"THE TEACHING OF GEOMETRY" AS SEEN BY TEACHERS OF SECONDARY SCHOOLS AND TEACHER TRAINERS

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ABSTRACT

This work looks into the geometry of the teacher, seen as a complex network among the disciplinary notion of the teacher as the result of their academic background, their appraisal of geometry as a content of learning and their didactic practice when teaching it. By means of a qualitative-interpretative research, these aspects are characterized and compared into two groups of teachers of Mathematics in the Province of Santa Fe, Argentina: teacher trainers and secondary school teachers. The study shows different approaches in both groups as regards criteria for selecting, hierarchizing, sequencing and scheduling geometric content in Secondary Education and as regards the importance granted to didactic materials and resources.

KEY WORDS: Geometry; Secondary School; Teacher training

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INTRODUCTION

As a consequence of the Boubaki revolution (Modern Mathematics, 1960s), elementary geometry and spatial intuition were scarcely included in education curriculum designs, especially in the training of Mathematics teachers. Freudenthal (1964) pointed that the word *geometry* was barely mentioned in university plans worldwide, and geometricians themselves avoided the term as they found it out-of-date. Atiyah (1976) questioned the lack of generation of new ways of teaching geometry in that period and the neglect of geometric intuition as a powerful support for understanding.

This trend is reflected in Argentina, where three great moments of geometry in the Secondary Education (SE) curriculum can be identified: 1950s and 1960s, with an approach based on statement, proof and problem-solving; 1970s and 1980s, with poor presence; and 1990s and 2000s, motivated in concrete problems. Thus, there is usually tension between the training of teachers and the demands towards their current pedagogical practices (Sgreccia, 2008).

Barrantes & Blanco (2006) condemn the poor geometric training of the majority of young teachers during their whole education. The latter often remember geometry as something difficult, with little time allocated for it only at the end of the course, and as the last chapter in books. As a result, teachers usually do not teach it as they are not familiar with disciplinary aspects and their importance (Jones & Fujita, 2001; Báez & Iglesias, 2007). In Argentina, paradoxical behaviors can be seen in the teaching of geometry: teachers, managerial staff and supervisors are worried because it is not dealt with because of lack of time, as it is left for the end of planning, but they neglect it themselves when setting criteria for letting students pass their exams (Broitman & Itzcovich, 2008).

Owens & Outhred (2006), in a review of research on geometry as from the mid 1970s, point out that research on spatial skills and relations among shapes has been often framed within Van Hiele levels (1986). Studies exploring the influence of other factors (materials, context, teachers) in the development of geometric thinking have shown that the meanings and mental representations built depend on the intentional processes oriented by teaching.

Numerous current applications of Mathematics, many of which are linked to technological advances, require substantial geometrical knowledge (Jones & Fujita, 2001; Jones, 2002; Jones & Mooney, 2003; Lindgren & Schwartz, 2009; Ramadas, 2009a,b; Sorby, 2009). Thus, we could mention situations related to computerized design, robotics, medical diagnosis imaging, computerized animation and visual presentations. Disciplines such as chemistry, material physics, biology, geographical information systems, and many fields of engineering resort to geometric modeling, specially three-dimensional (3D) (Whitely 1999, quoted in Jones, 2002). Liedtke, Grau & Growe (1995) identify a strong demand for 3D models of real objects in scenarios of flight developments, simulators, films, landscape planning, advertising, and education. According to Jones (2002), new computer advances make spatial thinking, visualization and interpretation of information of vital importance. Thus, geometrical education is deemed crucial in a world eminently technologized and visual.

Sharygin (2004) justifies the study of geometry in SE for its practical value and as knowledge which encourages cultural, spiritual, intellectual, creative, aesthetic and moral development. Broitman & Itzcovich (2008) claim that the study of geometry helps develop specific cognitive skills which allow us to reason on the representation of theoretical objects, separating from the merely perceptive or visual which originated them.
In view of all this, it is worth asking: how does teacher training affect the teaching of geometry? How has the relation of the teacher with geometry to teach it been modified? How does the SE teacher see the role of geometry in the education of adolescents starting to develop formal thinking?

The answers will help shape, by analogy with Halbwachs (1985), the *geometry of the teacher*, seen as the complex network of three components: (I) *their disciplinary conception resulting from their academic background;* (II) *their appraisal of geometry as a content of teaching;* (III) *their didactic practice when teaching it.* From this perspective, the *geometry of the teacher* is located between the *geometry of the mathematician* (based on an academic education oriented to the production of new knowledge and its transfer to other areas) and the *geometry of the student* (knowledge seen as a cultural component for social performance, which can also guide later professional decisions).
In order to analyze aspects related to the conformation of the *geometry of the teacher*, we carried out a qualitative-interpretative research in two groups of participants (SE Mathematics teachers and university teacher trainers). We aim to explore:

- The conceptions and appraisal on the teaching of geometry;
- The specific problems involved in its teaching in the first two years of SE;
- The links between class practices and their training as Mathematics teachers.
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This paper seeks to provide elements to identify characteristic traits of each of the three components of the *geometry of the teacher*, which have been analyzed by few studies so far (Jones, 2000; Moore-Russo & Schroeder, 2007).

THEORETICAL FRAMEWORK

For Jones (2000), the knowledge of the geometry teacher encompasses: what they know, how they organize it and how it may be supported by professional development. These aspects, in our opinion, are substantially marked by the way in which they learnt geometry, both in disciplinary and didactics terms, resulting in a *geometry of the teacher.*
Bass & Ball (2004) claim that the tasks of the Mathematics teacher require knowledge which is not usually taught in the training course. In the line of Shulman (1986), Ball, Thames & Phelps (2008) identify six *domains of mathematical knowledge for teaching.*

1. *Common content knowledge*: includes the knowledge, skills and mathematical language of people using Mathematics in diverse fields. Teachers should have a very good command of these content, in this case, geometrical.

2. *Horizon content knowledge*: includes knowledge which helps adopt a perspective on the way Mathematics content relates to other contents in the curriculum. They promote decision-making to respond to questions such as: can conflictive mathematical consequences arise from something that has been indicated explicitly or implicitly? Is this interesting or important in mathematical terms?

3. *Specialized content knowledge*: encompasses the mathematical knowledge and skills exclusive for teaching, including: finding an example to orient meaning; connecting the representations to underlying ideas and to other representations; explaining objectives and mathematical proposals to parents; appreciating and adapting the mathematical content of textbooks; modifying tasks to make them easier or more difficult; giving or evaluating mathematical explanations; choosing and developing useful definitions; using mathematical notations and language and being critical regarding its use; and formulating mathematically productive questions.

4. *Knowledge of content and students*: refers to knowledge which helps foresee answers, attitudes, difficulties, confusions and correct answers of their students in relation to mathematical knowledge. It implies an interaction between specific mathematical knowledge and knowledge of cognitive nature about their students.

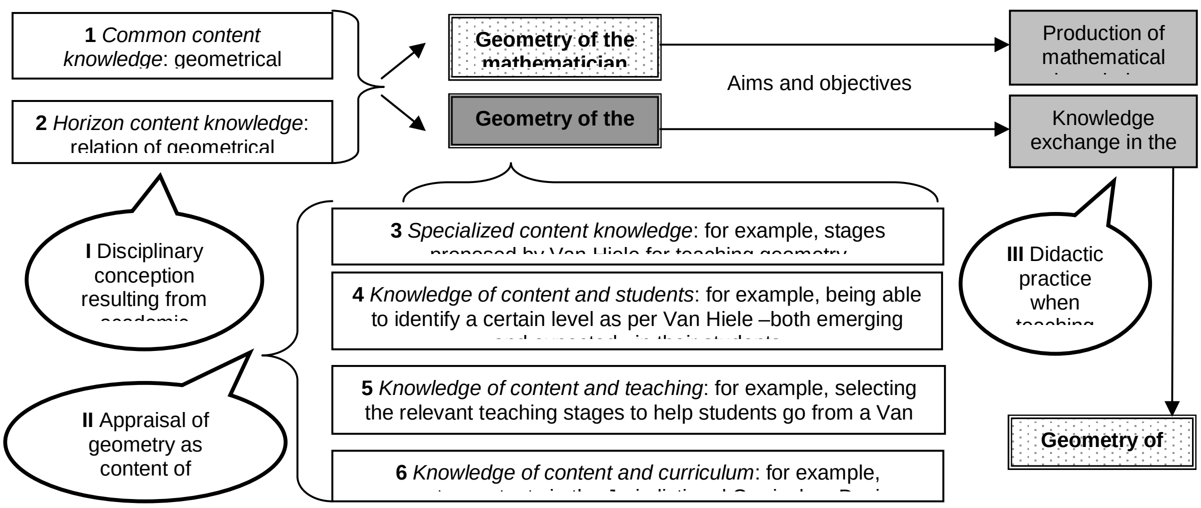
5. *Knowledge of content and teaching*: encompasses the specific aspects related to Mathematics didactics. Teachers express this knowledge when sequencing particular knowledge for teaching, when choosing examples in order to initiate the building of a concept and study content more in depth, and when assessing advantages and disadvantages of representations used in order to teach a specific idea. These tasks require an interaction between mathematical understanding and pedagogical understanding.

6. *Knowledge of content and curriculum*: according to Shulman (1986), the knowledge of the curriculum encompasses the complete range of programs designed for teaching particular subjects and contents in a certain education level, the variety of didactic materials available in relation to those programs, and the set of criteria for the use of particular curricular materials in specific circumstances.

7.

Fig. 1 shows the three components of the *geometry of the teacher* (I, II and III), integrated with these six domains of mathematical knowledge for teaching, contextualized for geometry. Also, the *geometry of the teacher* is situated in relation to the geometry of the mathematician and the geometry of the student, as the teacher is precisely in this place of mediator.

FIGURE 1. Components of the geometry of the teacher and domains of mathematical knowledge for teaching



As regards the conceptual contents of Geometry of the Jurisdiction Curriculum Design (*Diseño Curricular Jurisdiccional;* DCJ) (Ministry of Education of the Province of Santa Fe, 1999) for the first two years of SE (domain 6), the following objectives are outlined:

- To identify the properties of bi-dimensional and three-dimensional shapes and apply them in problem solving.

- To apply the concepts of measure, location and transformation in the study of space.

Their accomplishment depends on the real nuances acquired by components (I, II and III) of the *geometry of the teacher* and on the way the pedagogical practices are carried out in the classroom to explore relations, make guesses, analyze properties, solve problems, use different representations and view things from different perspectives.

In the structuring of domains 3, 4 and 5, the teacher's knowledge of Van Hiele's contributions (1986) to the learning of geometry is quite important, as associated with an inductive process of five levels, consistent with the building of space, whose observance (domain 5) enables meaningful, gradual learning.

VH1. *Visualization:* objects are perceived in their whole globally.
VH2. *Analysis*: some parts of the objects along with their properties are established.
VH3. *Informal deduction:* objects are defined, relations are established and properties are deduced.
VH4. *Formal deduction:* formal logical reasoning is carried out and the axiomatic structure is understood.
VH5. *Rigor:* the consistency, independence and integrity of geometrical axioms are understood.

As per Van Hiele (1986), the study of a geometrical content is not exhausted in a single level, so it would be expected that the student's reasoning and the teacher's guidance should be in tune with each other (domain 4). For this, Van Hiele defined five phases to be passed through by the learner in the teaching of geometry: inquiry, guided orientation, explanation, free orientation and integration (domain 3).

Within this framework, and in light of the meanings of the above aspects, domains 1 (common content knowledge) and 2 (horizon content knowledge) for teaching are associated with component I of the *geometry of the teacher* (disciplinary notion resulting from their academic background), as shown in Fig. 1, with differences in their objectives and aims of the geometry of the mathematician.

One of the main aims of the Mathematics teacher is the exchange of knowledge in the classroom, which encompasses component III (didactic practice when teaching geometry) of the *geometry of the teacher.* From this exchange among pedagogical actors, the so called *geometry of the teacher* is progressively shaped.

Domain 3 (specialized in content), 4 (of content and students), 5 (of content and teaching) and 6 (of content and curriculum) of mathematical knowledge for teaching are associated with component II (appraisal of geometry as a content of teaching) of the *geometry of the teacher*, as they are the very background required by teaching (and not necessarily by other professions) to teach geometry.

METHODOLOGY

An interpretative approach was adopted, focused on the educational phenomena in their "natural reality" and aimed at understanding the meanings granted by subjects to their practices and actions. A qualitative approach was used to analyze the complex network of the *geometry of the teacher* from the meanings they have for those involved in SE and teacher training (Taylor & Bogdan, 1986). The scope of the study is descriptive, aimed at seeking the characteristics of geometry for those involved in its teaching in the first two years of SE and establishing the relations between the modalities of emerging categories (Hernández Sampieri, Fernández Collado & Baptista Lucio, 2003).

In order to characterize the *geometry of the teacher* as set previously, two techniques were used: open semi-structured interviews and focus groups, with the same protocol being employed for both. Interviews were used to identify what the teacher knows (knowledge or information), what they like or dislike (values and preferences) and what they think (attitudes and beliefs). Focus groups were used to gather information from interventions enriched by the discourse exchange among people who have been working together during some time with a common purpose.
Subjects: two intentional groups of teachers of the province of Santa Fe, Argentina.

<u>Group 1 (G1, teacher trainers):</u> twelve specialists in Mathematics and Mathematical Education chosen because of their experience in the teaching of geometry. Nine of them were interviewed individually and the remaining three participated in the focus group.

Group 2 (G2, SE teachers): thirteen Mathematics teachers who teach geometry in the school stage under study. Ten of them were interviewed and the rest participated in the focus group.
Information processing: transcriptions of oral recordings were processed by means of content analysis techniques (Bernárdez, 1995) using five dimensions:

1 *Conceiving the teacher as a professional of Mathematical Education:* focused on the academic-professional characterization of participating groups.

2 *Locating geometry in the first two years of SE:* associated with domains 1, 2 and 6 of mathematical knowledge for teaching.

3 *Considering the students' learning process:* linked to domains 3 and 4 of mathematical knowledge for teaching and component III of the *geometry of the teacher*.

4 *Considering the teacher's planned didactics:* associated with domains 3 and 5 of mathematical knowledge for teaching.

5 *Considering the teacher's didactics in action:* associated with component III of the *geometry of the teacher.*

RESULTS

They are shown based on the different analysis categories, indicated in italics, associated with the five mentioned dimensions.

DIMENSION 1: Conceiving the teacher as a professional of Mathematical Education
Previous work experience

Group 1 (G1): comprises two subgroups which can be characterized as: *young*: teacher working experience equals 18 years or less; and *mature*: their experience is 35 years or less. Most of them have taught in SE and further education levels. Two teachers have worked in primary education. Only two teachers report to have worked exclusively in the University in the Mathematics training course. More than half of the respondents have taught in the first two years of SE. Almost half of the group has a strong background in the training of teachers in SE Mathematics.

Group 2 (G2): the years of teaching experience is distributed quite uniformly between 5 and 31 years. Their labor market entry was virtually immediate. They all have teaching experience in the first two years of SE and most of them have worked at university level.

Academic background

G1: for the most part they hold a BA in Mathematics, and senior teachers have completed postgraduate education. Most of them have a background integrating the disciplinary-academic with the didactic.

G2: there are more Mathematics teachers graduated from universities than graduates from higher education institutes. The tendency to pursue postgraduate education is very low.
Motivational factors to choose this course

G1: a marked liking of Mathematics as a science is observed, whether because of its epistemic traits as for the satisfaction of making use of it, already manifest since SE years. This defined the initial or later education as a Bachelor in Mathematics.

As regards the like for teaching, there is a variety of criteria, and four kinds of answers can be identified. Six respondents acknowledge that the very teaching practice as university professor has sparked their passion for teaching. Two mention a clear calling for teaching since their early education, from children's games; two mention the influence of a family environment with a teaching tradition, and two other mention teaching itself without further explanation.

G2: the group is characterized by a marked liking for teaching, which gradually arose from children games, during their basic education stages or from the feedback resulting from the teaching practice itself. A small number mentions family teaching tradition.

As regards the liking for Mathematics as a science, feelings range from assurance to challenge. One of the respondents points out the influence of SE Mathematics teachers who showed them the order in mathematical proofs.

DIMENSION 2: Locating geometry in the first two years of SE

Basic geometry content: it refers to the content of the Geometry area whose respondents consider priority in the above educational level.

G2 mentions a larger number of contents than G1, which suggests that teachers working in the first two years of SE point out only the ones they really teach, rather than a hierarchization of priorities for the education of citizens. G1 and G2 mostly point out contents related to measures and shapes. Two teachers of G1 and four of G2 grant importance to this school stage for introducing formal thinking processes, in line with ideas posed by Van Hiele (1986) in level 3. Nearly half of G1 considers attitudinal aspects close to scientific activity, which is not mentioned by G2.

Time allocated in a school year: an analysis is made regarding the time allocated to geometry against other curricular contents of Mathematics.

Both groups show differences in the grounds for justifying the allocation of time and moments assigned to geometry.

G1: they allocate time based on the characteristics of geometric content in relation to other Mathematics content and/or the students' cognitive characteristics.

G2: they base their responses on what they normally do as a teaching routine –without reflecting on or questioning the procedures used– or on grounds of *inability* –because their training in geometry was poorer than in other fields of Mathematics.

Geometry within Mathematics: accounts for the teachers' assumptions regarding the existing myths on the learning of the different areas of Mathematics.

The consensus shown in both groups is remarkable: learning geometry is easier than learning algebra for students in the first two years of SE because of its possibility of being related to the real. They also agree that mechanical applications are often made to reinforce contents. Their statements help infer some kind of confrontation between *what should be done theoretically* and *what is really done.* According to their comments: *theoretically*, if both algebra and geometry are taught from the signification of contents involved in the first two years of SE, geometry is seen as less abstract than algebra. But *really*, the signification of contents is not usually prioritized. For G2, what is prioritized is routine calculations lacking geometrical signification.

DIMENSION 3: Considering the learning process of students

Conceptual acquisition indicators: comprises the signals taken into account by the teacher to know if the student has grasped a geometrical concept, as well as the activities carried out to make those indicators effective. (The frequency in this the modality is presented in the responses of the respondents is indicated between brackets.)

Responses given can be grouped into four modalities (a, b, c, and d) based on their contents as indicated below:

a) Knowledge: In **G1**: (4) Resorts to previous knowledge; (2) Transposes knowledge to a new situation; (1) Has autonomy to decide on certain applications. In **G2**: (3) Solves new problems; (1) Recognizes shapes regardless of their spatial orientation; (1) Defines geometrical objects by their properties; (1) Uses concepts in new situations; (1) Distinguishes concepts.

b) Reasoning: In **G1**: (3) Faces the deduction of a theorem by themselves; (2) Makes anticipations with a right answer; (1) Recognizes the validity of the reciprocal of a property. In **G2**: (1) Recognizes missing or unnecessary data.

c) Argumentation: In **G1**: (2) Provides examples and counterexamples; (1) Provides simple explanations to solve problems; (1) Has right ideas to provide the argumentation. In **G2**: (1) Provides grounds and defends their ideas.

d) Language: In **G1**: (1) Represents a concept in different semiotic registers. In **G2**: (3) Verbalizes adequately; (1) Carries out a representation providing statements and relations without memorization.
It is observed that G1 grants more importance than G2 to skills related to logical reasoning as well as the independent application of contents. Both groups point out that questions made by students are indicators of conceptual acquisition.

Learning difficulties: refers to difficulties which may be encountered by some students to understand geometrical content, particularly, the ones related to didactic configurations of teachers.

G1: in general, they all point to teaching strategies as a potential obstacle; yet, they do not specifically mention the training they received in the teacher training course in this respect.

G2: four teachers acknowledge the poor training received in geometry and didactics. On this basis they account for the lack of criteria to choose the adequate strategies for teaching these contents. A teacher points out that the quick introduction of formulas becomes an obstacle. Two mention that it is not easy for the teacher to consider all the difficulties of all the students as they lack sufficient time and/or specific psycho-pedagogical training.
Five teachers of G2 mention issues specific to language and reading comprehension, which are not identified in G1, for learning in general and mathematical learning in particular.
Both groups again point to the lack of previous knowledge which students should credit to access the education level and the lack of study habits as external factors which hinder the learning of geometry. They agree they are closely associated with difficulties in the teaching of geometry.
Comprehension achieved: we analyzed whether the teacher thinks that good teaching is enough for the student to understand a given geometrical concept in class or other components are needed.

G1: they consider aspects (students' attitudes towards knowledge, contributions of teacher's practice in class, conditioning factors of the social context) which might favor students' comprehensive processes in Mathematics and in geometry in particular. As factors restricting such process, they recognize external factors (emotional, historical-contextual, social factors) rather than didactic factors.

G2: numerous aspects emerge, both personal, didactic and disciplinary, resulting from everyday classes.
None of the respondents in G1 point to explanation or teaching strategies as possible factors restraining comprehension, unlike observed in four statements made in G2 as regards explanations which are varied and repeated if necessary, as well as the selection of motivating activities to start a unit and problem scenarios to work on common mistakes made by students. As regards work after the class, clear differences are observed in the scope provided: G2 talks of "exercising" while G1 talks of "study" in a broader sense.

DIMENSION 4: Considering the planned didactics of the teacher
Specific teaching resources: accounts for the teacher's aim that the learner should use different senses to build geometrical knowledge.

Both groups show analogous considerations as regards the types of specific teaching resources mentioned. In G2 more specificities can be recognized regarding implementation, and respondents include accounts of their daily classroom activities. It is particularly remarkable that in both groups some teachers demand training in the use of teaching resources in the Mathematics Teacher course, which suggests poor work done with these resources. This is in contradiction with the importance granted by both groups to the use of teaching resources in geometry classes. The reasons put forward by both groups reflect the potential assigned to the use of teaching aids to reinforce geometrical contents from the concrete and to achieve a more solid abstraction.

Geometrical properties: refers to certain specific approaches chosen and provides insight on how the teacher organizes their activities, what their aims are for each stage and how they shape their geometry to be taught.
Both groups show similar responses as regards the demonstration ability as a non-automatic process acquired after years of study. It is observed that:

G1: both groups also value the demonstration of geometrical properties in student instruction processes significantly.

G2: most teachers report not using demonstrative processes due to time factors. Half of them value inductive processes in SE, with generalizations made almost immediately from a few examples (as pointed out by one respondent).

Didactic strategies selection: refers to teachers' didactic criteria to prepare their classes. Their choices hint their conception of teaching, geometry and of the teaching of geometry.
Similarities can be observed in both groups, both in the kind of aspects considered and in the frequency with which they are mentioned.

G1: (8) Student-centered; (7) Attention to what goes on in class; (6) Related to the mathematical content to be taught; (3) Available time.

G2: (10) Student-centered; (10) Attention to what goes on in class; (9) Related to the mathematical content to be taught; (6) Available time. They introduce two other aspects associated with ongoing education: (2) Renewal of strategies after the reading of specific literature and/or peer exchange; (2) Games and research strategies.

DIMENSION 5: Considering the teacher's didactics in action
Learning difficulties: provides insight on issues specific to geometry in which the teacher's self-assurance is conditioned or limited.

While all the references of G1 involve specific contents, G2 associates difficulties to the poor reflection to interpret and justify what is done. They demand work that departs from the concrete. A third part refers to the abstract concepts which contradict common sense; without being made explicit, references allude to the passing through successive levels proposed by Van Hiele (1986).

Problem resolution, a working methodology considered as priority and transversal –in line with DCJ proposals (1999)–, is highlighted as a difficulty by both groups.

G2 specifies the difficulty of presenting abstract concepts, but does not point out what happens in the transition stage from concrete to abstract, while G1 does.

Besides demonstration processes, which are also mentioned by G1, G2 mentions the understanding of applications. As regards daily life in the classroom, three teachers of G2 mention factors (such as previous knowledge) that lead to changes in the planned lesson, which is not mentioned in G1.
Good learning: the word "good" has both moral and epistemological force, in terms of teaching actions as seen from the morally and reasonably justifiable (Fenstermacher, 1989).
Generally speaking, both groups show similar aspects which can be grouped into four modalities:

- Concerning the disciplinary: in **G1**: (3) Knowledge of the discipline, and in **G2:** (13) Teacher confident of what they know.
- Concerning the teacher's attitudes: in **G1**: (3) Teacher as responsible for the process; (2) Willingness, calling for teaching. In **G2**: (13) Teacher's role and responsibility; (3) Willingness to teach; love for the subject; feels useful when teaching.
- Concerning the teaching strategies: in **G1**: (3) Teaching experience to see to the students' difficulties; (3) Observation of students' production; (1) Need of further experience in the training of teaching strategies. In **G2**: (13) Group management; adequate didactic transposition; (4) Attention to students' performance, to be considered for future interventions; (2) Theme planning in advance, knowledge of possible resolutions, class structure resulting from questions; (2) Class not purely expositive.

- Concerning conditioning factors: in **G1**: (2) Teaching of empty processes to move on with the syllabus. In **G2**: (2) Lack of time; the priority is moving forward in content presentation and reflections are not always promoted.

G2 presents more details of situations on the teacher's role and responsibility in the creation of a reflexive class of geometry in the first two years of SE. The omission of the knowledge of specific didactics reported in G1 is striking (only one respondent mentioned having experienced teaching strategies from the training stage), though some elements do appear in G2.

Evaluation: we focused more on specific school situations of decision-making and analyzed coherence in relation to answers to other questions.

Both groups agree on the need for individual written texts, as well as other evaluation instances, agreeing upon the hierarchization of basic contents. G1 points out that the student is expected to interpret the proposed situations. This aspect was not particularly mentioned by G2, though justified resolutions are expected. The third part of G2 refers to "the basics" to let students pass, but there remain doubts over this scope. Half of the teachers of G2 assess students' class work, an aspect not mentioned by G1.

Both groups grant little importance to attitudinal aspects when evaluating students.

DISCUSSION OF RESULTS

Figs. 2 and 3 synthesize the results obtained in both groups, related to the "Components of the geometry of the teacher and domains of mathematical knowledge for teaching" (Fig. 1). Comparatively, these results show the following:

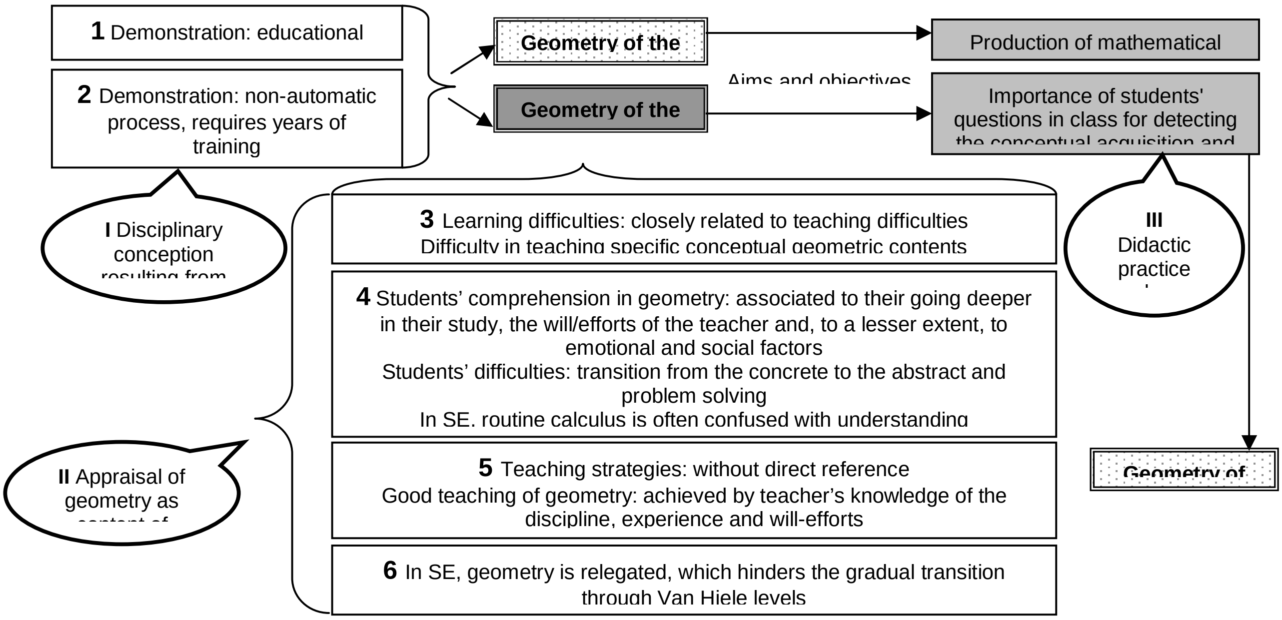
Component I: a distinction can be observed in the knowledge of the mathematical horizon (domain 2): G1 hierarchizes transversal skills of the whole curriculum, while G2 focuses its look and restricts it to what is normally done in a school year in order to cover the syllabus. A distinction is observed regarding the knowledge of the mathematical horizon, with a global perspective, in G1. G2 show a restriction in domains 1 and 2 of mathematical knowledge for teaching, supported in what is normally done in SE classes.

Component II: G2 report aspects of the knowledge of content and students (domain 4) with much more detail, as a result of the daily experience with the SE. This, in turn, favors the configuration of component III of the *geometry of the teacher.*

As regards specific teaching strategies, there appears a weakened knowledge of content and teaching (domain 5). In both groups, specialized knowledge of content (domain 3) and of content and teaching (domain 5) appear feeble, lacking an intentional planning. In order to characterize good teaching of geometry, G1 is more inclined towards the disciplinary mathematical than G2.

Differences have been found between the two groups regarding the didactic device for comprehension in geometry: G1 mentions *study* on the students' part, and G2, *exercising*; G2 refers to teachers' teaching strategies and G1 leaves it to the teacher's discretion (domain 3). It is inferred that this could indicate a distinction of teaching practices common in the classroom and maintained over time. G1 encourages studying, in the sense of understanding geometry from its theoretical-practical basis for establishing relations and arguing effectively. In G2 this practice is restricted to the reproduction and repetition of techniques, with no analysis of its basis. It would seem that G2, in its attempt to make geometry more accessible to students, promotes a mechanical, routine calculus. This is unlikely to contribute to progress in Van Hiele levels (1986). Difficulties seem to be framed within what group members usually prioritize in their teaching practices: the mathematical conceptual in G1 and the mathematical procedural in G2 (domain 4).

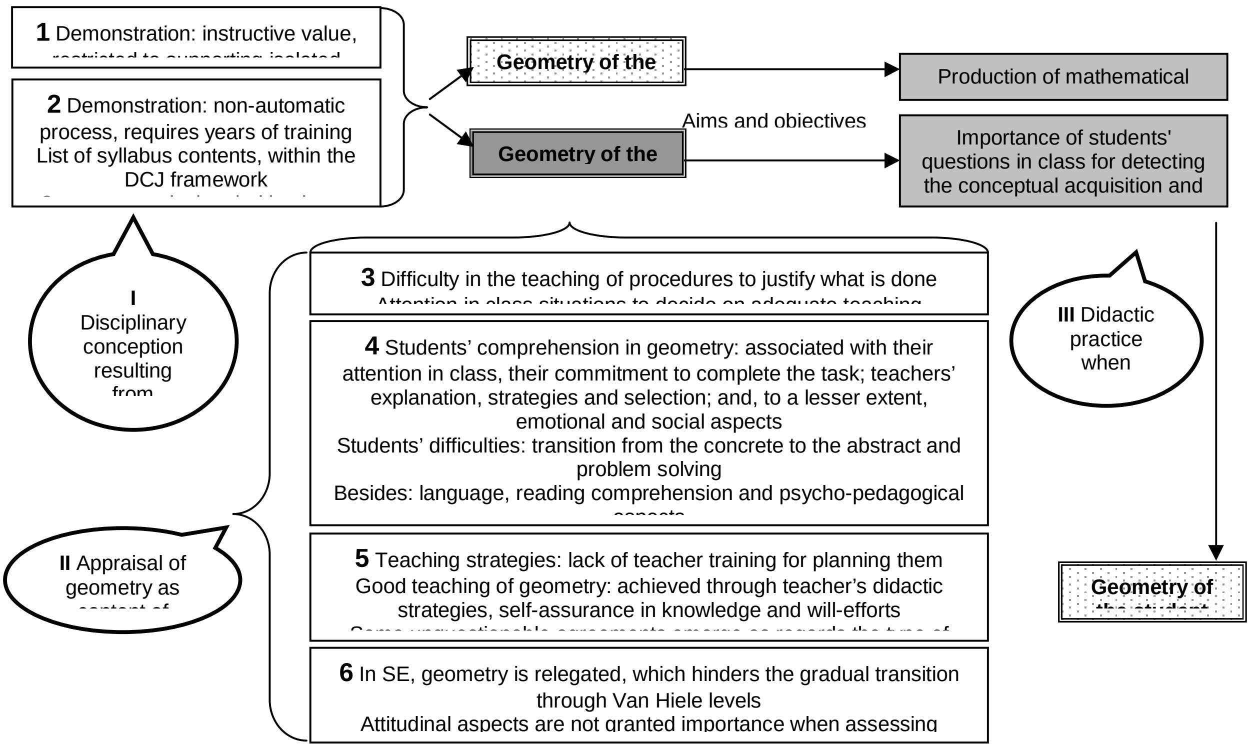
FIGURE 2. ***Geometry of the teacher*** **according to G1**



Significant differences in assessment criteria can also be observed, with direct references to domain 4 (knowledge of students). Teachers working in SE (G2) consider current social issues, which becomes a tool which can improve or delay a specific pedagogical proposal.

Component II: in both groups a certain weakening of component III of the *geometry of the teacher* can be detected, as teachers do not know how to build on cognitive-affective processes of learners, immersed in concrete, real and current contexts.

FIGURE 3. ***Geometry of the teacher*** **according to G2**



FINAL REFLECTIONS

References to geometrical skills and their sequential development appeared more frequently in G1. None of the teachers made a explicit reference to Van Hiele levels (1986) as a possible theoretical reference associated with such skills. The importance of *intuition* for geometrical knowledge in the first two years of SE was virtually not mentioned. Although in some occasions teachers used the word *visualization,* its meaning did not reach Van Hiele's conceptual linking (1986).

The little importance given to drawing and construction skills as an indicator of acquisition of concepts in geometry is striking, as these skills require, on a conceptual level, much more than motor skills. Constructions can prove very fruitful for promoting exploration, elaboration, discovery and application of properties in SE (Barrero, Beltrán, Bifano, Carpintero, Fioriti, Giuliani, Sessa & Veiga, 2007). Constructions are useful for students to interact with the properties of shapes, so that they can progress through Van Hiele levels (1986). The various ways of managing constructions in class imply diverse opportunities for students to develop geometrical knowledge. Such constructions may help to: revise some elements and relations characterizing shapes; visualize drawing not as a whole but through its properties; decide which information is necessary and enough to carry out a certain geometrical construction; elicit conceptions in relation to shapes under study; anticipate construction procedures; and validate their own production of drawings (Barrero et al, 2007).

In broad terms, G2 recognized the lack of training in geometry and its didactics to make essential decisions in their teaching tasks in SE. This restriction was not mentioned by G1, which also evidenced the lack of references to the specific didactics when being asked about the requirements for good teaching, with answers strictly associated with the teacher's will. Does this show a certain *indifference* for the didactic aspect by staff in charge of training teachers? Is teaching seen as an art or trade? G1 thus disregards the domains of knowledge for teaching related to the pedagogical (2 to 6), which make up component II of the *geometry of the teacher.* These results match conclusions drawn by Barrantes & Blanco (2006).

From the study, it is not clear what the G2 teachers base on to select, hierarchize, sequence and schedule geometrical content they teach in the first two year of SE, which is linked to the three components of the *geometry of the teacher*, as this is related to mathematical, didactic and situated-practice aspects.

Both groups mention previous lacking content to work efficiently in the classroom. The students they work with are not the ones they would *ideally* expect, and it is often *others* who are responsible for this deficit*:* teachers of the previous year, family, youth, etc. This result matches conclusions drawn by Barrantes & Blanco (2006).
In G2 there is evidence of a kind of appraisal poorly justified in epistemological and cognitive terms about the use of time allocated to geometry, an extremely important factor as an indicator of component II of the *geometry of the teacher*. Four teachers who consider it necessary to work before the number axis to "be able to operate in geometry" even show a shifting of the treatment of notions and skills strictly geometrical towards other axes (such as Measures or Numbers and Operations), also observed by Broitman & Itzcovich (2008).

From the wide range of skills mentioned when referring to the use of teaching aids, it can be inferred that the use of aids should occur in different moments, with different aims, promoting the development of different skills which go beyond the very manipulation and contribute to a spirit of inquiry.

In clear contrast is the importance given to teaching aids in geometry in the first two years of SE and the virtually null learning on its use during their initial teacher training. In terms of reflection, this does not promote the generation of knowledge inherent to domains 2 to 6, affecting the consolidation of component II of the *geometry of the teacher.*

G2 members hierarchize the *quantity* of content instead of the instruction in demonstrative processes. This evidences a contradiction between what *should be done theoretically* and what it is said to *be really done*, as stated by half the group. Teachers restrict the activity of demonstration of properties to the founding of statements; thus, the logical sequences of demonstrative processes are not being approached. This contrasts with what they consider right to reach a deep understanding of geometrical content.

The attention paid by the teacher in deciding the right strategies in the very class has a strong presence, its dynamics being characterized by the simultaneity of events and the immediacy of actions by the involved actors, which results in its complex nature. Half of the teachers in G2 recognize they lack the time to plan specific teaching strategies which could be more adequate. This suggests there is a risk of practicing the teaching profession as a mere art and shows the value granted to empirical learning of the very teacher based on their will or wish to teach ("the teacher *is made* in the classroom").

In G2, as so many daily aspects of their classes emerge, most of their comments can be thought of as framed within a specific *situated* didactics (seen as a teaching practice when teaching geometry in SE –component III of the *geometry of the teacher*) on the one hand, and on the other, as linked to the mere experience without a systematic theoretical reflection.

As for the didactic strategies, three teachers of G2 value the practice on the grounds that "you learn how to teach teaching". This leads us to think of a dynamic *geometry of the teacher*, meaning it is being constructed constantly, although we could run the risk of seeing it as a mere *art* in didactic terms.
Both groups consider time a conditioning factor of reflexive processes during classes, which again shows a tension between quality and quantity of content.
Thus, if we resume initial questions, it should be noted:

- *On the influence of the teacher training in the teaching of geometry:* it is crucial to grant instructive value to geometry and be able to select essential didactic strategies.
- *On the eventual modifications of the relation of the teacher with geometry to teach it:* these modifications do not appear very marked. It would seem most teachers teach "the way they have been taught".
- *On the conceptions of the SE teacher about the role of geometry in the instruction of an adolescent starting to develop formal thinking:* it is granted relevance in statements made, but not in practice, where other content axes prevail.

The study shows different approaches adopted by G1 and G2 in relation to teaching geometry in SE. Bringing the positions of teacher trainers and teachers working in the profession closer requires developing possibilities for a joint, reflexive work. Jaworski (2008) values this kind of experiences, and Shoenfeld & Kilpatrick (2008) claim that reflection is key for the teacher's professional development in different levels, going from the "what" to the "why", which goes through the different domains of mathematical knowledge for teaching. If the exchange of knowledge in the classroom was indicated as an aim of the *geometry of the teacher* (Fig. 1), it is considered that this exchange can be much more fruitful if the teacher themselves experiences instances of exchange of knowledge with other teachers. Such exchanges could contribute to good teaching of geometry (Jones, 2002) which, in short, entails understanding that a full, rich geometrical education is possible in SE and promoting versatile practice principles (Flores, 2007) which help the teacher to constantly renew their domains of mathematical knowledge for teaching in a conscious, well-grounded way.

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