SOME REFLECTIONS ON ARGUMENTATIONS AND MATHEMATICS IN SCENARIOS DEVOID OF ARISTOTELIAN INFLUENCE AND THEIR IMPORTANCE IN THE MATHEMATICS CLASSROOM

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Abstract

This paper presents a characterization of cultural scenarios that are devoid of Aristotelian influence and whose argumentations differ from those originated in Greece. In these cultures, mathematical concepts such as the zero and infinity - whose scientific acceptance and treatment required various centuries in Western culture - were addressed and elaborated. The cultural construction nature of forms of argumentation is evidenced, as well as the possibility of constructing mathematical concepts on the basis of other forms of thought. Additionally, certain forms of argumentation are detected in the mathematics classroom which are incorrect from the viewpoint of Aristotelian logic, and which are neither learned nor elaborated in scholastic scenarios.

Our Aristotelian-based culture has constructed specific forms of argumentation. The socioepistemological approach used in this research allows us to propose the need to pay attention to the non-classical forms of argumentation that arise in the classroom, and analyze how they could contribute to the construction of scholar mathematical concepts.

Keywords: argumentation, Aristotelian influence, socio-cultural construction, zero, infinitum

MSC classification : 97D20

1 INTRODUCTION

With the aim of understanding the socio-cultural nature of mathematical argumentations, in the present research we seek to display them as a result of the actions of a given community within a given socio-cultural scenario (Crespo Crespo, 2005, 2007a, 2007b; Crespo Crespo & Farfán, 2005, Crespo Crespo, Farfán & Lezama, 2010). Our culture has Aristotelian foundations and has constructed forms of argumentation that are based on this logic and which have been considered for centuries to be typical of human thought. However, on occasion certain situations arise in the mathematics classroom that reveal the quality of mathematical argumentation as a social construction, which in our opinion should be taken into account in scholastic mathematical discourse.

Ever since Aristotle systematized logical argumentations through the laws of classical logic, these laws have been identified as the laws of human thought and the laws that govern scientific developments and progress. These laws, which have been considered indisputable for centuries, have governed the paths of Western scientific thought. If they truly were typical of human reasoning, they should have been present in every culture and epoch. They should have been enunciated and accepted in different socio-cultural scenarios and, therefore, scientific developments in every culture should be governed by them. However, as stems from the descriptions presented below, this did not happen. Certain socio-cultural scenarios gave birth to expressions of logic that do not accept Aristotelian principles such as the principle of non-contradiction and the principle of the third excluded, and their forms of thought were not based on these principles. Evidently, these sociocultural scenarios did not give rise to argumentations such as the reduction to the absurd that are strongly rooted in the Aristotelian principles referred to.

2 CULTURES DEVOID OF ARISTOTELIAN INFLUENCE: THEIR LOGICAL THOUGHT AND SOME MATHEMATICAL CONCEPTS

In our present society it is difficult to imagine any form of scientific progress detached from the principles of logic identified by the Greeks, since our form of scientific thought has been constructed on those foundations. Some civilizations, however, did not have the same bases and yet were able to build mathematical concepts in a manner that differs from that of Western science. Below are some examples of this, which focus mainly on civilizations in which two concepts emerged whose construction in the West was arduous: the zero and infinity.

Mathematics is now understood as a kind of cultural knowledge, "which all cultures generate, but which need not necessary 'look' the same from one cultural group to another" (Bishop, 1988, 180).

2.1 Ancient Egypt

Although the birthplace of philosophy is attributed to Greece, it is possible to identify pre-philosophical characteristics in Ancient Egyptian thought, especially relative to certain conceptions of the universe and divinity. These are identified in the forms of thought of priests, who theorized and acted as the depositaries and transmitters of knowledge in that culture. Egyptian religious thought was physical and metaphysical, and gave rise to ritual techniques and non-religious instructions that lived on in ritual texts or propaganda aimed at ensuring the favorable progress of the Cosmos and guaranteeing social and agricultural prosperity, without the explicit aim of encouraging personal reflection (Parain, 2002).

In the Ancient Empire, it was necessary to coordinate the traditions that had arisen earlier, where myths narrated the same phenomena using different imagery and Gods took on different and contradicting identities. From an Aristotelian stand, this would have led to incoherencies and contradictions based on the principles of identity and non-contradiction. However, a polymorphism of deities developed unhindered in Egypt, without generating any inconsistencies or contradictions, and enabling the construction of theoretical explanations suited to the scientific conception of this scenario, encompassing the sacred and the profane facets alike. Consequently, it is possible to identify situations where non-contradiction was not necessary.

Egyptian culture left no traces of strict demonstrations of mathematical results or of logical argumentations that justify procedures presented in calculation techniques. Mathematical development in this culture reveals the relationship between the material needs of a society and the nature of the mathematics developed by it, without displaying any interest in the generalization or abstraction or systematical organization of knowledge. In its scientific vision, an orderly sequence of steps is offered to explain and define, and verification is added by way of conclusion that leads to a correct solution of the problem.

It has often been said that the concept of zero cannot be found in Ancient Egypt, however, certain mathematical historians believe that a predecessor of the zero concept lies in a symbol that was used to express the notions of beauty, completeness and perfection which appears in the buildings plans of temples, palaces and large buildings. These plans feature horizontal floor-level lines to guide the construction that symbolize ground level or zero level. Similarly, this symbol is also found in mathematical operations in the accounting records of the 13th. Dynasty of the Middle Kingdom when the accounts were balanced. These two uses of this symbol can be understood to be associated with the zero, as a manner of symbolizing equilibrium.

2.2 India

The oldest source of information on Indian thought that survived to the present day are the Vedas, a series of works that span various literary periods and which contain part of the religious and popular poetry existent during the Vedic period. These presented deities that personified the various forces of nature and were revered in a monotheistic conception presented through several deities. This current of Hindu thought gave birth most importantly to naturalism and two other schools of thought considered non-Vedic: Jainism and Buddhism.

By the IIIrd. or IVth. Century b.C. a significant amount of heterogeneous philosophical material had been gathered in the sũtras, whose function was to consolidate the doctrine of a specific school and criticize others that were opposed to it. Naturalist logic generates "*a science of trial as well as of discovery*" (Hiriyanna, 1960, p.52) and is carried out not through reasoning but rather through sensory perception. Naturalistic inference should not be confused with syllogistic forms, since it involves a process of search as the source of knowledge in terms of the perception of signs and their possible meaning, rather than in terms of logical argumentation. Nyaya logic values rational speculation as the basis of a coherent doctrine on knowledge. Although its basis was empirical, it generated a theory of rational reasoning based on causality.

Jainism, one of the most ancient forms of non-Vedic religion in India, is characterized by the belief in the independent and eternal existence of spirit and matter – "life" and "nonlife". Knowledge or conscience is the essence of the spirit, and empirical knowledge is one of its manifestations subject to the limitations of inanimate nature; true perceptions occur by intuition. Jainism was concerned with the relationship between the rationality and the coherence of thought, based on logics of a reconciling nature that differed from Greek thought, which accepted pluralism and skepticism (Ganeri, 2002).

The third stage of Indian philosophical thought corresponds to Buddhism, which originated as a religion and later was forced to become a philosophy in order to defend its stand relative to the Hindu and Jaina schools of thought. In philosophical terms, Buddhism conceived things as unstable and changing, and viewed stability as an illusion or a figment of the mind. Buddhists considered that nothingness and emptiness are not synonyms. They identified 25 different types of emptiness (Ifrah, 1997), which established the basis for the concept of zero - one of the legacies of India to the Western world. Nothingness and zero arise from a philosophical rather than a mathematical view: as the absence of something, as opposed to the result of an operation; as the cardinal of the empty set, which is characterized for containing no elements. Subsequently, it would become a figure, like a number, like the representation of an empty place in a number.

Nevertheless, references to the zero as a figure do not appear explicitly in the Vedas, nor does any word that identifies this concept. This happened later. Sunya was the representation of nothingness, emptiness, empty space; an empty, unoccupied place (de Mora & Jarocka, 2003). The relationship that arose during this period between Nothingness and the Being, and the possibility of one becoming the other, was notable. This differed radically from the Greek stand and from the eastern conception of nothingness, as opposed to Greek philosophy.

The Jaina, for their part, became familiar with numerical speculations using large figures, having classified numbers that comprised eighty and even one hundred figures as small. Concepts such as "impossible to count" our "countless", "innumerable" and "impossible to fathom" appeared and, finally, the concept of infinity (Ifrah, 1997). The Jaina classified numbers as: numerable, innumerable and infinite. Numerable numbers could be: minimums, intermediates and maximums; the innumerable: almost numerable, truly innumerable and innumerably innumerable; infinite numbers: almost infinite, truly infinite and infinitely infinite (de Mora & Jarocka, 2003). The Jaina recognized the zero and infinity as inverse concepts: dividing by zero was equivalent to infinity. They defined infinity as the quantity that is not modified at all when finite numbers are added to it or subtracted from it.

2.3 China

The primitive people of China adored and revered the forces of nature and also had a deeply-rooted cult for their ancestors whose purpose was to maintain communion between the present and the past. Religious activities were dominated from the very beginning by the appreciation of the human world and the natural world - a mission entrusted to the sovereign by the heavens and reflected in their ritual books. In ancient times, the Chinese scenario was characterized by the ideal of immutable institutions, with an impending concern for the preservation of family, political and social order. Symbols had a central part in the construction of the mental universe. Images were not considered as a mere pretense of the objects they related to, but as something that involved the entire reality of the object, containing all of its strength and energy and expressed through language and magic. As symbols, numbers played a special part in harmonious combinations, relationships, sequences and hierarchies that contributed to the universal order.

Chinese mathematics found increasing application in various disciplines through calendars, topography, chronology, architecture, meteorology, commerce, payment of taxes, etc. It displayed a notable preference for concrete matters. Operations with represented numbers were carried out on a board where the absence of a tenth potential on a number indicated a vacant hole on the board, called *wu*. That hole acted as a zero. Subsequently, this hole began to be filled with a dot, and later with a small circumference. Some historians consider this representation as indicative of Hindu mathematical influence, although others uphold the hypothesis of Chinese influence in India and others still consider that the two inventions were autonomous.

2.4 Pre-Columbian America

Centuries before the arrival of Christopher Columbus in America, the American Continent featured areas that were inhabited by a wide variety of peoples, many of which displayed a high level of cultural development. We have received from them an admirable artistic legacy. However, a significant portion of their knowledge was destroyed as a result of the clash between cultures that followed their discovery.

The Maya were one of the most ancient cultures in Meso-America. Their main gods were associated with agriculture and climatic conditions. They conceived man as a being that relied on the gods, who dominated the world. The cultural creations of the Maya were based on a religious conception of the cosmos, which considered that the universe had been born from multi-faceted sacred energies, aided by various natural beings that caused the various stages of creation. With this cosmic conception, religious activities were at the heart of the Maya culture. They developed knowledge relating to construction, counting, astronomical observation, painting and sculpture, and also carried out every-day activities such as agriculture and handicrafts based on this conception of the cosmos. The Maya numerical system is considered to be one of the most economical in terms of the number of symbols in it and the fact that it allows figures that reach millions of units to be recorded using positional numeration. The function of the zero in a number was used to identify the absence of a given order of units which lacked its cardinal value. Meso-American cultures, such as the Maya and the Aztecs, displayed a common feature with Indian culture in the construction of mathematical knowledge: the development of the notion, the symbol, the concept and the use of the zero. In this case, it referred to a tangible and concrete zero, typical of cultures that had developed around the careful observation of nature and its concrete manifestations.

"A strong similarity can be seen between their cosmovision and the notion of zero. In the case of Meso-American cultures, the absence of good-bad-type dichotomies significantly favored the establishment of the notion of zero" (Cantoral, 2001, p. 64).

2.5 About certain features of non-Aristotelian thought

It must be admitted that Aristotelian logic did not emerge in scenarios such as the ones described above. The Oriental and American scenarios differed completely from those that emerged in cultures that received Aristotelian influence.

The principle of contradiction and the principle of the third excluded are not valid in these scenarios (Crespo Crespo, 2007a). Antinomies fail to intimidate non-Aristotelian philosophies and additionally accept the fact that extremes complement each other and allow the emergence, evolution and development of certain mathematical concepts whose appearance in the Western world came many centuries later and was strongly debated.

In Ancient China, philosophical ideas were based on coexistence and equilibrium. This provided a symbolical foundation on which different types of numerical oppositions were constructed and additionally enabled the emergence of mathematical objects such as the zero. The symmetrical equilibrium that governs the Chinese paradigm differs radically from classical Greek philosophy. The Greeks considered that it was impossible to go from being to not-being, to change the gender or the nature of an object, that no identifiable object could be found on the border of being and not-being. Therefore, the zero with these characteristics did not emerge in Greece. The Greek vision of the world and use of logic to unravel how it worked became an impediment for the genesis of certain mathematical concepts, such as the zero and infinity. Their strict requirement for logical coherence and bivalence prevented the natural construction of these mathematical concepts, as occurred in other cultures that did not receive Aristotelian influence.

However, as shown above, these cultures constructed mathematical concepts and even developed some constructions that met resistance in the West and could not be approached or developed scientifically until many centuries later.

The presence of mathematical communities in very different scenarios indicates the existence of different demonstration strategies suited to the accepted forms of argumentation in each case. This additionally allows us to understand the possibility of accepting some of these strategies as valid while rejecting others, according to the basic features of the scenarios in which they emerged. Argumentation is constructed within a society. Therefore, the socio-cultural scenario gives argumentation its intrinsic features - which vary from one scenario to another - impregnating it with its thoughts and beliefs which flow from its epistemology.

3. STUDENTS AND NON-ARISTOTELIAN FORMS OF REASONING

What follows are two experiences that demonstrate that the forms of argumentation that arise in the classroom are not always based on Aristotelian logic, despite the fact that the latter is taught in classrooms as the foundation of all science (Crespo Crespo, 2007b).

3.1 Nyaya argumentations in the classroom

Based on features of Nyaya logic - which arose in India in opposition to Buddhism - Bruno D´Amore presents an experience consisting of examples taken from mathematics classes taught to students aged 14 and 15, in which he identifies certain types of argumentative behavior that display similarity with Nyaya argumentative structures (D´Amore, 2005). These forms of argumentation arose spontaneously, without being elicited by the researcher. From our viewpoint, these results suggest that the Aristotelian form of argumentation, which is generally considered a natural strategy in the mathematics classroom, is not so, but rather constitutes a socio-cultural construction which at times appears artificial to students.

We have recreated the research conducted by D'Amore, although having changed the scenario and the mathematical property that gives rise to the experimentation. The following enunciation was given to a teacher taking part in a training course on geometry:

If the diagonal lines of a quadrilateral figure are perpendicular and intersect at their middle point, the figure is a rhombus.

After reading the enunciation, the steps followed and the reasoning made by the teacher were as follows: after drawing a rhombus, naming its vertices and identifying the sides of the quadrilateral as having equal length, the teacher participating in the course affirmed that she knew that the diagonals were perpendicular and that they intersected at the middle point, and stated that given this information, the sides are alike and the figure is a rhombus and, since this is true in the example analyzed, the figure is a rhombus, which was the initial situation. The property was not demonstrated, however, the strategy followed by the student in the face of the problem posed is similar to the one reported by D'Amore. The strategy reveals the stages of Nyaya argumentation, which differs essentially from deductive thought: affirmation of the unproven conclusion, of the cause ascribed to it in order for it to occur, enunciation of the general proposition of which the thesis is a particular case, identification of the thesis in a particular case, general affirmation of the hypothesis or application and, finally, reaffirmation of the thesis. Thus, the property is assumed as a hypothesis and used in the reasoning. No doubt, if the correction of the reasoning is analyzed, it will be inferred that it is incorrect. However, the teacher who was taking the course and who made this reasoning, affirmed that she had tested the proposition, although not in a formal manner. She was aware that her reasoning would not be accepted from a mathematical viewpoint, however she accepted it as a form of testing the property. A non-Aristotelian argumentation strategy had come into play, which nevertheless satisfied its author.

3.2 Resistance to deductive argumentation

Another case that we detected shows that on occasion students prefer to apply non-deductive forms of reasoning, despite their knowledge of the deductive forms learned in class. This activity was presented to students during a written evaluation, requesting them to establish whether or not a form of reasoning based on the logic of predicates was valid by using Venn diagrams. This type of exercise had been solved in class on various occasions, with the subsequent discussion of the resolution and its justification.

One of the students proposed a variation of the invalidity test that is used in propositional logic – a method that is not applicable to the logic of predicates due to the presence of quantifiers. In her resolution, this student previously eliminated the quantifiers and applied a direct test, which is incorrect from the viewpoint of Aristotelian logic. We enquired why she had not used the requested method and why she preferred the one described, taking into account that in the previous class she had demonstrated her ability to perform the expected analysis. In order to clarify ideas, we resorted to an interview with the student, during which we confirmed that she not only possessed the expected knowledge but additionally defended the resolution she had carried out following the method of abduction. She had not taken into consideration the importance of the figures of analysis, rejected indirect argumentations and showed a clear preference for direct methods, even when they were not deductive.

4. COMMENTS

This article presents situations through which the principles of logic that for centuries have been identified as the laws of human thought can in actual fact be seen as socio-cultural constructions. On the one hand, it is evident that these principles are not present in every socio-cultural scenario and, furthermore, cultures in which they did not emerge were able to construct certain mathematical concepts whose characteristics differ from those that stem from Greek-based mathematics.

This paper presents examples of forms of argumentation that are present in the mathematics classroom which do not have Aristotelian characteristics. Therefore, the two forms of argumentation differ in terms of what they require in order to be considered valid.

The first case presented deals with a structure of argumentation that relates to those used in India during the Vedic period. Although it cannot be presumed that the person that used this form of reasoning had previous knowledge of it, it presents forms of argumentation that are devoid of Aristotelian influence and which, nevertheless, generate conviction among students. The other example presented shows how the response and explanations provided by a student reveal that although the latter initially responded to a problem posed within an academic scenario by providing a resolution that did not involve argumentations from Aristotelian logic, in a subsequent interview, the student demonstrated her knowledge of the latter, her reticence to apply them in certain cases and her opinion that the arguments she applied were better than these.

These examples strengthen the hypothesis that as socio-cultural constructions, forms of argumentation are not innate but have been constructed over time and constitute the foundation of the demonstrative social practices that characterize the mathematical community. It is evident that our form of argumentation and our mathematics have been constructed within a culture with a strong Aristotelian foundation; consequently, this type of logic is presumed to be innate. However, although in this paper we only have analyzed two examples of positions relative to certain forms of argumentation that exist in the classroom, these are not isolated examples; they present opinions that we have detected frequently in our classrooms and that we consider deserve careful analysis.

The difficulties that arise when conducting mathematical demonstrations in the classroom are often due to a failure to detect the existence of these types of argumentation and their characteristics, assuming Aristotelian reasoning as natural. As socio-cultural constructions, the forms of argumentation used in mathematics have not remained static in time. In our opinion, the comprehension of this socio-cultural nature of argumentations and of demonstrations as social practices will contribute to generate an increased perception of forms of argumentation in the classroom.

In order for students to comprehend the need for mathematical argumentation and, furthermore, for the demonstration of mathematical properties, it is essential for them to construct the significance of argumentation. This significance should be transmitted through the ability of teachers to adopt mathematical demonstration as a social practice. In our opinion, this will enable students to understand the importance of argumentation for justifying and granting validity to mathematical properties. The acceptance of this significance will imply the acknowledgment of the socio-cultural construction status of argumentations.

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