FINITE ELEMENT ANALYSIS OF LAMINATED COMPOSITE STRUCTURES WITH DISCRETE LAYER APPROACH

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ABSTRACT

This paper presents a parametric study on laminated plates including the effect of material anisotropy, laminate thickness and stacking sequence on the induced interlaminar stresses under uniaxial strain. By a rational analysis of the numerical work carried out, various factors to be considered while deciding the stacking order of different layers having specific orientations for a given laminate which leads to an optimum protection against delamination are addressed. The formulation follows Reddy's layerwise theory and laminates with finite dimensions are considered for the purpose of study.

Keywords: composite material, laminated plate, stacking sequence, interlaminar stresses, finite element analysis.

INTRODUCTION: The use of composite materials has increased manyfold in weight-sensitive applications such as aerospace and automotive structures because of their high specific strength and high specific stiffness properties. The increased use of laminated plates in various structures has stimulated considerable interest in their accurate analyses. This field has been very extensively studied for several decades and is still a subject of research interest as it is extremely difficult to find efficient modeling methods for multilayered structures with layers in the thickness direction and having mismatch in their elastic properties, thermal and hygroscopic expansion coefficients. Such a situation causes interlaminar stress concentrations near the free-edges of the laminate, where we have material discontinuities and these stresses can initiate heterogeneous damage in the form of delamination, transverse cracking and even may cause the damage to propagate to a substantial region of the laminate. Delamination sometimes called interlaminar cracking resulting from high interlaminar stress concentrations near the free edges is one of the most frequently encountered types of damage in advanced composite materials. The presence and growth of delamination cracks in composite laminates may lead to severe reliability and safety problems such as reduction of structural stiffness, exposure of the interior to the adverse environment and disintegration of the material which can cause the final failure. Several investigators have reported experimental data.
which indicates that the strength of symmetric composite laminates containing identical ply orientations can be strongly dependent on the detailed stacking sequence. Kaminski\textsuperscript{5} has reported data on the static tensile strength of a large sampling of $0^\circ$ / $90^\circ$ glass-epoxy coupons with the $0^\circ$ and $90^\circ$ layers on the outer surfaces and found, with low scatter in the data, that the strength of the latter group was 9\% higher. The solution presented by Puppo and Evensen\textsuperscript{6} indicate that while lamination theory (LT) gives a very realistic portrayal stress field in regions remote from a boundary, it fails in boundary layer regions, where significant interlaminar stresses are developed. It is therefore a strong possibility that this unique behavior can be attributed to the degradation caused by delamination triggered by interlaminar stresses. The possible mechanism which can explain strength dependence on stacking sequence is the constraining influence of adjacent layers on the propagation of a crack.* Corresponding author, E-mail: bandi.rambabu@gmail.com

on a given layer or at an interface. It has been hypothesized\textsuperscript{7} that the stress component primarily responsible for the unusual failure mode of fibrous composite laminates i.e. catastrophic delamination under a uniform axial extension is the interlaminar normal stress $\sigma_z$. Several approaches have been developed based mainly on either equivalent single layer (ESL) or layer by layer descriptions depending on the accuracy required for studying the behavior of laminated composite plates under different loading conditions\textsuperscript{8}. The equivalent single layer (ESL) theory will give a sufficiently accurate description of the global laminate response (e.g. transverse deflection, fundamental frequency and critical buckling load etc.), however these theories are often inadequate for determining 3D stress field at the ply level. In all the equivalent single layer theories it is assumed that the displacements are continuous function of thickness coordinate, which in turn results in continuous transverse strains and stresses computed using constitutive relations being discontinuous at the interface between two dissimilar layers. Unlike the equivalent single layer theory, the layerwise theory assumes separate displacement field expansions within each material layer that exhibits only $C^0$-continuity through the laminate thickness, thus providing a kinematically consistent representation of the strains in discrete layer laminates, and allowing for the calculation of ply-level stresses more accurately. Therefore, only layerwise theory can lead to consistent descriptions of localized effects.\textsuperscript{9}

Among the displacement-based refined theories that are available in the literature the first one is due to Basset\textsuperscript{10}. Basset assumed that the three displacement components in a shell can be expanded as a linear combination of the thickness co-ordinate and unknown functions of position in the reference surface of the plate. All laminate plate theories derived from the Basset type expansion assume that the displacements vary through the thickness of the laminate according to a single expression, not allowing for possible discontinuities in the slopes of the deflections at the interface of two individual laminae. Reddy\textsuperscript{11} presented a laminate plate theory that allows piecewise representation of displacements through individual laminae of a laminated plate. In the present work, the effect of aspect ratio on central deflection under the transverse
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loading condition, the distribution of interlaminar stresses through the thickness and through the width of the laminate is analyzed under a uniform axial extension for the symmetric laminates by applying finite element procedures to the Reddy’s layerwise theory. Four layered laminate is considered to study the effect of anisotropy, laminate thickness and an eight layered laminate is considered to study the effect of stacking sequence on the induced interlaminar stresses.

THEORY: A laminated plate composed of N orthotropic laminae, each being oriented arbitrarily with respect to the laminate (x, y) co-ordinates, as shown in Figure 1 is considered. The analysis is restricted to linear elastic material behavior, small strains and displacements. The displacement field expressed in Reddy’s\textsuperscript{12} full layerwise theory is given as

\[ u(x, y, z) = U_i(x, y) \phi_i(z), \]
\[ v(x, y, z) = V_i(x, y) \phi_i(z), \]
\[ w(x, y, z) = W_i(x, y) \psi_i(z). \]

In Eq. (1) \( u, v, w \) represents the displacement components in the \( x, y \) and \( z \) directions respectively, of a material point initially located at \((x, y, z)\) in the undeformed laminate. Also \( U_i(x, y), V_i(x, y) \) and \( W_i(x, y) \), where \((i=1, 2, \ldots, N+1)\), represent the displacement components of all points located on the \( i^{th} \) plane in the undeformed laminate, and \( \phi_i(z) \), \( \psi_i(z) \) are the continuous functions of the thickness coordinate \( z \) (global interpolation functions used for the discretization of the in-plane and transverse displacements respectively). Depending upon the polynomial order (linear, quadratic, cubic etc.) of the interpolation functions, Eq. (1) exhibit piecewise polynomial variation. It is to be noted that the repeated index indicates summation over all values of that index. Upon substitution of Eq. (1) into the linear strain-displacement relation\textsuperscript{13} of elasticity the following results will be obtained

\[ \varepsilon_x = \frac{\partial U_i}{\partial x} \phi_i, \varepsilon_y = \frac{\partial V_i}{\partial y} \phi_i, \varepsilon_z = \frac{\partial W_i}{\partial z} \psi_i, \]
\[ \gamma_{yz} = V_i \frac{d \phi_i}{dz} + \frac{\partial W_i}{\partial y} \psi_i, \gamma_{xz} = U_i \frac{d \phi_i}{dz} + \frac{\partial W_i}{\partial x} \psi_i, \]
\[ \gamma_{xy} = \left( \frac{\partial U_i}{\partial y} + \frac{\partial V_i}{\partial x} \right) \phi_i \]

EQUATIONS OF MOTION: The governing equations of motion for the present theory can be derived using the principle of virtual displacements

\[ \mathbf{O} = \delta \mathbf{U} + \delta \mathbf{V}. \]

Where \( \delta \mathbf{U}, \delta \mathbf{V} \) are the virtual strain energy and virtual work done by applied forces respectively. The \( 3(N+1) \) equations of equilibrium corresponding to \( 3(N+1) \) unknowns \( U_i, V_i \) and \( W_i \) can be given\textsuperscript{9} as
\[
\frac{\partial N^i_x}{\partial x} + \frac{\partial N^i_{xy}}{\partial y} - Q^i_x = 0,
\]

\[
\frac{\partial N^i_{xy}}{\partial x} + \frac{\partial N^i_y}{\partial y} - Q^i_y = 0,
\]  \hspace{1cm} \text{.................(4) where } i = 1, 2

\[
\frac{\partial R^i_x}{\partial x} + \frac{\partial R^i_y}{\partial y} - N^i_z = 0.
\]

...N+1.

In Eq. (4) \(N^i_x, N^i_y, N^i_z, N^i_{xy}, Q^i_x, Q^i_y, R^i_x\) and \(R^i_y\) are the generalized stress resultants. The linear constitutive relations for the \(k\)th orthotropic lamina with the fiber orientation \(\alpha\) with respect to the laminate coordinate axes (Figure 1) are given\(^{14}\) by

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yz} \\
\tau_{xz} \\
\tau_{xy}
\end{bmatrix}^k =
\begin{bmatrix}
-C_{11} & -C_{12} & -C_{13} & 0 & 0 & -C_{16} \\
-C_{21} & -C_{22} & -C_{23} & 0 & 0 & -C_{26} \\
-C_{31} & -C_{32} & -C_{33} & 0 & 0 & -C_{36} \\
0 & 0 & 0 & C_{44} & C_{45} & 0 \\
0 & 0 & 0 & C_{54} & C_{55} & 0 \\
-C_{61} & -C_{62} & -C_{63} & 0 & 0 & -C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{xz} \\
\gamma_{xy}
\end{bmatrix}
\]

In the Eq. (5), \(C_{ij}\) are the transformed elastic coefficients in the \((x, y, z)\) system, which are related to the elastic coefficients, \(C_{ij}\) in the material coordinate \((L, T)\) system. The laminate constitutive equations for the layerwise theory can be derived using the lamina constitutive equations Eq. (5), stress resultants and finally we obtain

\[
\begin{bmatrix}
N^i_x \\
N^i_y \\
N^i_{xy}
\end{bmatrix} = \sum_{j=1}^{N+1} \begin{bmatrix}
A_{11}^{ij} & A_{12}^{ij} & A_{13}^{ij} & A_{16}^{ij} \\
A_{12}^{ij} & A_{22}^{ij} & A_{23}^{ij} & A_{26}^{ij} \\
A_{13}^{ij} & A_{23}^{ij} & A_{63}^{ij} & A_{66}^{ij}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial U_j}{\partial x} \\
\frac{\partial V_j}{\partial x} \\
\frac{\partial V_j}{\partial y} \\
\frac{\partial U_j}{\partial y} + \frac{\partial V_j}{\partial x}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{\partial U_j}{\partial x} \\
\frac{\partial V_j}{\partial x} \\
\frac{\partial V_j}{\partial y} \\
\frac{\partial U_j}{\partial y} + \frac{\partial V_j}{\partial x}
\end{bmatrix}
\]  \hspace{1cm} \text{......}
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\[ Q_x^i = \sum_{j=1}^{N+1} \begin{pmatrix} A_{14}^{ij} & A_{15}^{ij} \end{pmatrix} \begin{pmatrix} V_j \end{pmatrix} + \begin{pmatrix} B_{14}^{ij} & B_{15}^{ij} \end{pmatrix} \begin{pmatrix} \frac{\partial W_j}{\partial x} \end{pmatrix} \]

\[ R_y^i = \sum_{j=1}^{N+1} \begin{pmatrix} B_{44}^{ij} & B_{45}^{ij} \end{pmatrix} \begin{pmatrix} V_j \end{pmatrix} + \begin{pmatrix} D_{44}^{ij} & D_{45}^{ij} \end{pmatrix} \begin{pmatrix} \frac{\partial W_j}{\partial y} \end{pmatrix} \]

\[ N_i^j = \sum_{j=1}^{N} \begin{pmatrix} A_{13}^{ij} \frac{\partial U_j}{\partial x} + A_{23}^{ij} \frac{\partial V_j}{\partial y} + A_{33}^{ij} \frac{\partial W_j}{\partial z} + A_{36}^{ij} \left( \frac{\partial U_j}{\partial y} + \frac{\partial V_j}{\partial x} \right) \end{pmatrix} \]

The stiffnesses of the laminate \( A_{ij} \), \( B_{ij} \), \( B_{ij} \), \( A_{ij} \), \( A_{ij} \) and \( D_{ij} \) are defined in Reddy12.

**FINITE ELEMENT FORMULATION:** The displacement finite element model is developed by substituting the assumed displacement field into the principle of virtual displacements Eq. (3) for a representative finite element of the plate. The displacement field is

\[ U_i(x, y) = \sum_{j=1}^{m} U_i^j N_j(x, y), \]
\[ V_i(x, y) = \sum_{j=1}^{m} V_i^j N_j(x, y), \]
\[ W_i(x, y) = \sum_{j=1}^{m} W_i^j N_j(x, y). \]

To approximate the in-plane displacements and transverse deflection, same number of nodes is considered for the in-plane 2-D element. Accordingly the Lagrangian interpolation polynomial, \( N_i(x, y) \) (where \( j = 1, 2, \ldots, m \)) can be written down for the 2-D element and \( U_i^j \), \( V_i^j \) and \( W_i^j \) which denote the values of the displacements \( U_i \), \( V_i \) and \( W_i \) at the \( j \)th node of the 2-D finite element representing the \( i \)th plane of the plate element can be represented. Eight-noded serendipity quadrilateral element with an isoparametric formulation in conjunction with quadratic 1-D Lagrangian interpolation through the thickness direction has been used. The same Quadratic 1-D Lagrangian interpolation function is used for in-plane as well as transverse displacements, which are given as
\[ \phi_i^j(z) = \psi_i^j(z); \quad z_1 \leq z \leq z_3 \\
\phi_{2i}^j(z) = \psi_{2i}^j(z), \quad z_{2i-1} \leq z \leq z_{2i+1}; (i = 1, 2,...N) \\
\phi_{2i+1}^{j+1}(z) = \begin{cases} 
\psi_{3i}^{j+1}(z), & z_{2i-1} \leq z \leq z_{2i+1} \\
\psi_{1i+1}^{j+1}(z), & z_{2i+1} \leq z \leq z_{2i+3} 
\end{cases}; (i = 1, 2,...N - 1) \quad \ldots \ldots \quad (11) \\
\phi_N^j(z) = \psi_{3i}^N(z), \quad z_{N-1} \leq z \leq z_{2i+3} \\

\text{where} \quad \psi_{1i}^k = \left(1 - \frac{z}{h_k}\right) \left(1 - \frac{2z}{h_k}\right), \\
\psi_{2i}^k = \frac{4z}{h_k} \left(1 - \frac{z}{h_k}\right), \\
\psi_{3i}^k = -\frac{z}{h_k} \left(1 - \frac{2z}{h_k}\right), \quad z_{2k-1} \leq z \leq z_{2k+1} \quad \ldots \ldots \quad (12) \\

\text{The shape functions } N_j (j = 1, 2,...,8) \text{ for an Eight-noded Quadrilateral serendipity element can be found in any standard book on finite element analysis, (R. D. Cook, David S. Malakus and Michael E. Plesha\textsuperscript{15}). In calculating the element stiffness matrices full quadrature rule for bending energy terms and one order less for the shear energy terms are considered in the present work.}

**RESULTS AND DISCUSSION:** For the purpose of numerical study the following example problems are chosen

**Example: 1.** Four layered \([0^0/90^0]\), simply supported square plate of equal layer thickness subjected to the doubly sinusoidal transverse load of intensity, \(q = q_0 \sin (\pi x / a) \sin (\pi y / b)\). The material (Graphite–epoxy) properties are \(E_L / E_T = 40, G_{LT} / E_T = 0.6, G_{TT} / E_T = 0.5, \nu_{LT} = 0.25\). This problem is studied for the purpose of comparing the central deflection obtained using layerwise theory with that of Suresh C Panda and R Natarajan\textsuperscript{16} as well as to verify the developed computer code of the finite element program. Figure 2 shows the variation of central deflection with respect to the plate aspect ratio and good agreement was found with the finite element results of Suresh C. Panda and R. Natarajan.

**Example: 2.** Four layered symmetric laminate \([45^0/-45^0]\), with the laminate configuration, \(a = 10b, b = 4h\) and \(h_k = h/4\) and with the given boundary conditions \(u(a, y, z) = u_0, u(0, y, z) = v(0, y, z) = w(0, y, z) = 0\) is considered to study the effect of material anisotropy on the interlaminar stresses. By varying the material anisotropy, \(E_L / E_T\) from 20 to 60 with \(E_T = 1 \times 10^6\) psi (6.894757 GPa), \(G_{LT} = G_{TT} = 0.85 \times 10^6\) psi (5.860544 GPa) and \(\nu_{LT} = \nu_{TT} = 0.21\) the interlaminar stresses are calculated. The results obtained are shown in Figures 3 to 5 and it is clear that as material anisotropy increases, the interlaminar normal and shear stresses behave in contradicting manners. The change in the interlaminar stresses for the two different anisotropy ratios 20
and 60 is quantified in Table 1. The primary reason for the delamination being the high interlaminar normal stress, it is necessary to use the laminae with higher material anisotropy to protect the laminate against delamination under uniaxial strain.

**Example: 3.** Four layered symmetric laminate [45°/-45°]_s, with the assumed laminate configuration, a = 10b, b = 16 and h₀ = h/4 is considered to study the effect of laminate thickness on the interlaminar stresses. By varying the laminate thickness, h from 1 to 5, the variation of interlaminar stresses through the width of the plate is shown in Figures 6 to 8 and the change in magnitude in the interlaminar stresses for the two laminate thicknesses 2 and 4 is quantified in Table 2. It is clear that, as the thickness increases for the given in-plane dimensions of the laminate, the induced interlaminar normal stress not only increases but also the stress concentration region extends from the free edge towards interior of the laminate.

**Example: 4.** An eight layered [90°/45°/-45°/0°]_s laminate with the assumption that a = 10b and b = 15h and h₀ = h/8 is considered to study the effect stacking sequence on the induced interlaminar stresses for a given laminate. By interchanging the layers within the laminate without disturbing mid-plane symmetry, the interlaminar stresses are calculated and the variation of these stresses for each of the stacking sequence through the thickness and along the width of the plate are shown in Figures 9 to 11 and 12 to 14 respectively. Also the change in the induced interlaminar stresses for the two stacking sequences obtained by interchanging 0° layer with the 90° layer is quantified in Table 3. From the Figures it can be found that the induced interlaminar stresses are smaller when the 0° layer is located near the mid-plane. By observing the variation of interlaminar stresses through the width of the laminate it is clear that the stress concentration effect extends over a region with its width approximately equal to the thickness of the laminate as it is generally observed in the literature.

For the example problems 3 and 4 the material properties are E_L = 20e⁶ psi (137.8951 GPa), E_T = 2.1e⁶ psi (14.47899 GPa), G_LT = G_TT = 0.85e⁶ psi (5.860544 GPa), and \( \nu_{LT} = \nu_{TT} = 0.21 \), and the boundary conditions are given as \( u(a, y, z) = u₀, u(0, y, z) = v (0, y, z) = w (0, y, z) = 0 \). It is to be noted that the data presented in Tables 1 to 3 correspond to the free edge of laminate.

**SUMMARY AND CONCLUSIONS:** Finite element isoparametric formulation for an eight-noded serendipity quadrilateral element is presented which uses a layerwise theory assuming separate displacement field expansions within each material layer that exhibits only C⁰-continuity through the laminate thickness. From the numerical studies conducted, various factors affecting the nature as well as the magnitude of induced interlaminar stresses in providing an optimum protection against delamination for the given laminate have been discussed. Based on the above discussions the following conclusions can be drawn

- Material anisotropy can optimally be put in to application by realizing the fact that, under uniaxial strain laminate with higher material anisotropy gives lesser values of interlaminar stresses in more explicitly; by using highly anisotropic material one will be able to reduce delamination.
As the thickness of the laminate increases the magnitudes of induced interlaminar stresses also increases. Moreover the free-edge effect extends over a region approximately equal to the thickness of the laminate. These observations are well addressed in the literature and the present study confirms the same.

In a symmetric laminate by interchanging the laminae within the laminate one can control the induced interlaminar stresses. By placing the 0° lamina near the mid-plane the magnitude of interlaminar normal stress can be minimized. By a proper analytical study the stacking sequence can be predicted.

**NOTATION:**

\( a, b = \) length and width of the plate respectively.

\( E_L, E_T = \) modulus of elasticity in L and T directions respectively.

\( G_{LT} = \) modulus of elasticity in L - T plane.

\( V_{LT} = \) Poisson’s ratio giving strain in \( T \) direction caused by a strain in \( L \) direction.

\( V_{TT} = \) Poisson’s ratio giving strain in \( T \) direction caused by a strain in \( T \) direction.

\( h_k = \) thickness of the \( k^{th} \) layer of the plate.

\( L, T = \) directions parallel and normal to the fiber direction respectively.

\( N = \) total no. of layers.

\( q = \) intensity of transverse load on the plate.

\( h = \) total thickness of the plate.

\( u, v, w = \) displacements along plate axes \( x, y \) and \( z \).

\( u_0 = \) axial displacement in \( x \)-direction.

\( \alpha = \) angle between fiber direction and \( x \) axis.

\( \varepsilon = \) normal strain. \( \gamma = \) shear strain.

\( \sigma = \) normal stress. \( \tau = \) shear stress.

The normalized quantities are

\[
\sigma^{*}_{XZ} = \sigma_{XZ\text{max}} \times 20 \times \frac{a}{u_0 / E_L}, \quad \sigma^{*}_{Z} = \sigma_{Z\text{max}} \times 20 \times \frac{a}{u_0 / E_L},
\]

\[
\sigma^{*}_{YZ} = \sigma_{YZ\text{max}} \times 20 \times \frac{a}{u_0 / E_L}, \quad w^* = 1e^3 \times w_{\text{max}} \times \frac{E_T \times h^3 / q_0}{a^4}.
\]
REFERENCES:


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<thead>
<tr>
<th>Stress</th>
<th>Anisotropy</th>
<th>% Change</th>
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<td>$E_L / E_T = 20$</td>
<td>$E_L / E_T = 60$</td>
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<tr>
<td>$\sigma^*_{z}$</td>
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<tr>
<td>$\sigma^*_{yz}$</td>
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<td>$\sigma^*_{xz}$</td>
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Table 2. Effect of Laminate ( [45/-45]s ) thickness, h

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<td>h = 4</td>
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Table 3. Effect of Stacking sequence

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<tr>
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<td>0.948985</td>
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<tr>
<td>-0.06738</td>
<td>-0.06421</td>
<td>4.70</td>
</tr>
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</table>

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Figure 5. Variation of interlaminar shear stress through the width of the plate.
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Figure 7. Variation of interlaminar shear stress through the width of the plate.

Figure 8. Variation of interlaminar shear stress through the width of the plate.

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Figure 11. Variation of interlaminar shear stress through the thickness of the plate.

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Figure 14. Variation of interlaminar shear stress through the width of the plate.