APPLICATIONS OF VERTEX (OR EDGE) CONNECTIVITY IN TRAFFIC CONTROL PROBLEMS

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ABSTRACT:
The main objective of this paper is to present the applications of vertex or edge connectivity in traffic control problems. Most of the traffic lanes can be conveniently modelled in terms of graphs by means of points (vertices) and lines (edges). Two examples of traffic lanes are used in this article to demonstrate how connectivity (vertex or edge) can be applied to traffic management problems in an efficient manner by reducing the amount of time that participants in the traffic must wait and the cost of placing the sensors in the right location to collect traffic data.

Key words: *Graph theory, Traffic Control Problem, Connectivity (vertex or edge), Modelling.*

1. INTRODUCTION:

Graph theory is now used extensively in all fields of science and technology. It is also frequently used in many other subjects, including sociology, biology, biochemistry (genomics), electrical engineering (communication networks and coding theory), computer science (algorithms and computation), and operations research (scheduling). It involves the study of molecules, the formation of bonds in chemistry, and the study of atoms. The most essential application of graph colouring is resource allocation and scheduling and many other applications can be found in [5 to 9]. In addition, pathways, walks, and circuits in graph theory are employed in several applications such as the travelling salesman problem, database design concepts, and resource networking, which leads to the development of new algorithms and theorems that may be used in a variety of applications [1]. Both edge connectivity and vertex connectivity are utilized as graph theoretic tools to investigate traffic control problems at intersections.

Owing to the massive population growth and urbanization of small towns and cities, there are more cars on the road, which increases traffic participant time losses, noise and environmental pollution, and the frequency of traffic accidents. With millions of people impacted, traffic congestion has emerged as one of the main barriers to the growth of many urban regions. While building additional roads could help the situation, doing so would be exceedingly expensive and frequently impractical because of the current infrastructure. The only option to manage traffic flow in such a scenario is to make better use of the existing road network. A strategy for effectively managing city traffic through appropriate traffic management of changing the infrastructure on the roads is researched by Dave and Jhala [4].

This study discusses the use of vertex and edge connectivity in traffic control problems at intersections to reduce traffic participants' waiting time and the cost of locating sensors to collect traffic data.

2. **Compatibility Graph and its connectivity:**

Vehicles approaching an intersection prepare to undertake a certain manoeuvre, such as driving through, turning left, or turning right at the intersection. The vehicles that undertake this manoeuvre are considered flow components. This type of arrival flow component is known as a traffic stream [10]. To analyse the traffic control problem at every intersection, it must be mathematically modelled using a simple graph for the traffic gathering data problem. The collection of edges in the underlying network will represent the communication link between the nodes, or traffic streams at an intersection.

The traffic streams that may cross an intersection simultaneously without encountering any conflicts are united by edges in the graph that depicts the traffic control problem, while the streams that are unable to move together are not connected by any edges. The resulting graph is a connected graph that will be known as the intersection compatibility graph for the traffic control issue.

In order to define a cut-set and the connectivity of the compatibility graph, the underlying graph G considered as G = (V, E) where V(G) denotes the set of vertices of G and E(G) denotes the set of edges of G. A **cut-set** F is a set of edges whose removal from G leaves G disconnected. Also it results in the increase in the number of components of G by one. Each cut-set of the compatibility graph G consists of a certain number of edges. The number of edges in the smallest cut-set i.e. the cut-set with fewest numbers of edges is defined as the e**dge connectivity** of G. The **vertex connectivity** of the compatibility graph G is defined as the
minimum number of vertices whose removal from G leaves the remaining graph disconnected. The details on these aspects for the graph G can be found in [2] and [11].

Both on each vertex of G specified by its vertex connection and on each edge in a cut-set of G determined by its edge connectivity can house the traffic sensors. The control system will receive comprehensive traffic data from these sensors. Therefore, by utilising the compatibility graph G's edge and vertex connectivity the best placements for the traffic sensors may be found.

3. An Example

The example considered is a traffic control problem at a seven leg intersection, with seven lanes as shown in **Fig. 1** below are taken from [3] and the corresponding compatibility graph which is drawn as shown in **Fig. 2**

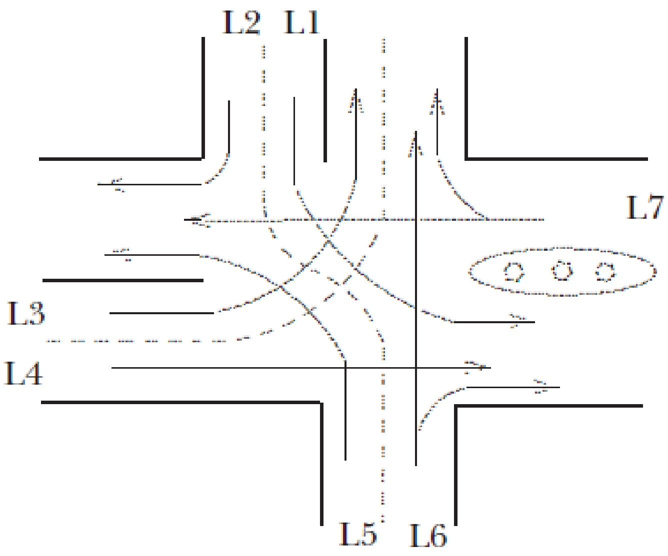

Fig 1: An intersection with seven traffic lanes

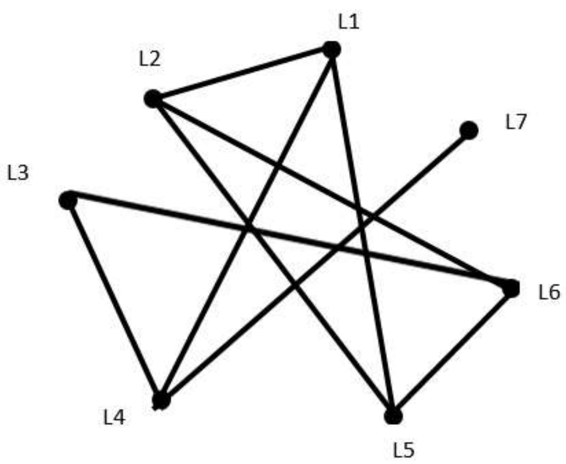

Fig 2: Modelling Traffic lanes at an intersection by means of a Graph G (Compatibility graph of Figure 1)

In the compatibility graph (**Fig. 2),** number of vertices n is 7 and number of edges e is 8. Following the discussion in Section 2, vertex connectivity as well as edge connectivity which can be achieved here are as high as 2. It is not difficult to find edge {L4, L7} corresponding to the edge connectivity as well as vertices L2 and L4 corresponding to the vertex connectivity whose removal disconnects the compatibility graph. The disconnected graphs having two components obtained after removing the edge {L4, L7} as well as vertices L2 and L4 from the compatibility graph are as shown in the **Fig. 3** and **Fig. 4** respectively as below:

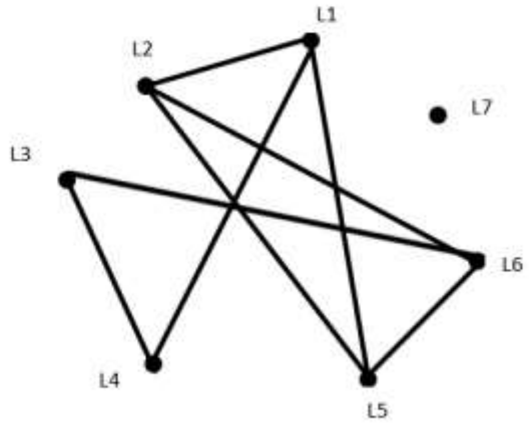

Fig. 3: Graph obtained after removal of the edge {L4, L7}

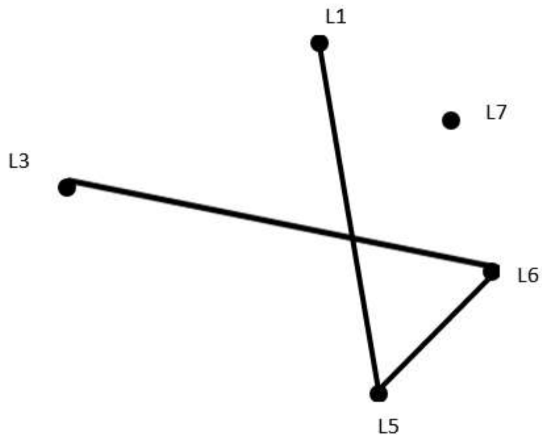

Fig. 4: Graph obtained after removal of the vertices L2 and L4

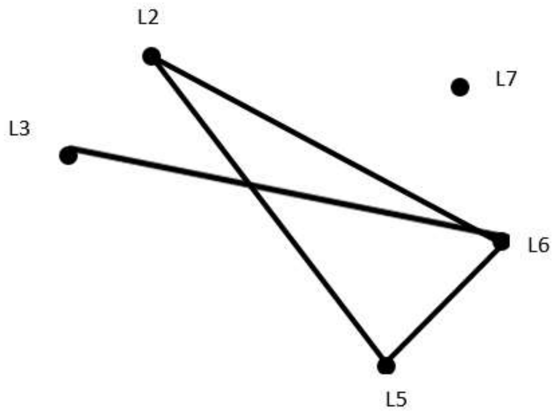

Fig. 5: Graph obtained after removal of the vertices L1 and L4

Another possible vertex connectivity set is the vertices L1 and 4 which is shown in Fig. 5

From the above discussion it is clear that sensors have to placed either in the edge {L4, L7} or in the vertices L1,L2 or L4, which will provide complete information and can be delivered to the traffic participants, regarding traffic flow, congestion etc.

4. An Example

Another example considered here is a traffic control problem with 9 lanes as shown in **Fig. 6** below are taken from [3] and the corresponding compatibility graph which is drawn as shown in **Fig. 7**.

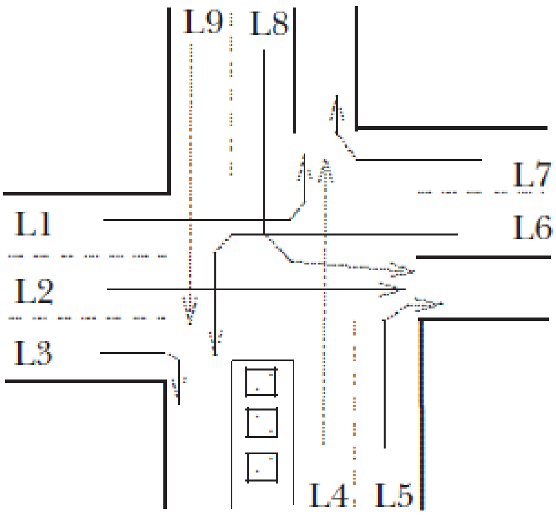

Fig 6: An intersection with nine traffic lanes

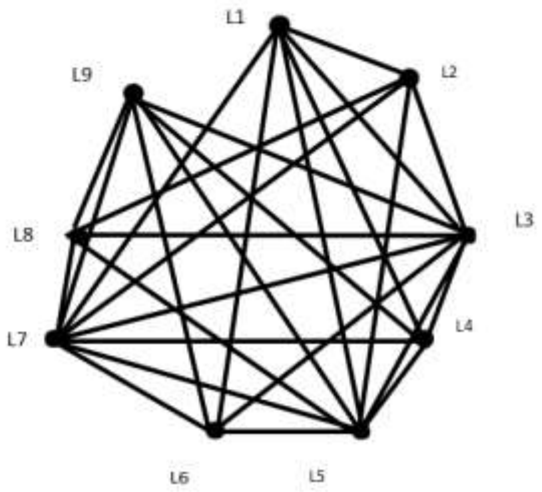

Fig 7: Modelling Traffic lanes at an intersection by means of a Graph G
(Compatibility graph of Figure 6)

In the compatibility graph (**Fig. 7),** number of vertices n is 9 and number of edges e is 28. Following the discussion in Section 2, vertex connectivity as well as edge connectivity which can be achieved here are as high as 5. It is not difficult to find edges incident with L6 corresponding to the edge connectivity as well as vertices L1,L3, L5,L7 and L8 corresponding to the vertex connectivity whose removal disconnects the compatibility graph. The disconnected graphs having two components obtained after removing the edges as well as vertices from the compatibility graph are as shown in the

Fig. 8 and **Fig. 9** respectively as below:

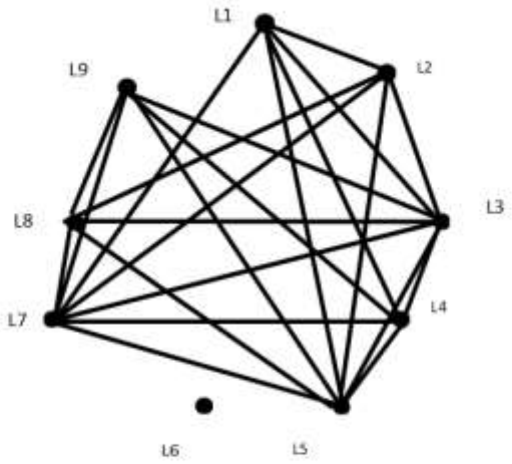

Fig. 8 : Graph obtained after removal of the edges incident with L6

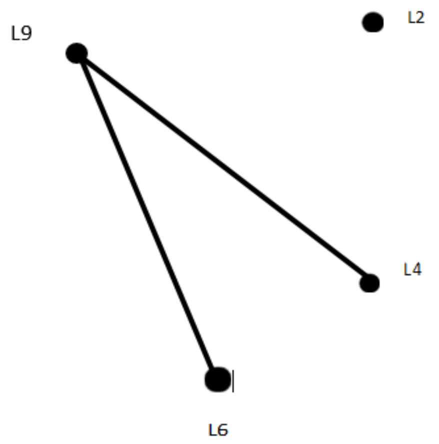

Fig. 9: Graph obtained after removal of the vertices L1, L3, L5, L7 and L8

Similarly the other possible set of vertices and edges can be found to disconnect the compatibility graph shown in Fig 7.

From the above discussion it is clear that sensors have to placed either in the edges incident with L4 or in the vertices L1,L3, L5, L7 or L8, which will provide complete information and can be delivered to the traffic participants, regarding traffic flow, congestion etc

5. APPLICATIONS:

There are several applications for edge and vertex connectivity in intersection traffic control issues. Either of the two connectivity's can be used to gather all of the data related to the traffic problem. The edges provided by the edge connectivity and the vertices offered by the vertex connection specify the precise locations where the sensors have to be put, minimising the overall cost [2].

These connectivity's are crucial to understanding the characteristics of transport and communication networks since they are required to determine the maximum flow rate between any two nodes in the network. There are a plethora of more uses for networks, including traffic-capable network representations of highways, data-transmission-capable computer network links, electric network currents, and industrial applications, among others.

6. CONCLUSIONS:

A few instances of graph theory applications are presented in this work. Specifically, the idea of a graph's vertex and edge connectivity can be applied in a variety of real-world problems and utilised as graph theoretic tools to examine the intersection of traffic control issues. By regulating the edges of the edge connectivity of the transportation network and installing traffic sensors on each of these edges, which will offer complete traffic network information, the waiting times of the traffic participants can be reduced. An option to the aforementioned would be to install sensors on every vertex in the transportation network's vertex connectivity to obtain all of the network's traffic data. Thus, it may be said that graph theory plays a crucial role in modern times.

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