ISSN: 2752-3829 Vol. 3 No.1, (June, 2023)

Stochastic Modelling and Computational Sciences

NEIGHBORHOOD AND CLOSED NEIGHBORHOOD PRIME LABELING OF DECOMPOSED GRAPHS

K. Sunitha[1], T.Revathi[2*]

[1]Assistant Professor, Department of Mathematics
Scott Christian College (Autonomous), Nagercoil, India - 629001.
E-mail: ksunithasam@gmail.com
[2] Research Scholar, Scott Christian College (Autonomous), Nagercoil, India- 629001.
Affiliated to Manonmaniam Sundaranar University,Tirunelveli- 627012.
E-mail: revathinathan77@gmail.com

Abstract

A decomposition of a graph G is a collection ψ_{NP} *of edge disjoint subgraphs* $H_1, H_2, \ldots, H_r$ *of G such that every edge of G belongs to exactly one* H_i. *If each* H_i *is a neighborhood prime graphs then* ψ_{NP} *is called a neighborhood prime decomposition of G. The minimum cardinality of a neighborhood prime decomposition of G is called a neighborhood prime decomposition number of G and it is denoted by* $\pi_{NP}(G)$. *In addition, If each* H_i *is a closed neighborhood prime graphs then* ψ_{CNP} *is called a closed neighborhood prime decomposition of G. In this paper, we investigate neighborhood prime decomposition of the W- graph* W_n, *generalized butterfly graph* BF_n, *regular caterpillar graph* $P_m \Theta nK_1$ *and closed neighborhood prime decomposition of cycle butterfly graph* $CB(n,m)$ *and H- graph* H_n.

Key words: *Generalized butterfly, Regular caterpillar**,** Neighborhood and Decomposition.*

***Subject Classification** : 05C78*

1. Introduction

A vertex u is a neighbor v in G, if uv is an edge of G. The set of all neighbors of v is denoted by $N(v)$; the set $N[v] = N(v) \cup \{v\}$ is the closed neighborhood of v in G. A graph is called finite if both V (G) and E(G) are finite. All graphs are considered as simple, finite, connected and undirected. Prime labeling was introduced by Tout at al [4]. Patel and Shrimali have introduced neighborhood prime labeling of graphs [6]. Rajeev Gandhi has introduced prime decomposition of graphs [5]. In this sequel, we investigate neighborhood prime decomposition of the W- graph, generalized butterfly graph BF_n, regular caterpillar graph $P_m \Theta nK_1$ and closed neighborhood prime decomposition of cycle butterfly graph $CB(n,m)$ and H- graph H_n.

Definition 1.1 Consider the two copies of the star graph $K_{1,n}$. If the last pendant vertex in the first copy of $K_{1,n}$ is merged with the initial pendent vertex in the second copy of $K_{1,n}$. Then the resulting graph is called the W- graph W_n.

Definition 1.2 A generalized butterfly graph BF_n is a double shell in which each shell has any n order and every shell has the same order. It has $2n+1$ vertices and $4n-2$ edges.

Definition 1.3 The regular caterpillar graph $P_m \Theta\, nK_1$ is obtained from the path P_m by joining nK_1 vertices to each vertex of the path P_m.

Definition 1.4. A cycle butterfly graph $CB(n,m)$ consists of two cycles of the same order n sharing a common vertex with an arbitrary number of m pendent vertices attached at the common vertex.

2. Main Results

Theorem 2.1 The decomposition of the W- graph W_n, $n \geq 3$ is a neighborhood prime graph.

Let $\psi_{NP} = \{SP(1^{n-1}, 2^1), K_{1,n-1}\}$ be a decomposition of W_n.

Let n be the positive integer and d be the decomposition number.

Let $\psi_{NP} = \begin{cases} (n-d-1)\, SP(1^{n-1}, 2^1) \,\&\, K_{1,n-1} & if\ n \equiv 1 \,(\text{mod } 2) \,\&\, d = 1, 3, 5, \ldots \\ (n-d-1)\, SP(1^{n-1}, 2^1) \,\&\, K_{1,n-1} & if\ n \equiv 0 \,(\text{mod } 2) \,\&\, d = 2, 4, 6, \ldots \end{cases}$

The decomposition of the W- graph W_n contains a spider $SP(1^{n-1}, 2^1)$ and a star $K_{1,n-1}$.

This implies that $\psi_{NP} \supseteq \{SP(1^{n-1}, 2^1) \,\&\, K_{1,n-1}\}$

That is $|\psi_{NP}| \geq |SP(1^{n-1}, 2^1)| + |K_{1,n-1}|$

Hence $\pi_{NP}[W_n] \geq 2$.

We claim that ψ_{NP} is a neighborhood prime decomposition of W_n.

Let 'α' be any vertex of $SP(1^{n-1}, 2^1)$ and $K_{1,n-1}$.

Case (i): Let $H_1 = SP(1^{n-1}, 2^1), n \geq 3$

Let $\{ v_0, v_1, v_2, \cdots, v_{n+1} \}$ be the vertices of H_1.

Define a function $\phi^+ : V(H_1) \to \{ 1, 2, 3, \cdots, n+2 \}$ by

$\phi^+(v_0) = 1$

$\phi^+(v_i) = i+1, \quad 1 \leq i \leq n+1$

Let $\alpha = v_0$ with $\deg(\alpha) \geq 3$.

Then $\gcd\{ \phi^+(w) / w \in N_V(\alpha) \} = 1$

Let $\alpha = v_n$ with $\deg(\alpha) = 2$.

Then $\gcd\{ \phi^+(w) / w \in N_V(\alpha) \} = 1$

Thus ϕ^+ admits neighborhood prime labeling.

Case (ii): Let $H_2 = K_{1,n-1}, n \geq 3$

Let $\{ u_0, u_1, u_2, \cdots, u_{n-1} \}$ be the vertices of H_2.

Define a function $\phi^+ : V(H_2) \to \{ 1, 2, 3, \cdots n \}$ by

$\phi^+(u_0) = 1$

$\phi^+(u_i) = i+1, \quad 1 \leq i \leq n-1$

Let $\alpha = u_0$ with $\deg(\alpha) \geq 2$.

Then $\gcd\{ \phi^+(w) / w \in N_V(\alpha) \} = 1$

Thus ϕ^+ admits neighborhood prime labeling.

Hence ψ_{NP} is a neighborhood prime decomposition of W_n.

Therefore the decomposition of the w- graph W_n is a neighborhood prime graph.

Theorem 2.2 The decomposition of the generalized butterfly graph $BF_n, n \geq 3$ is a neighborhood prime graph.

Proof.

Let $\psi_{NP} = \{ F_n, F_n \}$ be a decomposition of BF_n.

Let n be the positive integer and d be the decomposition number.

Then $\psi_{NP} = \begin{cases} (n-d) F_n & if \ n \equiv 1 \pmod 2 \ \& \ d = 1, 3, 5, \ldots \\ (n-d) F_n & if \ n \equiv 0 \pmod 2 \ \& \ d = 2, 4, 6, \ldots \end{cases}$

The decomposition of the generalized butterfly graph BF_n contains a Fan F_n.

This implies that $\psi_{NP} \supseteq \{ F_n, F_n \}$

That is $|\psi_{NP}| \geq |F_n| + |F_n|$

Hence $\pi_{NP}(BF_n) \geq 2$.

We claim that ψ_{NP} is a neighborhood prime decomposition of BF_n.

Let 'α' be any vertex of F_n.

Case (i): Let $H_1 = F_n, n \geq 3$

Let $\{ v_0, v_1, v_2, \cdots, v_n \}$ be the vertices of H_1.

Define a function $\phi^+ : V(H_1) \to \{ 1, 2, 3, \cdots, n+1 \}$ by

$\phi^+(v_0) = 1$

$\phi^+(v_i) = i+1, \quad 1 \leq i \leq n$

Let $\alpha = v_0$ with $\deg(\alpha) \geq 3$.

Then $\gcd\{ \phi^+(w) / w \in N_V(\alpha) \} = 1$

Let $\alpha = \{v_i / 1 \le i \le n\}$ with $\deg(\alpha) = 2\,(or)\,3$.

Then gcd$\{\ \phi^+(w)/w \in N_V(\alpha)\ \} = 1$

Thus ϕ^+ admits neighborhood prime labeling.

Hence ψ_{NP} is a neighborhood prime decomposition of BF_n.

Therefore the neighborhood prime decomposition of the generalized butterfly graph $BF_n, n \ge 3$ is neighborhood prime graph.

Theorem 2.3 The decomposition of the regular caterpillar graph $P_m \Theta\, nK_1, m \ge 3, n \ge 2$ is a closed neighborhood prime graph.

Proof.

Let $\psi_{CNP} = \left\{\ P_m, K_{1,n}, K_{1,n}, ..., K_{1,n}(m\, times)\right\}$ be a decomposition of $P_m \Theta\, nK_1$.

Let m, n be the positive integers and d be the decomposition number.

Then $\psi_{CNP} = \begin{cases} (n+m-d-1)\,K_{1,n}\ \&\ Pm \quad if\ m \equiv 1\,(\text{mod } 2)\,, n = 2, 3, ...\ \&\ d = 1, 2, 3, ... \\ (n+m-d-1)\,K_{1,n}\ \&\ Pm \quad if\ m \equiv 0\,(\text{mod } 2)\,, n = 2, 3, ...\ \&\ d = 1, 2, 3, ... \end{cases}$

The decomposition of the regular caterpillar graph $P_m \Theta\, nK_1$ contains a star graph $K_{1,n}$ and a path graph P_m.

This implies that $\psi_{CNP} \supseteq \{P_m, K_{1,n}, K_{1,n}, ..., K_{1,n}(m \sim times)\}$

That is $|\psi_{CNP}| \ge |P_m| + m\,|K_{1,n}|$

Hence $\pi_{CNP}(BF_n) \ge m+1$.

We claim that ψ_{CNP} is a closed neighborhood prime decomposition of $P_m \Theta\, nK_1$.

Let 'α' be any vertex of $K_{1,n}$ and P_m.

Case (i): Let $H_1 = K_{1,n}, n \ge 2$

Let $\{\ u_1, u_2, u_3, \cdots, u_{n+1}\ \}$ be the vertices of H_1.

Define a function $\phi^+ : V(H_1) \to \{\ 1, 2, 3, \cdots, n+1\ \}$ by

$\phi^+(u_1) = 1$

$\phi^+(u_{i+1}) = i+1, \quad 1 \le i \le n$

Let $\alpha = u_1$ with $\deg(\alpha) \ge 2$.

Then gcd$\{\phi^+(a),\ \phi^+(w)/w \in N_V[\alpha]\ \} = 1$

Thus ϕ^+ admits closed neighborhood prime labeling.

Hence ψ_{CNP} is a closed neighborhood prime decomposition of $P_m \Theta\, nK_1$.

Therefore the closed neighborhood prime decomposition of $P_m \Theta\, nK_1, m \ge 3, n \ge 2$ is closed neighborhood prime graph.

Theorem 2.4. The decomposition of the cycle butterfly graph $CB(n,m), m \geq 2, n \geq 3$ is a closed neighborhood prime graph.

Proof.

Let $\psi_{CNP} = \{ C_n, C_n, K_{1,m} \}$ be a decomposition of $CB(n,m)$.

Let m, n be the positive integers and d be the decomposition number.

Then $\psi_{CNP} = \begin{cases} (m-d+1)\, C_n \ \&\ K_{1,m} & if\ m \equiv 0 \,(\text{mod } 2),\, n = 3, 4, 5, \ldots \& \ d = 1, 2, 3, \ldots \\ (n+m-d-1)\, K_{1,n} \ \&\ Pm & if\ m \equiv 1 \,(\text{mod } 2),\, n = 3, 4, 5, \ldots \ \&\ d = 1, 2, 3, \ldots \end{cases}$

The decomposition of the cycle butterfly graph $CB(n,m)$ contains a cycle graph C_n and a star graph $K_{1,m}$.

This implies that $\psi_{CNP} \supseteq \{C_n, C_n, K_{1,m}\}$

That is $|\psi_{CNP}| \geq 2|C_n| + |K_{1,,m}|$

Hence $\pi_{CNP}(CB(n,m)) \geq 3.$

We claim that ψ_{CNP} is a closed neighborhood prime decomposition of $CB(n,m)$.

Let α be any vertex of C_n and $K_{1,n}$.

Case(i): Let $H_1 = C_n, n \geq 3$

Let $\{ v_1, v_2, v_3, \cdots, v_n \}$ be the vertices of H_1.

Define a function $\phi^+ : V(H_1) \to \{ 1, 2, 3, \cdots, n \}$ by

$\phi^+(v_i) = i, \quad 1 \leq i \leq n$

Let $\alpha = \{v_i / 1 \leq i \leq n\}$ with $\deg(\alpha) = 2$.

Then $\gcd\{\phi^+(a),\ \phi^+(w) / w \in N_V[\alpha]\} = 1$

Thus ϕ^+ admits closed neighborhood prime labeling.

Case(ii): Let $H_2 = K_{1,m}, m \geq 2$

Let $\{ w_0, w_1, w_2, \cdots, w_m \}$ be the vertices of H_2.

Define a function $\phi^+ : V(H_2) \to \{ 1, 2, 3, \cdots, m+1 \}$ by

$\phi^+(w_0) = 1,$

$\phi^+(w_i) = i+1, \quad 1 \leq i \leq m$

Let $\alpha = w_0$ with $\deg(\alpha) \geq 2$.

Then $\gcd\{\phi^+(a),\ \phi^+(w) / w \in N_V[\alpha]\} = 1$

Thus ϕ^+ admits closed neighborhood prime labeling.

Hence ψ_{CNP} is a closed neighborhood prime decomposition of $CB(n,m)$.

Therefore the closed neighborhood prime decomposition of $CB(n,m), m \geq 2, n \geq 3$ is a closed neighborhood prime graph.

3. Conclusion

In this paper , we investigate neighborhood and closed neighborhood prime decomposition of some graphs. In future, we will investigate various types of labeling using various graphs.

References

[1] Joseph .A. Gallian, "A dynamic survey of graph labeling", The Electronic Journal of Combinatorics (2018), #DS6.

[2] Sugumaran .A and Mohan .V ,Further Results on Prime Cordial Labeling, Annals of Pure and Applied Mathematics- Vol.14, No. 3, 2017, 489-496.

[3] Uma .R and Amuthavalli .D, Fibonacci Graceful Labeling of some Star Related Graphs , International Journal of Computer Applications-Volume 134-No.15,2016.

[4] Tout. A, Dabboucy and Hawalla . K, Prime labeling of graphs, Nat. Acad. Sci. Letters11(1982).

[5] Dr.Rajeev Gandhi .S, Decomposition of various graphs in to prime graphs, Mathematical Statistician and Engineering Applications-Vol.71 No.4(2022).

[6] S.K. Patel, N.P. Shrimali, "Neighborhood Prime labeling", International Journal of Mathematics and Soft Computing, Vol. 5, No. 2(2015),135-143.

[7] Sugumaran .A and Suresh .K ,A special class of prime cordial graphs, , International Journal of Pure and Applied Mathematics- Vol.114, No. 6, 2017.