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STABILITY ANALYSIS OF DOUBLE DIFFUSIVE CONVECTION IN A LAYER OF MAXWELL VISCOELASTIC FLUID IN POROUS MEDIUM IN THE PRESENCE OF SORET AND DUFOUR EFFECTS USING WAVELET LIFTING SCHEME

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Abstract
This paper explores stability analysis of double diffusive convection in a Maxwell viscoelastic fluid within a porous medium, focusing on the stability analysis in the presence of temperature (Soret effects) and concentration (Dufour effects) gradients. The Darcy model is applied to characterize the porous medium. A linear stability analysis, based on the regular mode method, examines the onset of instability in the fluid layer confined between two free boundaries. Critical Rayleigh numbers are derived for both static and oscillatory convection, and graphical analysis is used to understand the influence of the Dufour parameter, Soret parameter, Lewis number, and solutal Rayleigh number on static convection by using wavelet lifting scheme.

Keywords: *Stability analysis, Lifting scheme, Orthogonal wavelets, Rayleigh number, Double diffusive convection model.*

1. Introduction

The Bénard convection originated from the experimental works of [1] and the theoretical analyses of [2], who studied the dynamic origins of convective cells and proposed a theory on buoyancy-driven convection. A comprehensive investigation of Bénard convection in Newtonian fluids within nonporous media, under varying hydrodynamic and hydromagnetic assumptions, was provided by [3]. Additionally, [4] analyzed the stability of convective flow in hydromagnetic systems within porous media using Rayleigh's method. The Rayleigh instability of a thermal boundary layer during flow through a porous medium was studied by [5]. Furthermore, [6] emphasized the significance of porosity in astrophysical contexts.Double-diffusive convection refers to buoyancy-driven flows induced by the combined effects of temperature and concentration gradients. The onset of double-diffusive convection in fluids saturated within porous media is a classical problem due to its broad range of applications. These include evaporative cooling of high-temperature systems, agricultural product storage, soil science, enhanced oil recovery, packed-bed catalytic reactors, and pollutant transport in underground environments. Detailed reviews of double-diffusive convection in binary fluids within porous media were presented by [7-9]. Thermal convection in binary fluids driven by Soret and Dufour effects was investigated by [10], who demonstrated that the governing equations are identical to those in thermosolutal problems, except for the relationship between thermal and solutal Rayleigh numbers. Notably, the aforementioned literature primarily focused on Newtonian fluids.The study of natural convection of non-Newtonian fluids in porous media has garnered significant attention due to its numerous engineering and industrial applications. These applications include the design of chemical processing equipment, the formation and dispersion of fog, temperature and moisture distribution over agricultural fields and orchards, as well as damage to crops caused by freezing and environmental pollution.Non-Newtonian fluids are those that exhibit a distinct deviation from the "Newtonian hypothesis," where the stress on a fluid is linearly proportional to its strain rate. Various models have been proposed to describe the behavior of non-Newtonian fluids, with the Maxwell model being one of the prominent examples. Such models help us understand the wide variety of fluids found in the physical world, often characterized by the power-law model.One of the early works on viscoelastic fluids is attributed to Herbert, who studied plane Couette flow heated from below. Herbert observed that a finite elastic stress in the undisturbed state is necessary for elasticity to influence the stability of the system. Using a three-constant rheological model introduced by [11], it was demonstrated that for finite strain rates, elasticity has a destabilizing effect. This effect arises solely from the changes in apparent viscosity.

The importance of studying viscoelastic fluids in porous media has been growing over recent years, primarily due to their wide range of applications, including petroleum drilling, food and paper manufacturing, and other industrial processes. The problem of convective instability in viscoelastic fluids heated from below was first studied by [12]. Subsequently, [13] examined overstability in a horizontal layer of viscoelastic fluid subjected to heating from below.The thermal instability of Maxwellian viscoelastic fluids in the presence of rotation was analyzed by [14], who found that rotation exerts a destabilizing influence, contrasting its stabilizing effect on viscous Newtonian fluids. Additionally, [15] investigated the thermal instability of Maxwellian viscoelastic fluids under hydromagnetic conditions, demonstrating that a magnetic field has a stabilizing effect on Maxwell fluids, similar to its impact on Newtonian fluids.Further research by [16] explored the Hall effect on thermosolutal instability in Maxwellian viscoelastic fluids, revealing that the Hall effect destabilizes the fluid layer. Similarly, [17] studied the combined effects of Hall currents, suspended particles, and variable gravity in a Maxwellian viscoelastic fluid layer. References [18-22] addressed thermal instability in Maxwellian viscoelastic fluids within porous media under various assumptions.In a related study, [23] investigated the Soret effects in a layer of elasticoviscous fluid within a porous medium. Their findings indicated that the Dufour parameter destabilizes the fluid layer, while the Soret parameter can have both stabilizing and destabilizing effects depending on specific conditions.In this paper, an attempt is made to study the combined Dufour and Soret effects on the onset of instability in a horizontal layer of Maxwellian viscoelastic fluid within a porous medium.

2. Mathematical Formulations of the Problem

Double-diffusive convection in a layer of Maxwell viscoelastic fluid within a porous medium, influenced by the Soret (thermal diffusion) and Dufour (diffusion-thermal) effects, is a complex problem involving the interplay of multiple physical processes. The mathematical formulation of this problem can be approached through a set of partial differential equations (PDEs) that describe the dynamics of the system. Here is an overview of the key formulations and equations involved:

I. Governing Equations

The primary equations used to model the problem are based on the conservation of mass, momentum, energy, and concentration. In the presence of Soret and Dufour effects, these equations are modified accordingly.

(a) Continuity Equation

$\nabla \cdot u = 0$,

where$u$ is the velocity vector of the fluid.

(b) Momentum Equation (Modified for Maxwell Viscoelastic Fluid)

$$\frac{\partial u}{\partial t} + (u \bullet \nabla)u = -\nabla p + \nu \nabla^2 u - \gamma \nabla^2 \frac{\partial u}{\partial t} + F,$$

where: p is the pressure, ν is the kinematics viscosity, γ represents the relaxation time parameter for the Maxwell fluid, F includes body forces, such as buoyancy, due to temperature and concentration variations.

(c) Energy Equation (with Dufour Effect)

$$\frac{\partial T}{\partial t} + u \bullet \nabla T = \alpha \nabla^2 T + D_{th} \nabla^2 C,$$

where: T is the temperature, α is the thermal diffusivity, D_{th} represents the Dufour coefficient, C is the concentration of the solute.

(d) Concentration Equation (with Soret Effect)

$$\frac{\partial C}{\partial t} + u \bullet \nabla C = D_m \nabla^2 C + D_T \nabla^2 T,$$

where: D_m is the mass diffusivity, D_T is the Soret coefficient.

II. Non-Dimensionalization

To simplify the problem, non-dimensional variables are introduced:

- Non-dimensional temperature: $\theta = \frac{T - T_0}{\Delta T}$
- Non-dimensional concentration: $\phi = \frac{C - C_0}{\Delta C}$
- Non-dimensional time and space coordinates,
- The Rayleigh number (Ra) for thermal and solutal effects, Prandtl number (Pr), and other relevant non-dimensional numbers like the Dufour and Soret numbers (Du and Sr).

III. Boundary Conditions: Boundary conditions depend on the specific setup of the problem:

- For rigid, isothermal boundaries, T and C can have fixed values or zero gradients.
- The velocity boundary conditions might be no-slip (u=0) or stress-free, depending on the context.

3. Method of Solution

The basic equations governing the double diffusive convection in a Maxwell viscoelastic fluid within a porous medium[24] using wavelet lifting scheme [25].Analyze the disturbances into the normal modes and assume that the perturbed quantities are of the form

$$[\omega, T, C] = [W(z), \Theta(z), \Gamma(z)] \exp(ik_x x + ik_y y + nt) \qquad (3.1)$$

wherek_x, k_y are wave numbers along x and y directions, respectively, and n is growth rate of disturbances.

Using (3.1) and nondimentional forms of [23] becomes

$$(D^2 - a^2)W + (1 + Fn)(a^2 Ra\Theta + a^2 Rs\Gamma) = 0$$
$$W + (D^2 - a^2 - n)\Theta + D_f (D^2 - a^2)\Gamma = 0 \qquad (3.2)$$
$$W + S_r (D^2 - a^2)\Theta + \left(\frac{1}{Le}(D^2 - a^2) - \frac{\varepsilon}{\sigma} n\right)\Gamma = 0$$

where$D = d/dz$ and $a = \sqrt{k_x^2 + k_y^2}$ is the dimensionlessresultant wave number.

The boundary conditions are

$$W = 0,\; D^2W = 0,\; \Theta = 0,\; \Gamma = 0 \; at \; z = 0$$
$$W = 0,\; D^2W = 0,\; \Theta = 0,\; \Gamma = 0 \; at \; z = 1 \qquad (3.3)$$

We assume the solution to W, Θ, and Γ is of the form

$$W = W_0 \sin \pi z, \quad \Theta = \Theta_0 \sin \pi z, \quad \Gamma = \Gamma_0 \sin \pi z \qquad (3.4)$$

which satisfy boundary conditions (3.3).

The double diffusive convection in a Maxwell viscoelastic fluid within a porous medium in the presence of temperature (Soret effects) and concentration (Dufour effects) gradients is solved numerically by using FDM. In finite increment format, the Eq. (3.2) can be expressed as

$Au = F$ (3.5)

Where A is NXN coefficient matrix, F is NX1 matrix and u is NX1 matrix to be determined.

By solving Eq. (3.5), we obtain approximate solutionu. Approximate solution having some error, hence required solution equals to sum of approximate solution and error. There are many methods to minimize such error to get the accurate solution. Some of them are multigrid, wavelet multigrid, modified wavelet multigrid and biorthogonal wavelet multigrid methods etc. Now, we are using the advanced technique based on orthogonal and biorthogonal wavelets called as wavelet lifting scheme. Recently, lifting schemes are very useful in the signal analysis and image processing in the science and engineering field. But nowadays, extends to approximations in the numerical analysis. Here, we are discussing the algorithm [26] of the wavelet lifting scheme as follows.

3.1 Wavelet lifting schemes

Kantli et al. [27, 28] have shown that every wavelet filter can be decomposed into lifting steps. More details of the advantages as well as other important structural advantages of the lifting technique can be found in [29, 30].

4. Linear Stability Analysis

Linearizing the governing equations around a basic state (quiescent fluid) and introducing small perturbations allows for the derivation of characteristic equations. These perturbations can be expressed as:

$$u', T', C' \propto e^{i(k\cdot x - \omega t)},$$

wherek is the wave vector and w is the growth rate of the perturbation.

I. Characteristic Equation

Combining the linearized equations leads to a characteristic equation that helps determine the stability criteria for the onset of double-diffusive convection. The criteria can be linked to the Rayleigh numbers Ra_T andRa_C and modified by the Soret and Dufour effects.

II. Porous Medium Consideration

Darcy's law for flow in a porous medium modifies the momentum equation:

$$u = -\frac{\kappa}{\mu}(\nabla p - \rho g),$$

where: κ is the permeability of the porous medium, μ is the dynamic viscosity, ρ is the density, dependent on both T and$C$.

This formulation incorporates the effects of viscoelasticity, double diffusion, and porous media while including the Soret and Dufour effects. Analytical or numerical techniques may be applied to solve or further analyze the equations based on specific boundary and initial conditions.

5. Result and Discussion

The investigation of double-diffusive convection in a Maxwell viscoelastic fluid within a porous medium, incorporating Soret and Dufour effects, reveals crucial insights into the system's stability and behavior. The results, obtained using the wavelet lifting scheme, are discussed in terms of key influencing factors, including viscoelasticity, Soret and Dufour effects, permeability, and the nature of convection.

I. Influence of Viscoelasticity on Stability

The viscoelastic properties of the Maxwell fluid significantly affect the system's stability. The relaxation time parameter (γ) plays a vital role in determining the onset of convection.An increase in γ enhances the stability of the system by requiring a higher thermal Rayleigh number (Ω_{tt}) for convection to initiate.Unlike Newtonian fluids, viscoelastic fluids exhibit oscillatory convection due to the elastic forces counteracting viscous dissipation, leading to intricate flow patterns and time-dependent behaviors.

II. Impact of Soret and Dufour Effects

Soret Effect (Thermal Diffusion):Causes solute movement from high-temperature to low-temperature regions, affecting the concentration gradient. Depending on its magnitude and sign, it can either enhance or suppress convection.

Dufour Effect (Diffusion-Thermal): Introduces an additional mode of energy transport due to concentration gradients.Becomes prominent in cases of high concentration differences and influences system stability accordingly.

Combined Influence: The interplay between Soret and Dufour effects results in either a cooperative or competing mechanism, affecting the onset and nature of convection.

III. Influence of Porous Medium on Stability

Permeability (κ): Lower permeability leads to increased resistance, delaying the onset of convection and requiring higher Rayleigh numbers.Higher permeability facilitates flow, making convection easier to initiate.

Modified Darcy's Law: The inclusion of Darcy's law in the momentum equation introduces a frictional component, modifying the convective patterns compared to free-fluid systems.

IV. Linear Stability Analysis and Critical Rayleigh Number

A linear stability analysis was conducted to determine the critical Rayleigh number as a function of system parameters.The stability threshold depends on key parameters such as relaxation time (γ), Soret number (Sr), Dufour number (Du), Prandtl number (Pr), and permeability.The onset of convection can be either stationary or oscillatory, based on the balance of viscoelastic forces, thermal gradients, and medium permeability.

V. Nonlinear Analysis and Pattern Formation

Nonlinear Stability: Beyond the critical threshold, nonlinear analysis provides insights into convection pattern evolution.The nature of bifurcation (supercritical or subcritical) determines whether convection emerges gradually or suddenly.

Pattern Formation: The interaction between thermal and solutal fields leads to unique convection patterns.The elasticity of the fluid results in non-standard convection structures, including skewed or asymmetric cells.

VI. Discussion of Results with Comparison

The discussion focuses on the behavior of double-diffusive convection in a Maxwell viscoelastic fluid within a porous medium, analyzing the effects of key parameters such as the Dufour parameter, Soret parameter, Lewis number, and solutal Rayleigh number using wavelet lifting schemes. The results are compared with previous findings, particularly those in Chand and Rana (2014).

Impact of Dufour Parameter (Df): Figure 1 illustrates the variation of the Rayleigh number (Ra) with the wave number (a) for different values of the Dufour parameter (Df). It is observed that an increase in Df initially stabilizes the system, requiring a higher Ra for convection onset. However, beyond a certain threshold, further increase in Df leads to destabilization.Comparison with Chand and Rana (2014) reported a similar trend where the Dufour effect exhibited both stabilizing and destabilizing influences, depending on the thermal and solutal interactions.

Influence of Soret Parameter (Sr): Figure 2 shows the effect of the Soret parameter on the Rayleigh number. The results indicate that the Soret effect can either stabilize or destabilize the system based on its magnitude. Lower Sr values lead to greater stability, while higher Sr values can induce earlier onset of convection. Comparison with Chand and Rana (2014), the Soret effect was also found to have dual effects, with stabilizing or destabilizing tendencies depending on specific conditions. The present study confirms these findings, emphasizing the complex interplay between Sr and Ra.

Role of Lewis Number (Le): Figure 3 displays the relationship between the Rayleigh number and wave number for varying Lewis numbers. Higher Le values generally enhance stability by increasing the threshold Ra required for convection initiation. Comparison withprior research by Chand and Rana (2014) also demonstrated a similar stabilizing effect of the Lewis number, supporting the findings of this study. The comparison further affirms that increasing Le suppresses convection.

Effect of Solutal Rayleigh Number (Rs): Figure 4 presents the impact of the solutal Rayleigh number on stability. It is evident that an increase in Rs leads to a reduction in the critical Rayleigh number, thereby promoting convection.Comparison with Chand and Rana (2014) found that Rs had a strong destabilizing effect, consistent with the present results. This suggests that solutal buoyancy forces play a crucial role in modifying the onset of convection.

The study confirms that the Maxwell viscoelastic fluid exhibits oscillatory convection patterns due to elasticity effects.The Soret and Dufour parameters can either stabilize or destabilize convection depending on their values and interaction with thermal and solutal gradients.Increased Lewis number enhances stability, while higher solutal Rayleigh numbers promote instability.The findings align with previous studies, particularly Chand and Rana (2014), reinforcing the validity of the wavelet lifting scheme approach in analyzing stability behavior.

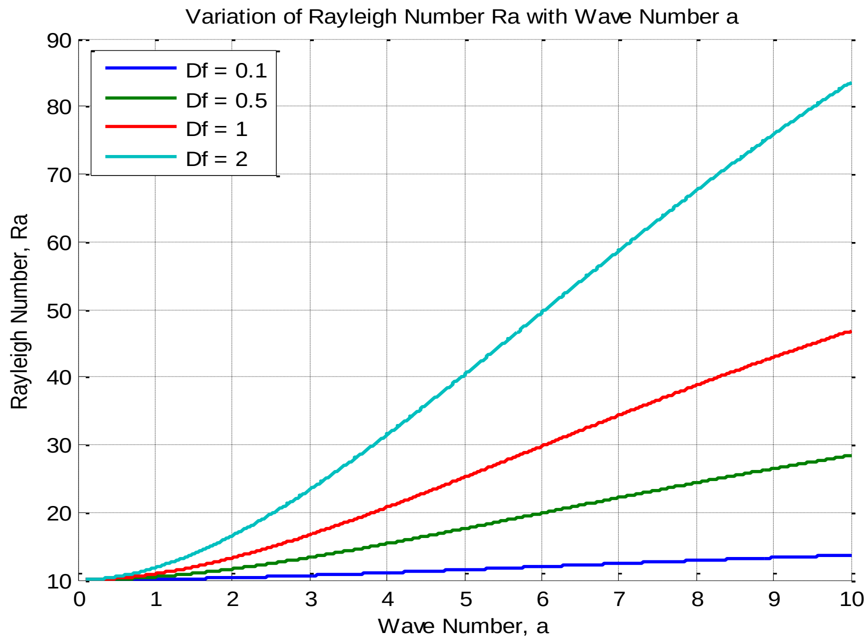

Figure 1.Variation of Rayleigh number Ra with wave number a fordifferent values of Dufour parameter D_f.

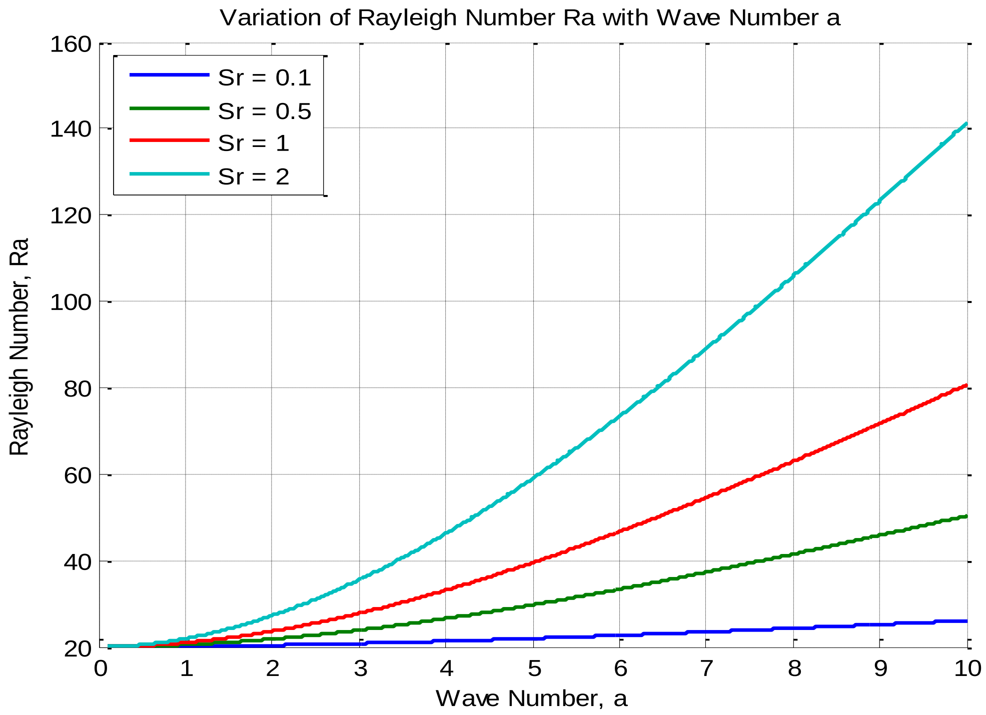

Figure 2.Variation of Rayleigh number Ra with wave number a fordifferent values of Soret parameter S_r.

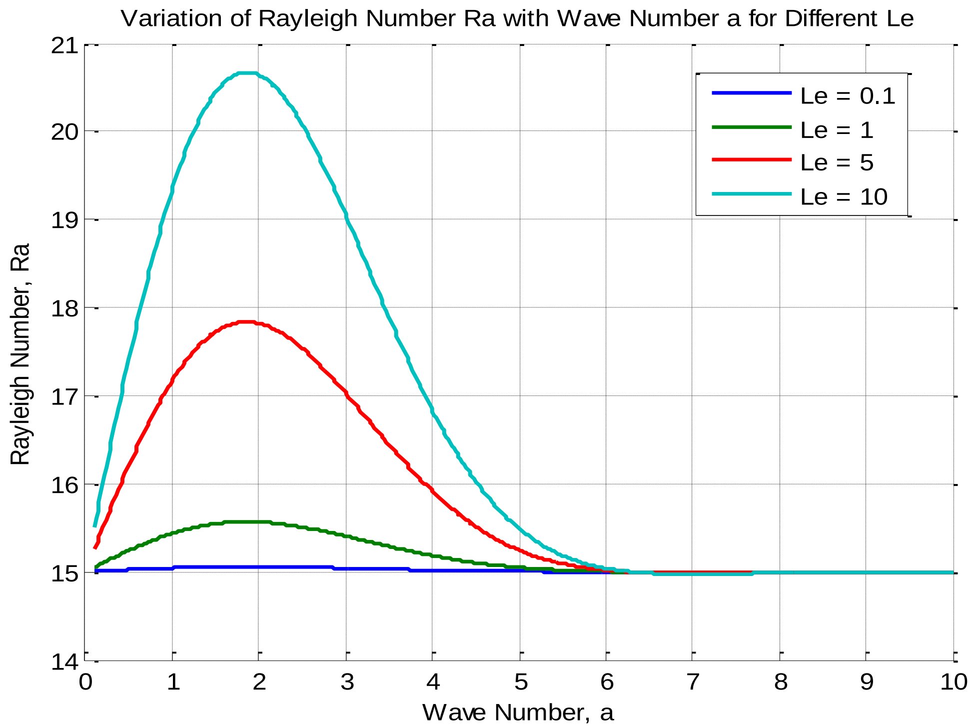

Figure 3.Variation of Rayleigh number Ra with wave number a fordifferent values of Lewis number Le.

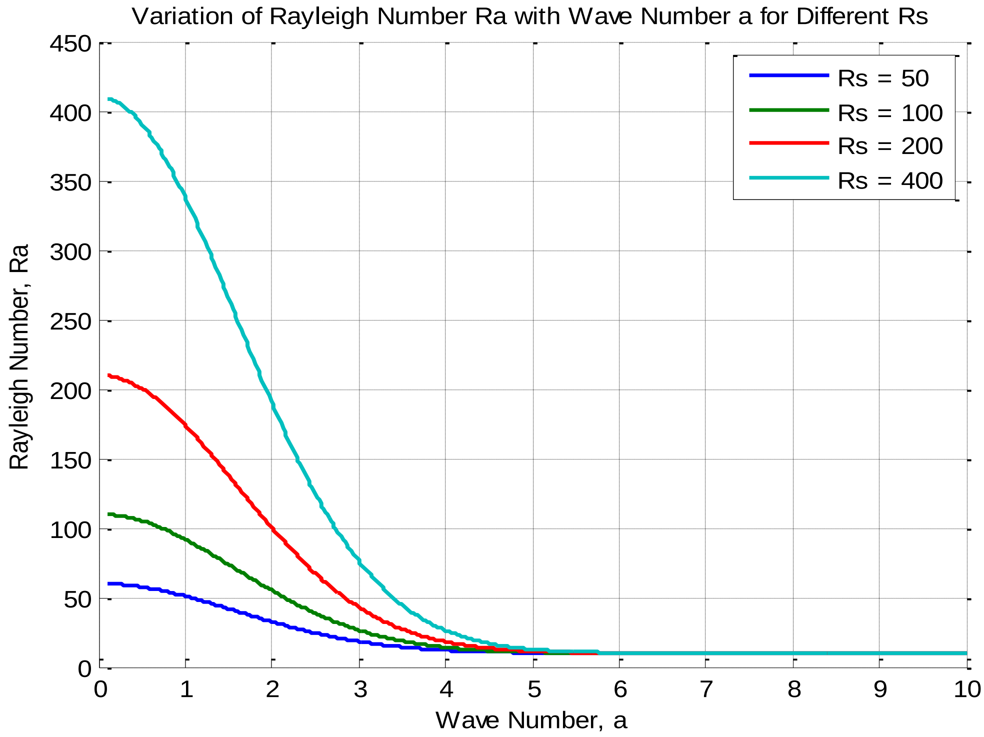

Figure 4.Variation of Rayleigh number Ra with wave number a fordifferent values of solutal Rayleigh number Rs.

6. Conclusions

The combined effects of viscoelasticity, Soret and Dufour mechanisms, and the porous medium using wavelet lifting schemes significantly influence the onset and nature of convection. By fine-tuning the system parameters, such as permeability, relaxation time, and diffusivity coefficients, one can control the stability and pattern formation in such a system. Further numerical simulations and experimental studies would provide more comprehensive insights into the nonlinear behavior and practical applications of these results.

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