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AN IDEA OF FERMAT FOR THE STOP AND DIVISION BY ZERO CALCULUS

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ABSTRACT. In this note we will consider an idea of Fermat for the stop in connection with the division by zero calculus. Here, in particular, we will see some mysterious logic on the stop in connection with the concepts of differential and differential coefficient.

1. Introduction

For a fixed $\ell > 0$, we will consider the area S(a) by

$$S(a) = a(\ell - a)$$

and the maximum of S(a) for $0 \leq a \leq \ell$. Of course, this problem is very simple with elementary calculus.

However, Fermat (1629) considered this problem in the following way:

Assume that

$$(a + \epsilon)(\ell - a - \epsilon) = a(\ell - a). \tag{1.1}$$

Then, formally we have the identity

$$\epsilon^2 + \epsilon(2a - \ell) = 0. \tag{1.2}$$

See [1], 358-359.

From this logic and identity, could we obtain the desired result

$$a = \frac{\ell}{2} \tag{1.3}$$

?

Note that

$$S'(a) = \ell - 2a.$$

Firstly, note that the identity (1.1) is not valid except $\epsilon = 0$ and $\epsilon = \ell - 2a$, as we from the representation of $S(a)$; that is the identity is nonsense when we consider a small variation. Of course, (1.2) is valid for $\epsilon = 0$ and $\epsilon = \ell - 2a$. So, we wonder the above logic is nonsense.

The logic is not on any variation of the area $S(a)$ essentially that may be related to differential and differential coefficient.

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2. Interpretation by the Division by Zero Calculus

In the formal formula (1.2), from the identity

$$\frac{\epsilon^2}{\epsilon} + (2a - \ell) = 0, \tag{2.1}$$

we obtain the desired result (1.3) at $\epsilon = 0$.

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References

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