

# ON FUZZY SUPRA P-SPACES

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## Abstract

*In this paper we introduced and studied the concepts of fuzzy supra P-spaces. Also we discuss several characterizations and properties of fuzzy supra P-spaces are introduced. Several examples are given to illustrate the concepts.*

**Keywords:** Fuzzy supra  $F_\sigma$ -set, Fuzzy supra  $G_\delta$ -set, Fuzzy supra nowhere dense set, Fuzzy supra dense set.

## 1. Introduction

In the year 1965, L. A.Zadeh [12] was first introduced the concept of fuzzy sets and fuzzy set operations in his classical paper. In 1968, the theory of fuzzy topological spaces was introduced and developed by C.L.Chang [3].

In 1983, Mashhour.A.S.et.al., [5] introduced and studied the concept of Supra topological spaces. In 1987, AbdEl-monsef et.at [1] introduced the concepts of fuzzy supra topological as a natural generalization of the notion of supra topological spaces.

In 1972, Mishra.A.K [6] introduced the concepts of P-spaces as a generalization of  $\omega_\alpha$ -additive spaces of Sikorski [10] and Cohen.L.W and Goffman.C [4]. The concept of P-spaces in fuzzy setting was introduced by Thangaraj.G and Balasubramanian.G [11].

In this paper introduced and studied the concepts of fuzzy supra P-spaces. Several examples are given to illustrate the concepts. It is established that fuzzy supra  $\sigma$ -boundary sets are fuzzy supra closed sets, fuzzy supra co- $\sigma$ -boundary sets are fuzzy supra open sets in fuzzy supra P-spaces.

## 2. Preliminaries

### Definition 2.1 [1].

A collection  $T^*$  of fuzzy sets in a set  $U$  is called fuzzy supra topology on  $U$  if the following conditions are satisfied:

- 1)  $\mathbf{0}$  and  $\mathbf{1}$  belongs to  $T^*$ .
- 2)  $g_\chi \in T^*$  for each  $\chi \in \Lambda$  implies  $(\vee_{\chi \in \Lambda} g_\chi) \in T^*$ .

The pair  $(U, T^*)$  is called a fuzzy supra topological space. The elements of  $T^*$  are called fuzzy supra open sets and the complement of a fuzzy supra open set is called fuzzy supra closed set.

### Definition: 2.2 [9]

Let  $(U, T^*)$  be a fuzzy supra topological space and  $\mu$  be a fuzzy set in  $U$ , then the fuzzy supra closure and fuzzy supra interior of  $\mu$  defined respectively as

$$cl^*(\mu) = \wedge \{ g / g \text{ is a fuzzy supra closed set in } U \text{ and } \mu \leq g \}$$

$$int^*(\mu) = \vee \{ g / g \text{ is a fuzzy supra open set in } U \text{ and } g \leq \mu \}$$

**Definition: 2.3 [9]**

Let  $(U, T)$  be a fuzzy topological space and  $T^*$  be a fuzzy supra topology on  $U$ . We call  $T^*$  a fuzzy supra topology associated with  $T$  if  $T \leq T^*$ .

**Remark: 2.4 [9]**

- 1) The fuzzy supra closure of a fuzzy set  $\mu$  in a fuzzy supra topological space is the smallest fuzzy supra closed set containing  $\mu$ .
- 2) The fuzzy supra interior of a fuzzy set  $\mu$  in a fuzzy supra topological space is the largest fuzzy supra open set contained in  $\mu$ .
- 3) If  $(U, T^*)$  is an associated fuzzy supra topological space with the fuzzy topological space  $(U, T)$  and  $\mu$  is any fuzzy set in  $U$ , then  $int(\mu) \leq int^*(\mu) \leq \mu \leq cl^*(\mu) \leq cl(\mu)$

**Lemma 2.5 [3]**

For a fuzzy set  $\mu$  in a fuzzy topological space  $U$ ,

$$(i) 1 - int(\mu) = cl(1 - \mu),$$

$$(ii) 1 - cl(\mu) = int(1 - \mu).$$

**Definition 2.6 [7]**

A fuzzy supra open set  $\mu$  in fuzzy supra topological space  $(U, T^*)$  is called fuzzy supra  $F_\sigma$ -set in  $(U, T^*)$  if  $\mu = \vee_{i=1}^\infty (\mu_i)$ , where  $1 - \mu_i \in T^*$  for  $i \in I$ ,

**Definition 2.7 [7]**

A fuzzy supra open set  $\mu$  in fuzzy supra topological space  $(U, T^*)$  is called fuzzy supra  $G_\delta$ -set in  $(U, T^*)$  if  $\mu = \wedge_{i=1}^\infty (\mu_i)$ , where  $\mu_i \in T^*$  for  $i \in I$ .

**Definition 2.8 [7]**

A fuzzy set  $\mu$  in a fuzzy supra topological space  $(U, T^*)$  is called a fuzzy supra dense set if there exists no fuzzy supra closed set  $\beta$  in  $(U, T^*)$  such that  $\mu < \beta < 1$ . That is,  $cl^*(\mu) = 1$ , in  $(U, T^*)$ .

**Definition 2.9 [8]**

A fuzzy set  $\mu$  in fuzzy supra topological space  $(U, T^*)$  is called a fuzzy supra nowhere dense set if there exists no non-zero fuzzy supra open set  $\delta$  in  $(U, T^*)$  such that  $\delta < cl^*(\mu)$ . That is,  $int^* cl^*(\mu) = 0$ , in  $(U, T^*)$ .

**Definition 2.10 [8]**

A fuzzy set  $\mu$  in a fuzzy supra topological space  $(U, T^*)$  is called a fuzzy supra first category set if  $\mu = \vee_{i=1}^\infty (\mu_i)$ , where  $(\mu_i)$ 's are fuzzy supra nowhere dense set in  $(U, T^*)$ . Any other fuzzy set in  $(U, T^*)$  is said to be fuzzy supra second category space.

### 3. Fuzzy Supra P-Spaces

**Definition 3.1**

A fuzzy supra topological space  $(U, T^*)$  is called a fuzzy supra P-space if countable intersection of fuzzy supra open sets in  $(U, T^*)$  is fuzzy supra open. That is., every non- zero fuzzy supra  $G_\square$ -set in  $(U, T^*)$  is fuzzy supra open set in  $(U, T^*)$ .

### Example 3.1

Let  $U = \{x, y, z\}$ . The fuzzy sets  $\mu_1, \mu_2$  and  $\mu_3$  are defined on  $U$  as follows:

$\mu_1: U \rightarrow [0, 1]$  is defined as  $\mu_1(x)=0.2; \mu_1(y)=0.3; \mu_1(z)=0.1.$

$\mu_2: U \rightarrow [0, 1]$  is defined as  $\mu_2(x)=0.3; \mu_2(y)=0.4; \mu_2(z)=0.4.$

$\mu_3: U \rightarrow [0, 1]$  is defined as  $\mu_3(x)=0.1; \mu_3(y)=0.2; \mu_3(z)=0.3.$

Then,  $T^* = \{0, \mu_1, \mu_2, \mu_3, (\mu_1 \square \mu_2), (\mu_1 \square \mu_3), (\mu_2 \square \mu_3), (\mu_1 \square \mu_2), (\mu_1 \square \mu_3), (\mu_2 \square \mu_3), [\mu_1 \square (\mu_2 \square \mu_3)], [\mu_1 \square (\mu_2 \square \mu_3)], (\mu_1 \square \mu_2 \square \mu_3), (\mu_1 \square \mu_2 \square \mu_3), 1\}$  is a fuzzy supra topology on  $U$ . Now the fuzzy sets  $\mu_1 \square \mu_3 = \{\mu_1 \square \mu_1 \square \mu_2 \square \mu_2 \square \mu_1 \square \mu_2 \square \mu_2 \square (\mu_2 \square \mu_3)\}$  and  $\mu_1 \square \mu_2 = \{\mu_2 \square (\mu_2 \square \mu_3) \square \square [\mu_1 \square (\mu_2 \square \mu_3)] \square \square [\mu_1 \square (\mu_2 \square \mu_3)]\}$  are fuzzy supra  $G_\square$ -sets in  $(U, T^*)$  and  $\mu_1 \square \mu_3, \mu_1 \square \mu_2$  are fuzzy supra open sets in  $(U, T^*)$ . Hence,  $(U, T^*)$  is a fuzzy supra P-space.

### Example 3.2:

Let  $U = \{x, y, z\}$ . The fuzzy sets  $\mu_1, \mu_2$  and  $\mu_3$  are defined on  $U$  as follows:

$\mu_1: U \rightarrow [0, 1]$  is defined as  $\mu_1(x)=0.3; \mu_1(y)=0.6; \mu_1(z)=0.2.$

$\mu_2: U \rightarrow [0, 1]$  is defined as  $\mu_2(x)=0.5; \mu_2(y)=0.7; \mu_2(z)=0.4.$

$\mu_3: U \rightarrow [0, 1]$  is defined as  $\mu_3(x)=0.7; \mu_3(y)=0.3; \mu_3(z)=0.4.$

Then,  $T^* = \{0, \mu_1, \mu_2, \mu_3, (\mu_1 \square \mu_2), (\mu_1 \square \mu_3), (\mu_2 \square \mu_3), (\mu_1 \square \mu_2), (\mu_2 \square \mu_3), (\mu_1 \square \mu_2 \square \mu_3), 1\}$  is a fuzzy supra topology on  $U$ . Now the fuzzy set  $\lambda = \{\mu_1 \square \mu_2 \square \mu_3 \square (\mu_1 \square \mu_2) \square \square (\mu_1 \square \mu_3) \square \square \square (\mu_2 \square \mu_3) \square \square \square (\mu_1 \square \mu_2) \square \square (\mu_2 \square \mu_3) \square \square (\mu_1 \square \mu_2 \square \mu_3)\}$  is a fuzzy supra  $G_\square$ -set in  $(U, T^*)$ . But  $\lambda$  is not a fuzzy supra open set in  $(U, T^*)$ . Hence the fuzzy supra topological space  $(U, T^*)$  is not a fuzzy supra P-space.

### Proposition 3.1.

If  $\mu$  is a non-zero fuzzy supra  $F_\sigma$ -set in a fuzzy supra P-space  $(U, T^*)$ , then  $\mu$  is a fuzzy supra closed set in  $(U, T^*)$ .

#### Proof.

Since  $\mu$  is a non-zero fuzzy supra  $F_\sigma$ -set in  $(U, T^*)$ ,  $\mu = \bigvee_{i=1}^{\infty} (\mu_i)$ , where  $(\mu_i)$ 's are fuzzy supra closed in  $(U, T^*)$ . Then  $1 - \mu = 1 - (\bigvee_{i=1}^{\infty} (\mu_i)) = \bigwedge_{i=1}^{\infty} (1 - \mu_i)$ . Now,  $(\mu_i)$ 's are fuzzy supra closed set in  $(U, T^*)$ , implies that  $(1 \square \mu_i)$ 's are fuzzy supra open set in  $(U, T^*)$ . Hence, we have  $1 - \mu = \bigwedge_{i=1}^{\infty} (1 - \mu_i)$ , where  $1 \square \mu_i \in T^*$ . Then  $1 \square \mu$  is a fuzzy supra  $G_\delta$ -set in  $(U, T^*)$ . Since  $(U, T^*)$  is a fuzzy supra P-space,  $1 \square \mu$  is fuzzy supra open in  $(U, T^*)$ . Therefore  $\mu$  is a fuzzy supra closed set in  $(U, T^*)$ .

### Proposition 3.2.

If the fuzzy supra topological space  $(U, T^*)$  is a fuzzy supra P-space, then  $cl^* (\bigvee_{i=1}^{\infty} (\mu_i)) = \bigvee_{i=1}^{\infty} cl^* (\mu_i)$  where  $(\mu_i)$ 's are non-zero fuzzy supra closed sets in  $(U, T^*)$ .

#### Proof.

Let  $(\mu_i)$ 's be non-zero fuzzy supra closed sets in a fuzzy supra P-space  $(U, T^*)$ . Then  $\mu = \bigvee_{i=1}^{\infty} (\mu_i)$ , is a non-zero fuzzy supra  $F_\sigma$ -set in  $(U, T^*)$ . By proposition 3.1,  $\mu$  is a fuzzy supra closed set in  $(U, T^*)$ . Hence  $cl^*(\mu) = \mu$ , which implies that  $cl^* (\bigvee_{i=1}^{\infty} (\mu_i)) = \bigvee_{i=1}^{\infty} (\mu_i) = \bigvee_{i=1}^{\infty} cl^* (\mu_i)$  [since  $(\mu_i)$ 's are fuzzy supra closed,  $cl^*(\mu_i) = (\mu_i)$ ]. Therefore  $cl^* (\bigvee_{i=1}^{\infty} (\mu_i)) = \bigvee_{i=1}^{\infty} cl^* (\mu_i)$ , where  $(\mu_i)$ 's are fuzzy supra closed sets in

$(U, T^*)$ .

**Proposition 3.3.**

If  $(\mu_i)$ 's are fuzzy supra regular closed sets in a fuzzy supra P-space  $(U, T^*)$ , then  $cl^* \left( \bigvee_{i=1}^{\infty} (\mu_i) \right) = \bigvee_{i=1}^{\infty} (\mu_i)$ .

**Proof.**

Let  $(\mu_i)$ 's be fuzzy supra regular closed sets in a fuzzy supra P-space  $(U, T^*)$ . Then  $(1-\mu_i)$ 's are fuzzy supra open sets in  $(U, T^*)$ . Let  $\lambda = \bigwedge_{i=1}^{\infty} [1-\mu_i]$ . Then  $\lambda$  is a non-zero fuzzy supra  $G_{\delta}$ -set in  $(U, T^*)$ .

Since the fuzzy supra topological space  $(U, T^*)$  is a fuzzy supra

P-space,  $\text{int}^*(\lambda) = \lambda$ , which implies that  $\text{int}^* \left( \bigwedge_{i=1}^{\infty} [1-\mu_i] \right) = \bigwedge_{i=1}^{\infty} [1-\mu_i]$ . Then,

$$1 - cl^* \left( \bigvee_{i=1}^{\infty} (\mu_i) \right) = 1 - \bigvee_{i=1}^{\infty} (\mu_i). \text{ Hence, we have } cl^* \left( \bigvee_{i=1}^{\infty} (\mu_i) \right) = \bigvee_{i=1}^{\infty} (\mu_i).$$

**Proposition 3.4.**

If the fuzzy supra topological space  $(U, T^*)$  is a fuzzy supra P-space and if  $\mu$  is a fuzzy supra first category set in  $(U, T^*)$ , then  $\mu$  is not a fuzzy supra dense set in  $(U, T^*)$ .

**Proof.**

Assume the contrary. Suppose that  $\mu$  is a fuzzy supra first category set in  $(U, T^*)$  such that  $cl^*(\mu) = 1$ .

Then,  $\mu = \bigvee_{i=1}^{\infty} (\mu_i)$ , where  $(\mu_i)$ 's are fuzzy supra nowhere dense sets in  $(U, T^*)$ . Now  $1-cl^*(\mu_i)$  is a

fuzzy supra open set in  $(U, T^*)$ . Let  $\lambda = \bigwedge_{i=1}^{\infty} [1-cl^*(\mu_i)]$ . Then  $\lambda$  is a non-zero fuzzy supra  $G_{\delta}$ -set in

$(U, T^*)$ . Now we have  $\bigwedge_{i=1}^{\infty} [1-cl^*(\mu_i)] = 1 - \bigvee_{i=1}^{\infty} cl^*(\mu_i) \leq 1 - \bigvee_{i=1}^{\infty} (\mu_i) = 1 - \mu$ . Hence  $\lambda \leq 1 - \mu$ .

Then  $\text{int}^*(\lambda) \leq \text{int}^*(1-\mu) = 1-cl^*(\mu) = 1-1 = 0$ . That is,  $\text{int}^*(\lambda) = 0$ . Since  $(U, T^*)$  is a fuzzy supra P-space,  $\lambda = \text{int}^*(\lambda)$  which implies that  $\lambda = 0$ , a contradiction to  $\lambda$  being a non-zero fuzzy supra  $G_{\delta}$ -set in  $(U, T^*)$ .

Hence  $cl^*(\mu) \neq 1$ . Therefore  $\mu$  is not a fuzzy supra dense set in  $(U, T^*)$ .

**Proposition 3.5.**

If the fuzzy supra topological space  $(U, T^*)$  is a fuzzy supra P-space and if  $\mu$  is a fuzzy supra first category set in  $(U, T^*)$ , then  $\mu$  is not a fuzzy supra nowhere dense set in  $(U, T^*)$ .

**Proof.**

Let  $\mu$  be a fuzzy supra first category set in a fuzzy supra P-space  $(U, T^*)$ . Then, we have  $\mu = \bigvee_{i=1}^{\infty} (\mu_i)$ ,

where  $(\mu_i)$ 's are fuzzy supra nowhere dense sets in  $(U, T^*)$ . Now

$\text{int}^* cl^*(\mu) = \text{int}^* cl^* \left( \bigvee_{i=1}^{\infty} (\mu_i) \right) \geq \text{int}^* \left( \bigvee_{i=1}^{\infty} cl^*(\mu_i) \right)$  and  $\left[ \bigvee_{i=1}^{\infty} cl^*(\mu_i) \right]$  is a fuzzy supra

$F_{\sigma}$ -set in  $(U, T^*)$ . Since  $(U, T^*)$  is a fuzzy supra P-space, by Proposition.3.1,  $\left[ \bigvee_{i=1}^{\infty} cl^*(\mu_i) \right]$  is a non-zero

fuzzy supra closed set in  $(U, T^*)$ . Also interior of a fuzzy supra closed is a fuzzy supra regular open set,

$\text{int}^* \left[ \bigvee_{i=1}^{\infty} cl^*(\mu_i) \right]$  is a non-zero fuzzy supra regular open set in  $(U, T^*)$ . Hence, we have

$0 \neq \text{int}^* \left[ \bigvee_{i=1}^{\infty} cl^*(\mu_i) \right] \leq \text{int}^* cl^*(\mu)$  implies that  $\text{int}^* cl^*(\mu) \neq 0$ . Therefore,  $\mu$  is not a fuzzy supra

nowhere dense set in  $(U, T^*)$ .

**Proposition 3.6.**

If  $\mu$  is a fuzzy supra first category set in a fuzzy supra P-space  $(U, T^*)$  such that  $\lambda \leq 1 - \mu$ , where  $\lambda$  is a non-zero fuzzy supra dense and fuzzy supra  $G_{\delta}$ -set in  $(U, T^*)$ , then  $\mu$  is a fuzzy supra nowhere dense set in  $(U, T^*)$ .

**Proof.**

Let  $\mu$  be a fuzzy supra first category set in  $(U, T^*)$ . Then,  $\mu = \bigvee_{i=1}^{\infty} \mu_i$ , where  $(\mu_i)$ 's are fuzzy supra nowhere dense sets in  $(U, T^*)$ . Now,  $1 - cl^*(\mu_i)$  is a fuzzy supra open set in  $(U, T^*)$ . Let  $\lambda = \bigwedge_{i=1}^{\infty} [1 - cl^*(\mu_i)]$ . Then  $\lambda$  is a non-zero fuzzy supra  $G_\delta$ -set in  $(U, T^*)$ . Now, we have  $\bigwedge_{i=1}^{\infty} [1 - cl^*(\mu_i)] = 1 - \bigvee_{i=1}^{\infty} cl^*(\mu_i) \leq 1 - \bigvee_{i=1}^{\infty} \mu_i = 1 - \mu$ . Hence,  $\lambda \leq (1 - \mu)$ . Then we have  $\mu \leq (1 \square \lambda)$ . Now,  $int^* cl^*(\mu) \leq int^* cl^*(1 \square \lambda)$ , which implies that  $int^* cl^*(\mu) \leq 1 - cl^* int^*(\lambda)$ . Since  $(U, T^*)$  is a fuzzy supra P-space, the fuzzy supra  $G_\delta$ -set  $\lambda$  is fuzzy supra open set in  $(U, T^*)$  and  $int^*(\lambda) = \lambda$ . Therefore,  $int^* cl^*(\mu) \leq 1 - cl^*(\lambda) = 1 - 1 = 0$  (since  $\lambda$  is fuzzy supra dense). Then,  $int^* cl^*(\mu) = 0$  and hence  $\mu$  is a fuzzy supra nowhere dense set in  $(U, T^*)$ .

### Theorem 3.1.[8]

Let  $(U, T^*)$  be a fuzzy supra topological space. Then the following are equivalent:

- (1)  $(U, T^*)$  is a fuzzy supra Baire space.
- (2)  $int^*(\lambda) = 0$ , for every fuzzy supra first category set  $\lambda$  in  $(U, T^*)$ .
- (3)  $cl^*(\mu) = 1$ , for every fuzzy supra residual set  $\mu$  in  $(U, T^*)$ .

### Proposition 3.7.

If  $\mu$  is a fuzzy supra first category set in a fuzzy supra P-space  $(U, T^*)$  such that  $\lambda \leq 1 - \mu$ , where  $\lambda$  is a non-zero fuzzy supra dense and fuzzy supra  $G_\delta$ -set in  $(U, T^*)$ , then  $(U, T^*)$  is a fuzzy supra Baire space.

#### Proof.

Let  $\mu$  be a fuzzy supra first category set in  $(U, T^*)$ . By Proposition.3.6, we have  $int^* cl^*(\mu) = 0$ . Then  $int^*(\mu) \leq int^* cl^*(\mu)$  implies that  $int^*(\mu) = 0$  and hence by Theorem 3.1,  $(U, T^*)$  is a fuzzy supra Baire space.

### Proposition 3.8.

If the fuzzy supra topological space  $(U, T^*)$  is a fuzzy supra P-space and if  $\mu$  is a fuzzy supra dense and fuzzy supra first category set in  $(U, T^*)$ , then there is no non-zero fuzzy supra  $G_\delta$ -set  $\lambda$  in  $(U, T^*)$  such that  $\lambda \leq 1 \square \mu$ .

#### Proof.

Let  $\mu$  be a fuzzy supra first category set in  $(U, T^*)$ . By Proposition.3.6, we have a fuzzy supra  $G_\delta$ -set  $\lambda$  in  $(U, T^*)$  such that  $\lambda \leq 1 \square \mu$ . Then,  $int^*(\lambda) \leq int^*(1 \square \mu)$  implies that  $int^*(\lambda) \leq 1 - cl^*(\mu) = 1 - 1 = 0$  [since  $\mu$  is fuzzy supra dense,  $cl^*(\mu) = 1$ ]. That is,  $int^*(\lambda) = 0$ . Since  $(U, T^*)$  is a fuzzy supra P-space,  $int^*(\lambda) = \lambda$  and hence we have  $\lambda = 0$ . Hence, if  $\mu$  is a fuzzy supra dense and fuzzy supra first category set in  $(U, T^*)$ , then there is no non-zero fuzzy supra  $G_\delta$ -set  $\lambda$  in  $(U, T^*)$  such that  $\lambda \leq 1 \square \mu$ .

### Definition 3.2.

A fuzzy set  $\mu$  in a fuzzy supra topological space  $(U, T^*)$  is called a fuzzy supra  $\sigma$ -boundary set, if  $\mu = \bigvee_{i=1}^{\infty} \mu_i$  where  $\mu_i = cl^*(\mu_i) \wedge (1 - \mu_i)$  and  $(\mu_i)$ 's are fuzzy supra regular open sets in  $(U, T^*)$ .

### Proposition 3.9.

If  $\mu$  is a fuzzy supra  $\sigma$ -boundary set in a fuzzy supra topological space  $(U, T^*)$ , then  $\mu$  is a fuzzy supra  $F_\sigma$ -set in  $(U, T^*)$ .

#### Proof.

Let  $\mu$  be a fuzzy supra  $\sigma$ -boundary set in a fuzzy supra topological space  $(U, T^*)$ . Then  $\mu = \bigvee_{i=1}^{\infty} \mu_i$ , where  $\mu_i = cl^*(\mu_i) \wedge (1 - \mu_i)$  and  $(\mu_i)$ 's are fuzzy supra regular open sets in  $(U, T^*)$ . Since  $(\mu_i)$  is a fuzzy supra regular open set in  $(U, T^*)$ ,  $(1 - \mu_i)$  is a fuzzy supra regular closed set in  $(U, T^*)$ . This implies that  $(1 - \mu_i)$  is a fuzzy supra closed set in  $(U, T^*)$  and hence  $cl^*(1 - \mu_i) = 1 - \mu_i$  in  $(U, T^*)$ . Then  $\mu_i = cl^*(\mu_i)$

$\bigwedge (1 - \mu_i)$  is a fuzzy supra closed set in  $(U, T^*)$ . Thus  $\mu = \bigvee_{i=1}^{\infty} (\mu_i)$ , where  $(\mu_i)$ 's are fuzzy supra closed sets in  $(U, T^*)$ , implies that  $\mu$  is a fuzzy supra  $F_{\sigma}$ -set in  $(U, T^*)$ .

**Proposition 3.10.**

If  $\mu$  is a fuzzy supra  $\sigma$ -boundary set in a fuzzy supra P-space  $(U, T^*)$ , then  $\mu$  is a fuzzy supra closed set in  $(U, T^*)$ .

**Proof.**

Let  $\mu$  be a fuzzy supra  $\sigma$ -boundary set in  $(U, T^*)$ . Then, by Proposition 3.9,  $\mu$  is a fuzzy supra  $F_{\sigma}$ -set in  $(U, T^*)$  and  $1 - \mu$  is a fuzzy supra  $G_{\delta}$ -set in  $(U, T^*)$ . Since  $(U, T^*)$  is a fuzzy supra P-space, the fuzzy supra  $G_{\delta}$ -set,  $1 - \mu$  is a fuzzy supra open set in  $(U, T^*)$  and hence  $\mu$  is a fuzzy supra closed set in  $(U, T^*)$ .

**Definition 3.3.**

A fuzzy set  $\mu$  in a fuzzy supra topological space  $(U, T^*)$  is called a fuzzy supra co- $\sigma$ -boundary set if  $\mu = \bigwedge_{i=1}^{\infty} (\mu_i)$ , where  $\mu_i = \text{int}^*(1 - \mu_i) \wedge (\mu_i)$  and  $(\mu_i)$ 's are fuzzy supra regular open sets in  $(U, T^*)$

**Proposition 3.11.**

If  $\mu$  is a fuzzy supra co- $\sigma$ -boundary set in a fuzzy supra P-space  $(U, T^*)$ , then  $\mu$  is a fuzzy supra open set in  $(U, T^*)$ .

**Proof.**

Let  $\mu$  be a fuzzy supra co- $\sigma$ -boundary set in  $(U, T^*)$ . Then,  $1 - \mu$  is a fuzzy supra  $\sigma$ -boundary set in  $(U, T^*)$ . Since  $(U, T^*)$  is a fuzzy supra P-space, by Proposition 3.10,  $1 - \mu$  is a fuzzy supra closed set in  $(U, T^*)$  and thus  $\mu$  is a fuzzy supra open set in  $(U, T^*)$ .

**4. Conclusion**

In this paper, we introduced and studied the concepts of fuzzy supra P-spaces. Several characterizations and properties of these spaces are established. Suitable examples are given to illustrate the concepts. It is established that fuzzy supra  $\sigma$ -boundary sets are fuzzy supra closed sets and fuzzy supra  $F_{\sigma}$ -set, fuzzy supra co- $\sigma$ -boundary sets are fuzzy supra open sets in fuzzy supra P-spaces.

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