Int. Jr. of Contributions to Mathematics Teaching
Vol. 1 No. 1 (June, 2019)

Cross thematic activities in physics and mathematics are performed while students use dynamic geometry to simulate their own constructions

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Abstract

We present three cross thematic activities in the fields of physics and mathematics that were carried out during the academic year 2009 – 2010 in a small private school on the island of Rhodes, Greece. The three activities have to do with simulating gravitational free fall, exploring the centre of mass of a non regular body and modelling a moving bead on a rotating rod respectively. Students use The Geometer's Sketchpad dynamic geometry environment to simulate, verify and utilize their results or to make conjectures about theoretical properties of these models, while Euclidean geometry is paramount throughout. A thematic design triangle that takes into consideration the three basic axes of computer supported collaborative learning, scientific models and cross thematic approach was used to design these activities in which students are encouraged through strong usage of guidance and dialogue to discover several notions and guess physical laws by themselves.

Keywords: cross thematic activity, geometer's sketchpad, young scientist, simulation, scientific model

MSC classification: 97U70

Introduction

We focus on the main points, process and methodology of three cross thematic activities in the fields of physics and mathematics that were carried out during the academic year 2009 – 2010 in a small private school on the island of Rhodes, Greece. Three eager and promising young students of the 1st year of high school (Α΄ Λυκείου) volunteered to be guided through physical notions that were entirely or almost new to them as well as to relate the relevant physical models to mathematics of their school curriculum. Working with such a small sample of students provides the advantage of a closer look at the methodology's potential that we want to examine and exploit. Therefore, this is an initial opening study from which we can use the gained experience to extend the methodologies to a larger number of students in the near future.

As teachers and potential educational researchers we ask: Are students able to provide their own solutions to problems that arise in a natural setup of the physical world? How can we evoke hidden abilities in the process of guiding them through an activity? In what way can ICT be used so that students can guess and make conjectures, simulate or verify their results? How can we best prepare our themes in order to achieve the above goals?

Computer supported collaborative learning

One can employ contemporary didactic theories that efficiently model computer supported collaborative learning (CSCL) which "is focused on how collaborative learning supported by technology can enhance peer interaction and work in groups" (Lipponen, 2002). A list of main such models can be found in Karasavvidis, 2006. Specifically, Hakkarainen and his colleagues at the University of Helsinki, in their pedagogical 'progressive inquiry' model propose didactic phases that resemble actual scientific research: setting up the context, presenting research problems, creating working theories and critical evaluation, among others (Muukkonen et al, 2004). Similar concepts can be found in the model of 'knowledge building' (Scardamalia & Bereiter, 2003). In the 'knowledge integration' model, interactive dialogue is predominant in all of its design patterns (Linn, 2006). Indeed, Linn argues that "instruction that helps students develop criteria to evaluate ideas promotes cohesive understanding". The teacher can integrate didactic activities in the 'knowledge creation' framework of learning, in the sense that "collaborative activities are organized around shared objects rather than take place through immediate interaction between participants (Lipponen et al, 2004).

Scientific models

Another important part of these activities is properly introducing the students to the concept of a scientific model. Indeed, the didactic CSCL theories described above are very closely related to student group work around a scientific model or simulation of the real world. In their detailed account, Hartmann and Frigg note that "models are vehicles for learning about the world. Significant parts of scientific investigation are carried out on models rather than on reality itself because by studying a model we can discover features of and ascertain facts about the system the model stands for" (Hartmann & Frigg, 2006). Specifically for physics, several aspects of an instructional theory are very well described in Hestenes, 1987: he analyzes the components of a mathematical model, a scientific theory and the procedural knowledge of the scientific method from a pedagogical point of view. He describes four types of model structure (Hestenes, 1996) among which quite relevant to our activities are the components: configuration (geometric relations among the parts) and descriptive models (they represent change by explicit functions of time). The use of models as a means to teach physics is very popular in several schools around the globe, sometimes supported by government grants. For example, based on the Modelling Instruction Program, teachers at a workshop offer that model making in science education has to be graphical, mathematical, diagrammatical and verbal. They also suggest that "a good model must be simple, appropriate to the student's capabilities (and may therefore include inaccuracies or approximations as long as they form a stepping stone to more advanced models), identifies facts subject to experimental verification, identifies definitions created to ease communication, does not contain untestable or unverifiable claims and is acknowledged as an approximation to reality" (Schober, 2003).

Cross thematic approach

The Pedagogical Institute (Π.Ι.), opting to "upgrade the quality of education" uses a cross thematic approach to knowledge in order to "encourage the interconnection of cognitive disciplines through appropriate extensions of taught subjects" via its Cross Thematic Integrated Curriculum Frame (Δ.Ε.Π.Π.Σ.) (Alachiotis, 2003), while "the aid of a computer and the appropriate dynamic simulations can prove very useful to the student so that they can apprehend and better understand concepts and procedures" (Pedagogical Institute, 2003). They describe specific methodologies to accomplish these ends (exploration and discovery, dialogue and discussion, etc), as well as forms of evaluation (feedback).

With the aim of designing and performing the activities to the best of our abilities, we took into consideration these three basic axes of CSCL, scientific models and cross thematic approach as they form a triangle in figure 1.

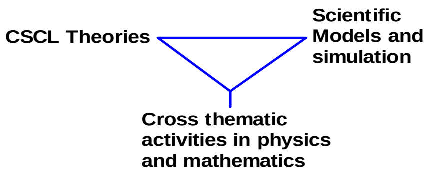

Figure 1. The three activities are based on this thematic design triangle.

Prior to the activities, we had to have a very good idea of the exact mathematical knowledge and to which level this was possessed by the involved students (especially when some of the physical notions are extracurricular). During the activities, we emphasized on the following: a good initial presentation of the aim and purpose of each upcoming activity, interactive student dialogue and team work regarding the physical notions, and guidance towards a notion. There was minor interference by the teacher for the software simulations.

The present three activities unavoidably fall under the category of mini research as far as the students are concerned (or all of us for that matter) (Pipinos, 2010). At key points throughout this paper, students have come up with original (to them) ideas. There are exact student comments and impressions. Actually, the dynamic characteristic of ICT helped our students to get rid of basic misconceptions as is noted in the core below.

About the activities

We present all three activities in one paper since the same students were involved during the same school period. Hence, we can have a good initial indication about the methodology's potential.

The students used the familiar to them dynamic geometry emulator The Geometer's Sketchpad in Greek (GSP) (Manual, 2000) for the ruler and compass constructions and the physical motion simulations.

The first activity, "simulating Newtonian free fall by the use of Euclidean geometry", precedes the other two (Pipinos, 2010). In this paper we describe its main phases. The idea is to employ a similarity ruler and compass construction to simulate the 2nd degree equation of a free fall exploiting the possibilities of GSP, imminently simulating gravity. Duration: 2 hours.

The second activity, "exploring the centroid of non regular bodies", opts to provide students with a good understanding of the centre of mass and where it is located, how to construct it with ruler and compass, that is. As a prerequisite, we had to familiarize our students with the bisector theorem (Argyropoulos et al, 2007). A main centroid theorem is used recursively to construct the centroid of various shapes, like three uniform rods forming a triangle, directly in GSP environment. Duration: 2 + 2 hours in two days, one week apart.

The third activity, "a moving bead on a rotating rod" (Pipinos, 2010), is possibly extracurricular. However, the students use familiar mathematics to encounter polar coordinates for the first time (without knowing that they do so). They also provide a mathematical condition that the simulated composite physical motion is periodic in quite an elementary way. ICT plays a central role in this activity, as it is used both to analyze the polar coordinate equation of motion that naturally arises from the physical setup itself, as well as to verify the derived condition for periodicity by creating controlled 'flower' curves. Duration: 2 ½ hours.

Simulating Newtonian free fall by the use of Euclidean geometry

Introducing the aim and the reason to consider a scientific model

We began by reminding students the basics of a Newtonian free fall. They recalled the gravity constant g and the formulae that describe how the velocity and distance from the ground change with respect to time. We explained to them that $d = h - \frac{1}{2}gt^2$ is the formula to hold on to for the rest of the activity. We talked to them a bit about scientific models and that what we actually want is to find ourselves a model on which to base our geometrical simulation of the above formula. One way to do a simulation is to simply feed a computer with this formula and let it plot the graphics, but we don't want to do it this way! So, an interactive dialogue followed in which we concluded that, mathematically, it will suffice to simulate $y = x^2$ from a given x playing the role of time and y playing the role of distance. How are we to go from an algebraic relation to geometry? A small discussion on the previous school year notions of similarity followed whence the students used algebraic inverse thinking abilities to quickly come up with ratio equation $y/x = x/1$.

Main student constructions

The students worked on their own for 10 minutes with the objective to use similarity to find a geometric construction whose result was the above ratio. The construction had to work in the sense that y has to be found as a segment in terms of the given segment x. Both students came up initially with sketches that correctly produced the ratio equation. However, both efforts failed to be useful, as they didn't produce segment y from given segment x. So some more time was used during which the two students successfully used their geometric inverse thinking abilities to correctly modify their constructions. We must note here that the two proposed constructions were different and this is important.

Dynamic software simulation – the bouncing ball

The final part of the activity was to simulate their constructions. One of the constructions is shown in Figure 2. Thus, a similarity construction based on a simple algebraic ratio provides a bouncing ball in GSP environment.

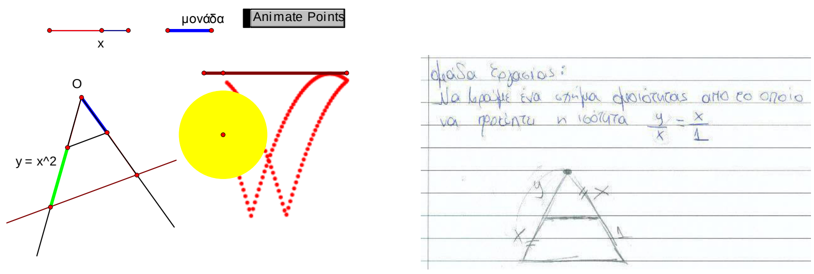

Figure 2. A student simulation of the parabolic trajectory of a 'geometric' bouncing ball and an initial idea failing to be useful

Student comments and impressions

Foteini pointed out that "it is the traveling from one discipline to the other that I enjoyed the most". She said, "it was very impressive because from something that we did in physics we ended up in mathematics, initially algebra, and then geometry". Konstantina said, "it seemed to me a bit weird, I had not seen that from one subject we can end up in another and it was very nice…. things that we did not know…". Foteini mostly liked "the switch that happened, from physics to mathematics and from algebra to geometry, the fact that we can connect them, that there are ways". Konstantina commented, "the geometric simulation, the movement of the segment for x^2 helped me realize the difference from what I thought, I thought that $(2x)^2 = 2x^2$". Their teacher and my helping hand, Kyriaki, pointed out that "I was happy to see that each of you produced a different solution to the same problem!"

Exploring the centroid of non regular bodies

First part

We began with a discussion of what the centre of mass of a body should be. From their every day experience, the students suggested that this is the point where the body will balance when hanged from a ceiling for instance. An experiential sub activity followed: a non regular randomly cut piece of hard paper was hanged by two separate points of its surface on which two respective vertical lines where drawn. The centre of mass was found simply as the intersection of these two lines (Shepherd & Lovat, 1997). The situation of two equal masses and then of two unequal masses M and m placed at the endpoints of a weightless rod of length l balancing on a pivot at distances x, y from the masses respectively was proposed to the students. Although the students had never been taught the condition for balance, they quickly suggested it: $x/y = m/M$. This ratio together with $x + y = l$ provides the exact point of where the pivot should be. This is also where the centre of mass of this system of bodies should be. There followed a small talk about the notion of torque that they will encounter in forthcoming school years. This helped to further strengthen the notion of the centroid as the point of zero torque. Then, we related the previous discussion to the following main theorem that is to be used repeatedly for the rest of the activity: If two bodies of masses M and m have centroids at points A, B respectively, then the centroid of the system of the two masses necessarily lies on segment AB at distances x, y from the two respective masses so that $x/y = m/M$. Direct application of the theorem followed by the students in GSP: Using the controls 'midpoint' and 'dilate' (Patsiomitou, 2009), the students constructed the centroid of three equal masses and realized that it is just the familiar barycentre G of their forming triangle. They also found the centroid of four equal masses and proved easily that this is just the intersection point of the segments that join the midpoints of the opposite sides of the forming quadrilateral.

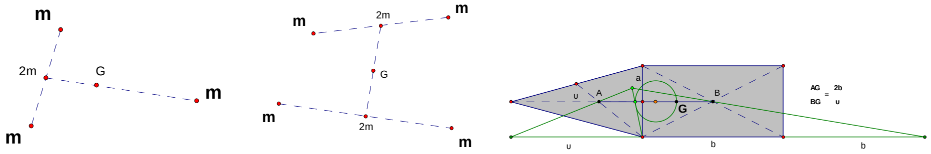

Figure 3. Using the main theorem, a student can easily produce and understand the position of the centroid of various configurations of equal or unequal masses in space

Second part

What about shapes comprising of rods of uniform density? This is the main part of the activity where the students were presented with three such rods forming a triangle $ΑΒΓ$. They worked together to find a way to determine the centroid G of the uniform perimeter of a triangle. They simplified their search by first finding the centroid of the two sides only. This is done as follows: The centroid of each rod is just its midpoint, since the rods have uniform density. So let $Μ, Ν$ be these midpoints. Then, by our main theorem, the centroid of two of the rods must lie on segment $ΜΝ$ at a point $Λ$ for which $ΜΛ/ΛΝ = β/γ$. In order to construct point $Λ$ we take advantage of the bisector theorem. If $Κ$ is the intersection of $ΜΝ$ and the bisector of $\angle Α$, then $ΜΚ/ΚΝ = γ/β$. To obtain the required inverse ratio, point $Λ$ is just the symmetric point to $Κ$ with respect to the midpoint of segment $ΜΝ$ (in fact, this was conjectured and proved by the students). Now that we have the centroid of any two rods, how are we to locate G? According to our main theorem, G must lie on the segment joining $Λ$ to the midpoint $Δ$ of rod $α$. Here is a clever way to locate G (with direct correlations to the experiential subactivity): why not perform the same construction as above but with another pair of rods, say $β, α$. Then we will produce another such segment on which G must also lie on. Hence, G is just the intersection point of these two segments! (Figure 4).

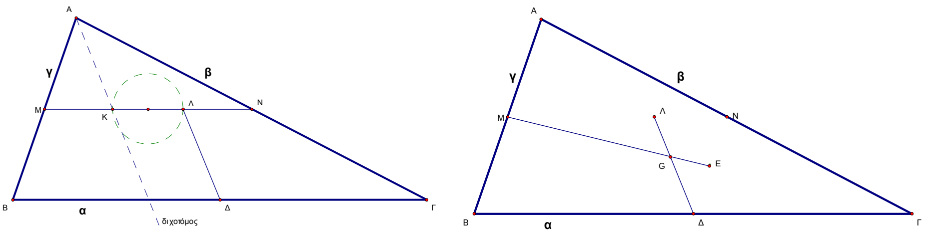

Figure 4. The centroid of three uniform rods as the intersection of two basic segments

Note that G will not coincide with the barycentre of the triangle! ICT can play an important role so that students can obtain firsthand knowledge and thus distinguish the differences in scientific constructions, avoiding oversimplified statements.

A moving bead on a rotating rod

The students were instructed to construct a free point P rotating on a fixed circle (O, R) (with constant angular velocity ω) and the diameter from that point in order to simulate a rotating rod of length l. Then, they created another free point A on the rod that was to travel back and forth along the rod at a constant velocity v. Thus, we had a model of a moving bead along a rotating rod. By using the command 'trace' on the point on the rod and by creating an 'action button/animation' for the simultaneous motion of the rod and the point on it, the students were able to directly observe the flowerlike curves that emerged for various values of the two velocities.

When asked to find a way to determine the equation for the points of the curve, it was no surprise that they mentioned the usual Cartesian coordinates from the school curriculum (Andreadakis et al, 1998). However, with little encouragement and discussion on why it is natural here to consider distance $r = OA$ and angle θ at time t, they quickly deduced the parametric polar equation $r = R - vt,\ \theta = \omega t$ that naturally led to the polar $r = R - \frac{v}{\omega}\theta$ by eliminating time t. We verified this equation (which holds until the moving point on the rod travels a half rod length) by the 'graph/plot new function/equation/$r = f(\theta)$' tool of GSP.

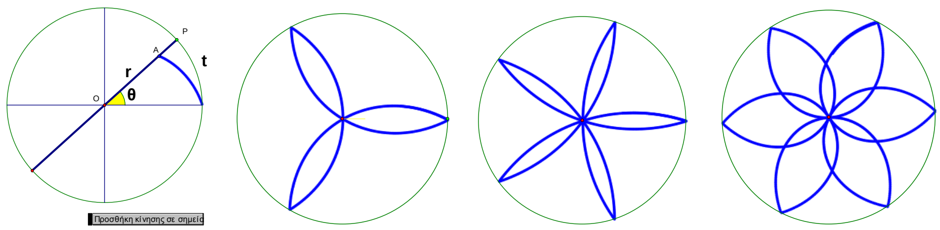

Figure 5. Deducing the polar equation of the curve and (1, 3), (1, 5), (2, 3) flowers are created by using the derived periodicity condition

Now the students were introduced to a challenge: to find a mathematical condition for our motion to be periodical in the sense that both free moving points must coincide at the original starting point at some future time. Working together, the students decided that periodicity means that "both the rod and the bead have completed a number of independent complete moves" but no mathematics had been written down yet. So we had to help them consider that the rod will have completed n turns and the bead m rod lengths at coincidence time t. We actually created a table for various values of n, m. They came up with necessary and sufficient equations $\theta = \omega \cdot t = \pi \cdot n$ and $v \cdot t = m \cdot l$. (There are some details in that n, m must have the same parity, but let us not get into this for didactic reasons. Besides, without this detail, periodic motion will occur in another sense anyway). It is notable that they derived these equations themselves after the teacher lead them to 'consider' n, m. By eliminating t again, a condition for periodicity is found: $\omega \cdot l / \pi \cdot v \in \mathbb{Q}$! Moreover, if u is the circular velocity of point P then this condition becomes $u/v = \pi/2 \cdot n/m$, i.e. a rational multiple of $\pi/2$. This enabled us to simulate any desired periodic 'flower' by setting the right ratio of velocities in submenu 'properties/animate/speed/other' of our animation button for corresponding desired values of n, m (Figure 5). Interesting closed shapes and other flowers occur for other values for n, m like $(2, 3)$ or $(3, 1)$. The students extremely enjoyed the fact that scientific considerations enabled them to "combine my favourite subjects (i.e. physics and geometry) to totally control the motion, creating something beautiful, when in the beginning it was just chaotic", as Polyvios notes. They also realized a connection between rational numbers and periodicity. This connection appears in mathematics at university level when one considers when the sum of two periodic functions is periodic (Olmsted & Townsend, 1972) and at several other instances.

Conclusions

In our opinion, the presentation of the problem in itself has a vast impact for the rest of any activity. It is important to rely on the imagination, natural curiosity and thirst for discovery of our students. Equally indispensable are dialogue, discussion and verbal communication. These form the basis of any effective pedagogical approach of today and are after all common elements in all three axes presented in this paper.

It is not enough to simply read about CSCL theories implementing ICT in the classroom: these are offered to teachers to try their hands on. These models are "not meant to be taken prescriptively, as an ideal path to be followed rigidly; rather they offer conceptual tools to describe, understand and take into account the critical elements in collaborative knowledge-advancing inquiry." Besides, practise and empirical data are most valuable in order to validate these theories.

It is notable that pupils reacted positively to three unrelated themes in physics, with the connection that their results had to be modelled in GSP. They used GSP to construct and test the validity of their results, to simulate physical notions and observe whether their simulations resemble reality, to observe the behaviour of a real physical system of composite motion, to make and verify mathematical conjectures. As teachers, we found that pupils were able to produce original ideas on which to work on either as a group or on their own. From their performance, comments and impressions we conclude that the cross thematic approach of simulating physical models based on an ICT setup can help them visualise better, invent laws and conditions, create geometry that works, and even get rid of previous misconceptions.

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