following parameter are used to calculate $\tilde{w}$.

Table 5: $\beta^*=0$, $K/K_0=0$, time $\tilde{T}=0\tilde{K}$ , time $\tilde{T}=5\tilde{K}$ and values in bold indicate deflection at $\tilde{T}=5\tilde{K}$

Table 6: $\beta^*=0.4$,$K/K_0=0.4$ ,time $\tilde{T}=0\tilde{K}$, time $\tilde{T}=5\tilde{K}$ and values in bold indicate deflection at $\tilde{T}=5\tilde{K}$

**TABLE5:- DEFLECTION ($\tilde{w}$) OF A CSS RIGHT TRIANGULAR PLATE FOR DIFFERENT VALUES OF $X'$ AND $Y'$, A CONSTANT ASPECT RATIO (a/b=3/2) AND $\beta^*=0,K/K_0=0$AND TIME $\tilde{T}=0\tilde{K}$ AND $5\tilde{K}$**

| $X'$ | 0.2 | | 0.4 | | 0.6 | |
|---|---|---|---|---|---|---|
| $Y'$ | MODE 1st | MODE 2nd | MODE 1st | MODE 2nd | MODE 1st | MODE 2nd |
| 0.2 | 0<br>**0** | 0<br>**0** | 0.0039429125<br>**9.86271× $10^{-3}$** | 0.0831407182<br>**27.966× $10^{-3}$** | 0.0059555843<br>**5.69662× $10^{-3}$** | 0.0470211131<br>**44.9765× $10^{-3}$** |
| 0.4 | -0.0182353<br>**-0.894× $10^{-3}$** | -0.6987498<br>**-34.2896× $10^{-3}$** | 0<br>**0** | 0<br>**0** | 0.011911168<br>**11.3932× $10^{-3}$** | 0.094042226<br>**89.9530× $10^{-3}$** |
| 0.6 | -0.08205928 | -3.144374 | -0.035486213 | -0.748266464 | 0 | 0 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | **-4.2688×** $\mathbf{10^{-3}}$ | **-154.303×** $\mathbf{10^{-3}}$ | **-8.87644×** $\mathbf{10^{-3}}$ | **-187.170×** $\mathbf{10^{-3}}$ | **0** | **0** |
| 0.8 | -0.21882476 **-17.383×** $\mathbf{10^{-3}}$ | -8.384998 **-411.475×** $\mathbf{10^{-3}}$ | -0.126173203 **-31.5607×** $\mathbf{10^{-3}}$ | -2.660502984 **-665.492×** $\mathbf{10^{-3}}$ | -0.047644674 **-45.5729×** $\mathbf{10^{-3}}$ | -0.37616890 **-359.812×** $\mathbf{10^{-3}}$ |

**TABLE 6:- DEFLECTION ($\tilde{w}$) OF A CSS RIGHT TRIANGULAR PLATE FOR DIFFERENT VALUE OF $X'$ AND $Y'$, A CONSTANT ASPECT RATIO (a/b=3/2) AND $\beta^*$=0.4,$K/K_0$=0.4 AND TIME $\tilde{T}$= 0$\tilde{K}$ AND 5$\tilde{K}$**

| $X'$ | 0.2 | | 0.4 | | 0.6 | |
|---|---|---|---|---|---|---|
| $Y'$ | MODE 1st | MODE 2nd | MODE 1st | MODE 2nd | MODE 1st | MODE 2nd |
| 0.2 | 0 **0** | 0 **0** | 0.003398 **0.067×** $\mathbf{10^{-6}}$ | 0.072377969 **1.428×** $\mathbf{10^{-6}}$ | 0.005673 **0.225×** $\mathbf{10^{-6}}$ | 0.041440 **1.6451×** $\mathbf{10^{-6}}$ |
| 0.4 | -0.13553499 **-0.1989×** $\mathbf{10^{-6}}$ | -0.60626995 **-5.363×** $\mathbf{10^{-6}}$ | 0 **0** | 0 **0** | 0.011346 **0.450×** $\mathbf{10^{-6}}$ | 0.082880 **3.2903×** $\mathbf{10^{-6}}$ |
| 0.6 | -0.060990746 **-0.5395×** $\mathbf{10^{-6}}$ | -2.72821480 **-24.134×** $\mathbf{10^{-6}}$ | -0.03058232 **-0.604×** $\mathbf{10^{-6}}$ | -0.65140172 **-12.856×** $\mathbf{10^{-6}}$ | 0 **0** | 0 **0** |
| 0.8 | -0.162641989 **-1.4387×** $\mathbf{10^{-6}}$ | -7.27523948 **-64.357×** $\mathbf{10^{-6}}$ | -0.10873716 **-2.146×** $\mathbf{10^{-6}}$ | -2.31609501 **-45.710×** $\mathbf{10^{-6}}$ | -0.045384 **-1.80×** $\mathbf{10^{-6}}$ | -0.331523 **-13.161×** $\mathbf{10^{-6}}$ |

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## REFERENCES

1. A.W.Leissa, Vibration of plates.NASA SP-160(1969).

2. Cowper, G. R., Kosko, E., Lindberg, G. M., & Olson, M. D. (1969). Static and dynamic applications of a high- precision triangular plate bending element. AIAA Journal, 7(10), 1957–1965,,https://doi.org/10.2514/3.5488

3.Mirza, S., & Bijlani, M. (1985). Vibration of triangular plates of variable thickness. Computers & Structures, 21(6), 1129–1135, https://doi.org/10.1016/0045-7949(85)90167-1

4. Mirza, S., & Bijlani, M. (1983). Vibration of Triangular Plates. AIAA Journal, 21(10), 1472–1475, https://doi.org/10.2514/3.60149.

5. Gorman, D. J. (1983). A highly accurate analytical solution for free vibration analysis of simply supported right triangular plates. Journal of Sound and Vibration, 89(1), 107–118. https://doi.org/10.1016/0022-460X(83)90914-8

6. Gorman, D. J. (1986). Free vibration analysis of right triangular plates with combinations of clamped-simply supported boundary conditions. Journal of Sound and Vibration, 106(3), 419–431.https://doi.org/10.1016/0022-460X(86)90189-6

7. Christensen, R. M. (1963). Vibration of a $45^0$ right triangular cantilever plate by a Gridwork method. AIAA Journal, 1(8), 1790–1795. https://doi.org/10.2514/3.1926

8. Bhat, R. B. (1987). Flexural vibration of polygonal plates using characteristic orthogonal polynomials in two variables. Journal of Sound and Vibration, 114(1), 65–71. https://doi.org/10.1016/S0022-460X(87)80234-1

9. Liew, K. M., Lam, K. Y., & Chow, S. T. (1990). Free vibration analysis of rectangular plates using orthogonal plate function. Computers & Structures, 34(1), 79–85, https://doi.org/10.1016/0045-7949(90)90302-I

10. Lam, K. Y., Liew, K. M., & Chow, S. T. (1990). Free vibration analysis of isotropic and orthotropic triangular plates. International Journal of Mechanical Sciences, 32(5), 455–464, https://doi.org/10.1016/0020-7403(90)90172-F

11. Gupta, A. K., & Khanna, A. (2007). Vibration of visco-elastic rectangular plate with linearly thickness variations in both directions. Journal of Sound and Vibration, 301(3-5), 450–457,doi:10.1016/j.jsv.2006.01.074