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The Performance of a Supply Chain System with Three Successive Suppliers

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ABSTRACT

In this paper, we develop a general framework for the performance of one specific supply chain type. The supply chain consists of three successive suppliers, operating in a make-to-order environment. The customer demand is immediately fed back to the first supplier. All suppliers work along the same lines and produce the goods according to the first-come–first-served discipline. Once each supplier finishes its production, the goods are immediately sent to the next supplier. The performance of this supply chain is measured in terms of lead time and its constituent parts, and output. By simulation, we investigate the effect of varying different input parameters on these performance measures. While the average value of the performance measures reveals a clear pattern, the squared coefficient of variation of the performance measures does not. A specific relation between the average lead time and the output rate is also observed.

* * *

Deze paper beoogt het opstellen van een algemeen raamwerk voor de performantie van een bepaald type supply chain. De supply chain bestaat uit 3 opeenvolgende leveranciers die opereren binnen een make-to-order omgeving. De vraag van de klant wordt onmiddellijk teruggekoppeld naar de eerste leverancier. Alle leveranciers werken op een uniforme manier en produceren de goederen volgens de first-come–first-served discipline. Telkens een leverancier zijn productie beëindigt, worden de goederen doorgezonden naar de volgende leverancier. De performantie van dit type supply chain wordt gemeten in termen van doorlooptijd en zijn onderdelen en output. We onderzoeken het effect van het wijzigen van verschillende inputparameters op deze performantiemaatstaven met behulp van simulatie. We observeren een duidelijk patroon voor de gemiddelde waarde van de performantiemaatstaven in tegenstelling tot de kwadratische variatiecoëfficiënt. We merken ook een duidelijk verband op tussen de gemiddelde doorlooptijd en de output ratio.

I. INTRODUCTION

Over the past few years, supply chain management has become a key to the competitiveness of manufacturing and service companies. The main building blocks of a supply chain are procurement, production, distribution, and sales. Improvement in competitiveness relies on two factors: closer integration of the organizations involved and better coordination of materials, information, and financial flows (Stadtler and Kilger (2000)).

Along the same lines as Suri, Sanders and Kamath (1993) report on manufacturing systems, analysing and improving supply chain systems are vital to their functioning. That is why we will analyse the *coordination* of the *production* part of ‘one-of-a-kind products’.

Detailed analysis of the production part of the supply chain environment shows that time to market and high output rates are crucial factors in maintaining competitive advantage. Simchi-Levi, Kaminsky and Simchi-Levi (2000) show that the shorter the lead time and the higher the output rate, the more service customers can be offered. They also report the following advantages due to lead time reduction: reduction in the bullwhip effect, more accurate forecasts, and reduction in finished goods inventory. In addition, both the lead time and the output rate should be reliable. Short lead times and high output rates with high coefficients of variation (the coefficient of variation of a variable is the standard deviation of that variable divided by its mean) lower customers’ service levels.

It is well known that exception and variability have the greatest impact on business performance (Stadtler and Kilger (2000)). Therefore, we should focus on short, reliable lead times and high, reliable output rates. As a *first stage* in this research we should investigate the effect of the different input parameters on the average and the squared coefficient of variation of the lead time, on its constituent parts, and on the output. However, it is known from the literature that short lead times are generally not compatible with high output rates (Hopp and Spearman (2000)). Therefore, in a *second stage* we should examine how to deal with these conflicting goals. Hence, we look for those input parameters that influence lead time and output in order to decide how to change these parameters to find an acceptable ‘balance’ for the supply chain system.

We are not aware of a framework within supply chain management that describes the performance of a supply chain and tackles the relationship between the average lead time and the output rate. This is one of the main reasons for performing this analysis. Another important reason relates to our earlier statement that analysing and improving supply chain systems constitutes a vital part in their functioning.

Most books on supply chain management (see, e.g., Stadtler and Kilger (2000); Simchi-Levi, Kaminsky and Simchi-Levi (2000)) do not tackle this production analysis. This is probably because of the extensive literature on queueing systems (see, e.g., Kleinrock (1975); Walrand (1988); Suri, Sanders and Kamath (1993)) that can be used for this type of analysis.

However, we will use simulation as the basis for our production analysis for reasons that will become clear in Section IV.

This paper is organized as follows. Section II describes the supply chain. We give an overview of the input parameters that influence such a supply chain system and the performance measures in Section III. In Section IV, we search for the impact of altering these input parameters on the performance measures of the supply chain by means of simulation. In Section V, we lay the foundation for finding a framework describing the effect of the input parameters on the performance measures and explain how to deal with the conflict between the average lead time and the output rate. The paper concludes with the most important findings embedded in the framework in Section VI.

II. DESCRIPTION OF THE SUPPLY CHAIN

This paper is limited to product flows within a single chain (we do not assume divergent or convergent flows). The supply chain consists of three successive suppliers. The first delivers the finished raw material, the second delivers the semi-finished product, and the third delivers the finished product. Each supplier operates in a make-to-order environment. This means that production commences only when there is a customer order for a finished product. The customer demand is immediately fed back to the first supplier by the third supplier, for example, through EDI.
The three suppliers operate along the same lines:

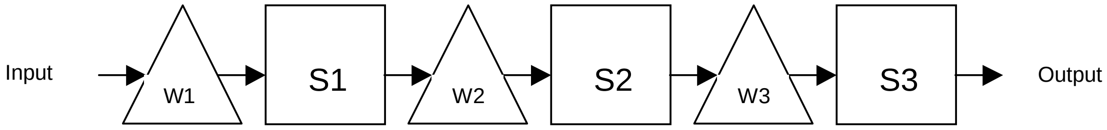
- each produces its goods (finished raw material, semi-finished products, and finished products, respectively) according to the first-come first-served discipline; and
- ‘finished’ goods are immediately sent to the next supplier.

Customer orders arrive individually and are processed individually by each of the three suppliers. To simplify the model, we assume that the customer inter-arrival time distribution is exponential and all suppliers have an equally distributed supplier lead time. This lead time includes the distribution (transportation) time from the first supplier to the second supplier, from the second supplier to the third supplier, and from the third supplier to the finished goods warehouse. We initially assume a triangular distribution for all supplier lead times. To investigate the effect of higher coefficients of variation for all supplier lead times, we switch to a lognormal distribution while keeping the same average supplier lead times. The reason for using these distributions for customer inter-arrival time and supplier lead times will be explained later. Further, we assume a warehouse at each supplier to stock the products coming from the previous supplier in the chain. Note that the warehouse of the first supplier represents the order book. Therefore, it is always assumed to have an infinite ‘capacity’. The warehouses of the second and third supplier have an equal finite capacity that may vary from 0 to 10.

A supply chain as described above can be represented as shown in Figure 1, in which the symbols W1, W2, and W3 represent the warehouses of suppliers 1, 2, and 3, and the symbols S1, S2, and S3 represent suppliers 1, 2, and 3.

FIGURE 1

A representation of the production part of the supply chain



When a customer order arrives, it is sent to supplier S1. If S1 is busy, it is kept in the order book W1 until S1 has finished processing the previous order. At that moment the order starts production at S1. S1 processes the order and the order leaves for W2 after processing. If W2 has reached its capacity limit, the order remains stored at S1 until W2 releases the previous order. If the order enters W2, it waits in W2 until S2 is idle. Then it enters S2 and is processed. After processing, the order leaves S2 for W3. It enters W3 when this warehouse has capacity. Otherwise, the order remains in S2 until W3 has released the previous order. Then it enters W3 and waits there till S3 is idle. S3 then processes the order. Thereafter, the finished product leaves the system by being stored in the finished goods warehouse.

III. INPUT PARAMETERS AND PERFORMANCE MEASURES OF THE SUPPLY CHAIN SYSTEM

First, we examine the main *performance measures* of the supply chain. These are twofold:

1. the total lead time and its components; and
2. the output.

Total lead time is defined as the time that starts at the moment the customer order enters the supply chain at the first warehouse and ends at the moment the finished product leaves the supply chain by entering the finished product warehouse. Total lead time can be divided into the following three *components*:

1. the waiting time at each warehouse;
2. the lead time at each supplier; and
3. the blocking time at the first and second supplier.

Output will be measured by *inter-departure time*, representing the time between two successive departures from the last supplier in the supply chain (Supplier 3). We focus on inter-departure time, and not on output rate, simply because the output rate, which is the reciprocal of the average inter-departure time, is only an average measure. However, inter-departure time also allows the variability in the departure stream to be quantified.

The law of conservation limits the highest output rate to the input rate because the supply chain system cannot output more than is put into it. In a system in which the output rate equals the input rate, the system may not be blocked or starved.

We now proceed to examine the *input parameters* that influence these two performance measures:

- customer order arrival process;
- lead time of all suppliers;
- warehouse capacity;
- initial utilization of all suppliers; and
- order processing.

The customer order arrival process characterizes the arrivals of customer orders at the supply chain. It is determined by the inter-arrival time distribution of the orders. These are assumed to be exponentially distributed (because this distribution is widely used for interarrival times (Law and Kelton (2000))) with a mean value of 12 time units. Hence, on average, a rate of 0.08333 orders per time unit arrives at the supply chain.

The *lead time of all suppliers* is set by the distribution of the supplier lead time. The lead time of each supplier is defined as the processing time plus the distribution time to the next supplier for the first and second supplier, and the processing time plus the distribution time to the finished product warehouse for the third supplier. To simplify the model, equally distributed supplier lead times are assumed for all suppliers. In the initial scenario, a triangular distribution is assumed because it is frequently used in the absence of real data (Law and Kelton (2000)). In other scenarios, the squared coefficient of variation of the supplier lead times is changed by changing the variance of the supplier lead times, while keeping the average supplier lead times the same. The variance is altered to reach squared coefficients of variation of 0.014 (triangular distribution), 0.5, 1, 1.5, 2, and 2.5, respectively. For squared coefficients of variation above or equal to 0.5, the supplier lead times are assumed to be lognormally distributed because the triangular distribution cannot handle high squared coefficients of variation. With the same average supplier lead time, a higher squared coefficient of variation implies that the difference between the minimum value and the maximum value of the triangular distribution increases. Thus, negative values will be encountered for the minimum value above a specific squared coefficient of variation. In general, negative values are not allowed for supplier lead times. The lognormal distribution is widely used as an approximate lead time distribution (see, e.g., Law and Kelton (2000); Vandaele (1996)).

Warehouse capacity highly influences performance. With a finite capacity, an order can be blocked when its downstream warehouse has reached its capacity limit. It is then blocked at the preceding supplier until the warehouse releases one order leading to a higher supplier lead time than the 'natural' supplier time. This additional supplier lead time will be defined as the blocking time. Note that the third supplier can never be blocked since it has no succeeding supplier. Blocking may also cause the downstream warehouses to starve since they receive no more products as long as this blocking lasts. In contrast, infinite warehouse capacity causes no blocking. Processed orders can always leave the supplier as there is enough storage capacity in the downstream warehouse. As stated earlier, the warehouse of the first supplier corresponds to the order book. Therefore, its capacity always equals infinity. The other two warehouses have an equal capacity of 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 customer orders, respectively.

Initial utilization of all suppliers is determined by the average inter-arrival time of the customer orders and the average supplier lead times. It represents the percentage of time that the supplier is processing (and distributing) an order. The word 'initial' means when no blocking or starving occurs and the utilization of the supplier simply equals the average supplier lead time divided by the average inter-arrival time of the customer orders (provided that the system is stable). When warehouse capacity is limited then blocking may occur, increasing average supplier lead times and leading to a higher utilization of the suppliers. The higher utilization ratio is referred to as effective utilization. In our case, we will alter the initial utilization ratio by changing the average supplier lead times while keeping the average inter-arrival time of the customer orders the same. Our initial utilization for all suppliers is 0.5, 0.75, 0.889, 0.95, 0.975, and 0.99, respectively.

Order processing is the way an order proceeds through the supply chain. In this paper, we assume a make-to-order environment as well as the first-come first-served discipline. A make-to-order environment is a mix of a pull system and a push system; processing starts only when a customer order has arrived (pull), but once an order has arrived it is pushed through the system (push) (Hopp and Spearman (2000)). First-come–first-served means that orders arriving first will be processed first and distributed first for each part of the supply chain.

IV. SIMULATION MODEL

A supply chain system, as described in Section II, may be modelled by queueing (Shantikumar, Yao and Zijm (2003)) or by simulation (Law and Kelton (2000)). The reason we prefer simulation over queueing for our analysis is because of the way our supply chain would be modelled within queueing theory. By assuming finite warehouse capacity, we would need to rely on queueing models with finite buffers. These models are approximate, taking into account the assumptions in the previous section. That is why we will use simulation. If we take care about the verification and validation of our simulation model, we will obtain more reliable results than with the queueing models. Models with infinite buffer capacity can only be used if the warehouses rarely reach their capacity limit (see, e.g., Vandaele, De Boeck and Callewier (2002)).

The model of our supply chain system is developed in Arena. The symbols are shown in Table 1.

TABLE 1

The symbols for the performance measures of the supply chain

E(Var)	Average value of the variable Var
SCV(Var)	Squared coefficient of variation of the variable Var
WT_S	Waiting time at supplier S (= waiting time in warehouse W of supplier S)
BT_S	Blocking time at supplier S
TLT	Total lead time
ELT	Effective lead time (= TLT – WT1)
D_S	Inter-departure time from supplier S

The model is built to obtain easily the average value and the ‘average’ squared coefficient of variation of the following performance measures:

- waiting time (in warehouses 1, 2, and 3);
- blocking time (at suppliers 1 and 2);
- total lead time;
- effective lead time (= total lead time – waiting time in warehouse 1); and
- inter-departure time (at supplier 3).

In all scenarios, the customer order inter-arrival time is assumed to be exponentially distributed with a mean of 12. Further, we assume the first-come–first-served discipline for all entities.

Different scenarios are developed by combining different values for the warehouse capacities: Cap, the initial utilization ratio; Rho; and the squared coefficient of variation of the supplier lead times, SCV, as shown in Table 2.

TABLE 2
The different parameter values for the different scenarios

Parameters	Values
Warehouse capacity (Cap)	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10
Utilization ratio (Rho)	0.5, 0.75, 0.889, 0.95, 0.975, and 0.99
Squared coefficient of variation of the supplier lead time (SCV)	0.014, 0.5, 1, 1.5, 2, and 2.5

The warehouse capacity for the second and third supplier is varied from zero to ten. The initial utilization ratio is altered by changing the average supplier lead times while keeping the customer order inter-arrival times the same. The squared coefficient of variation of the supplier lead times is modified by changing the variance of the supplier lead times while keeping the average supplier lead times the same. Table 3 shows the supplier lead time distributions for all combinations of Rho and SCV.

TABLE 3
The different parameter values for the supplier lead times to obtain different values for Rho and SCV

	SCV					
Rho	0.014 triangular	0.5 lognormal	1 lognormal	1.5 lognormal	2 lognormal	2.5 lognormal
0.5	(8,10,14)	(6,4.24)	(6,6)	(6,7.35)	(6,8.49)	(6,9.49)
0.75	(6.75,8.4357,11.8125)	(9,6.36)	(9,9)	(9,11.02)	(9,12.73)	(9,14.23)
0.889	(8,10,14)	(10.67,7.54)	(10.67,10.67)	(10.67,13.06)	(10.67,15.08)	(10.67,16.87)
0.95	(8.55,10.6875,14.9625)	(11.4,8.06)	(11.4,11.4)	(11.4,13.96)	(11.4,16.12)	(11.4,18.02)
0.975	(8.775,10.96875,15.35625)	(11.7,8.27)	(11.7,11.7)	(11.7,14.33)	(11.7,16.55)	(11.7,18.50)
0.99	(8.91,11.1375,15.5925)	(11.88,8.40)	(11.88,11.88)	(11.88,14.55)	(11.88,16.80)	(11.88,18.78)

A triangulation distribution is used for the lowest value of SCV. The lognormal distribution is used for larger values of SCV, as explained above. The triangular distribution is characterized by its minimum, mode, and maximum, and the lognormal distribution by its average value and its standard deviation. There are a total of 396 scenarios because there are 11 values for Cap, 6 for Rho and 6 for SCV. For each scenario, 500 replications of 20,000 time units were performed.

Three steps were used to obtain the results.

- *Step 1*: The entities to be used for the performance measures were identified by determining the lower bound and the upper bound to be taken into account. The warm-up period was checked to obtain the lower bound. It was found that 199 first entities of each replication for each scenario could be cancelled. The scenario with the lowest average number of entities processed during the simulation for each scenario was identified as the upper bound. This scenario had a Cap value equal to zero, a Rho value equal to 0.99, and an SCV value equal to 2.5. Its average inter-departure time was 23.09 time units. Hence, there were on average 20,000/23.09 ≈ 866 entities processed within the replications of this scenario. For each replication of this scenario, the number of values for each X^{th} entity was checked. From entity 748 on, there were less than 500 values, meaning that not all replications processed 748 entities. To build in some safety, results were restricted to entities equal to 700 and smaller. Therefore, results were based on the values for the performance measures of entities 200 to 700 for each scenario.
- *Step 2*: The average value and the squared coefficient of variation for each performance measure PM and each entity X ($200 \leq X \leq 700$) were calculated over all replications of each scenario S. These values, for a performance measure PM of entity X in scenario S, are denoted as $E(PM_{XS})$ and $SCV(PM_{XS})$, respectively.
- *Step 3*: To obtain one number for the average value and the squared coefficient of variation of each performance measure for each scenario, the following formulas were used:

$$\mathrm{E}(\mathrm{PM}_{\mathrm{S}}) = \frac{\sum_{\mathrm{X}=200}^{700} \mathrm{E}(\mathrm{PM}_{\mathrm{XS}})}{501};$$

$$\mathrm{SCV}(\mathrm{PM}_{\mathrm{S}}) = \frac{\sum_{\mathrm{X}=200}^{700} \mathrm{SCV}(\mathrm{PM}_{\mathrm{XS}})}{501}.$$

V. RESULTS

We now discuss the results of the simulation study. The impact of the input parameters on the performance measures are discussed first followed by a discussion of the relationship between the average lead time (total and effective) and the output rate.

A. *Impact of the input parameters on the performance measures*

Figures 2a, 2b, 3a, and 3b show the plots of the simulation results for the average value of the blocking time at the first supplier, the waiting time at the first supplier, the total lead time, and the effective lead time.

FIGURE 2a

The average blocking time at the first supplier (left-hand side)

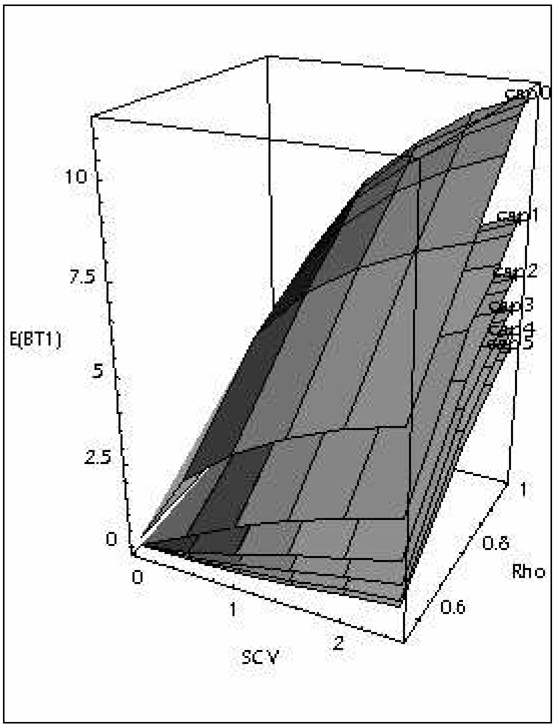


FIGURE 2b

The average waiting time at the first supplier (right-hand side)

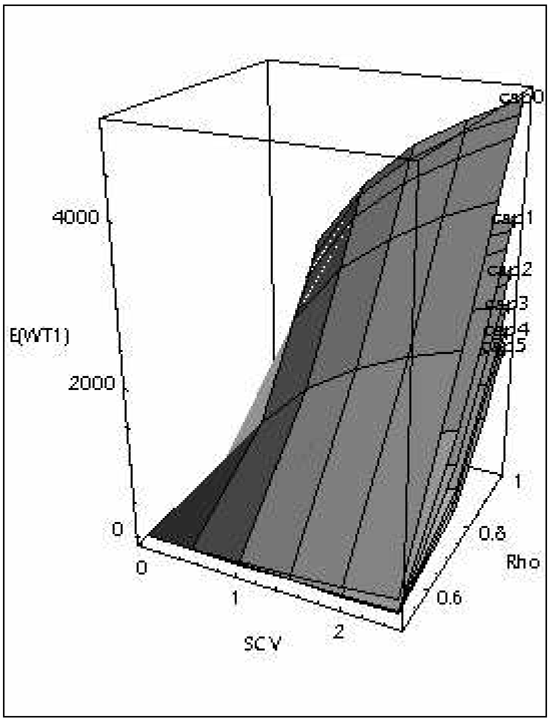


FIGURE 3a
The average total lead time (left-hand side)

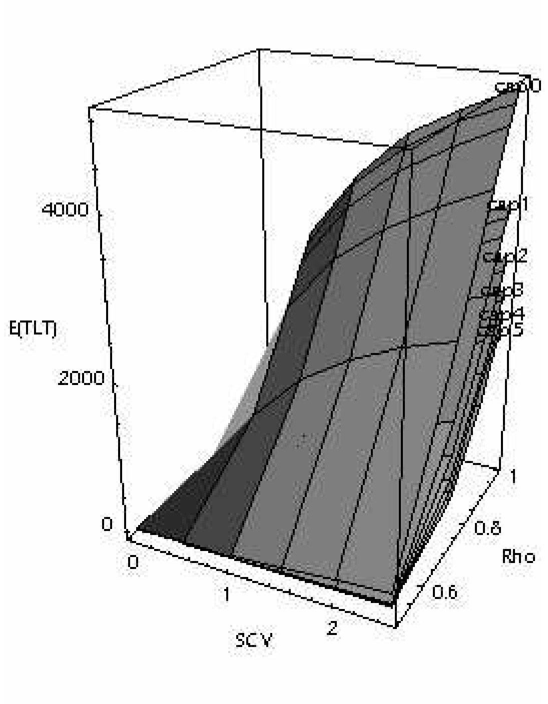
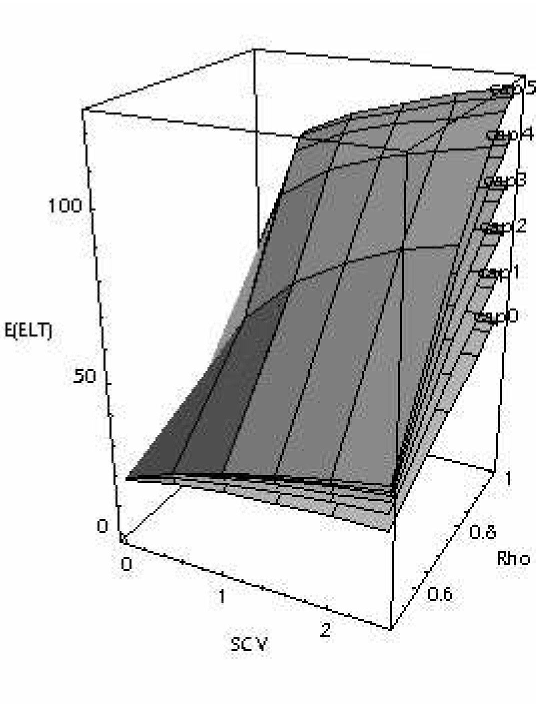


FIGURE 3b

The average effective lead time (right-hand side)



Figures 4a and 4b show the plots of the simulation results for the squared coefficient of variation of the blocking time at the first supplier and the effective lead time.

FIGURE 4a

The squared coefficient of variation of the blocking time at the first supplier (left-hand side)

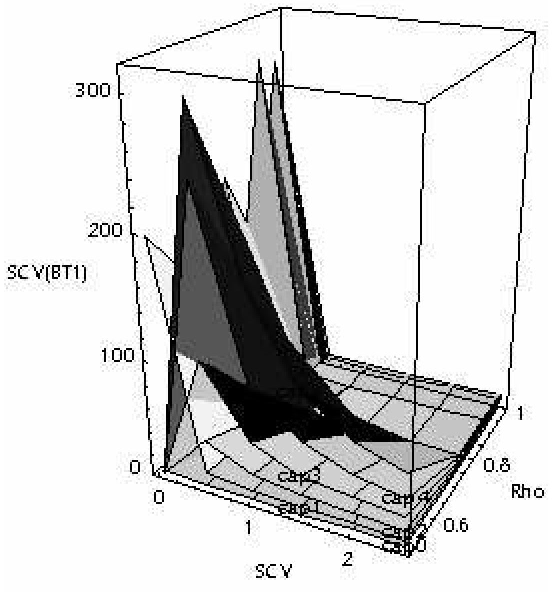
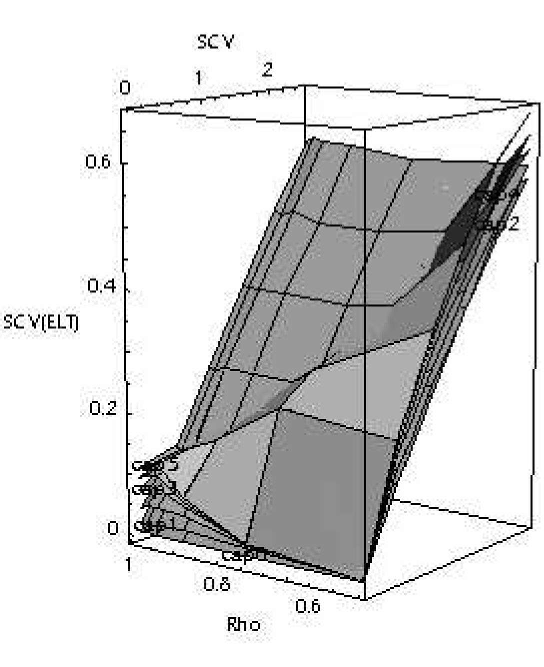


FIGURE 4b
The squared coefficient of variation of the effective lead time (right-hand side)



To keep the figures readable, only the plots for Cap equal to 0, 1, 2, 3, 4, and 5 are shown. The performance measures are expressed as a function of Rho and SCV. A surface is also drawn for each value of Cap. This enables the impact of all input parameters (Cap, Rho, and SCV) on the performance measures to be clearly shown.

To obtain a clearer view of the behaviour in Figures 2a, 2b, 3a, and 3b, two types of cross-sections for all values of Cap are also shown:

- cross-sections for SCV equal to 1 for all values of Rho and Cap are shown in Figures 5a, 5b, 6a, and 6b;
- cross-sections for Rho equal to 0.889 for all values of SCV and Cap are shown in Figures 7a, 7b, 8a, and 8b.

FIGURE 5a

A cross-section of the average blocking time at the first supplier for SCV equal to 1 (left-hand side)

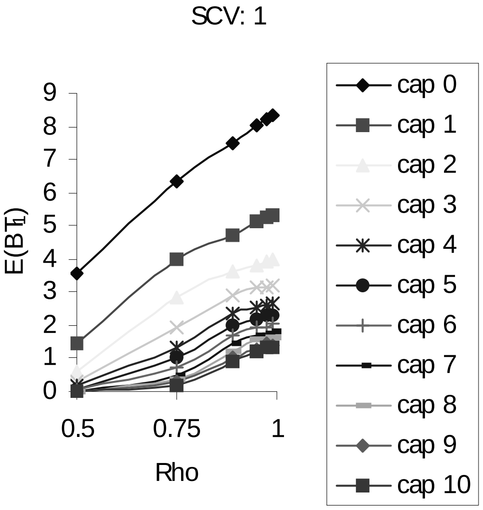


FIGURE 5b
A cross-section of the average waiting time at the first supplier for SCV equal to 1 (right-hand side)

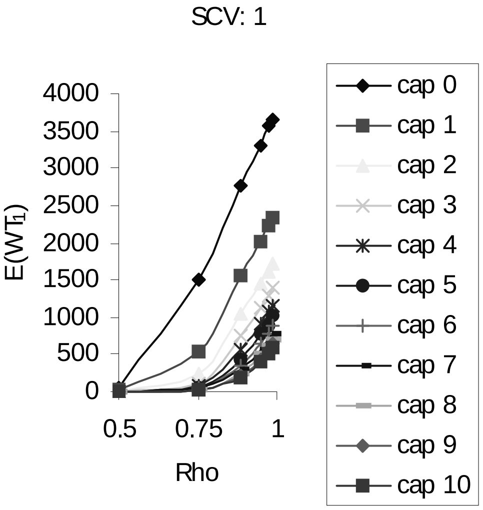


FIGURE 6a
A cross-section of the average total lead time for SCV equal to 1

(left-hand side)

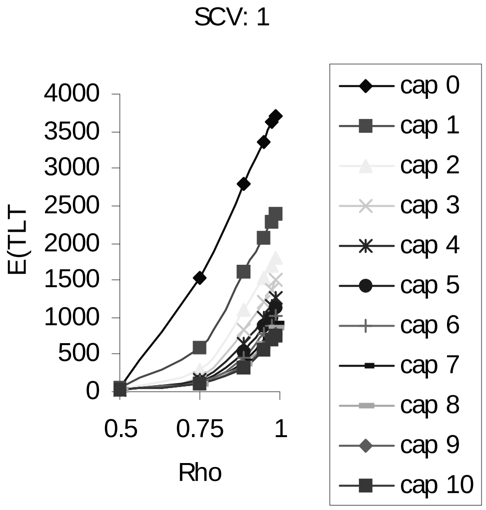


FIGURE 6b

A cross-section of the average effective lead time for SCV equal to 1 (right-hand side)

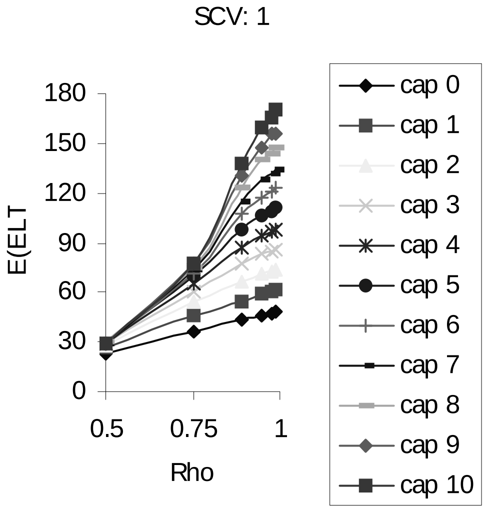


FIGURE 7a

A cross-section of the average blocking time at the first supplier for Rho equal to 0.889 (left-hand side)

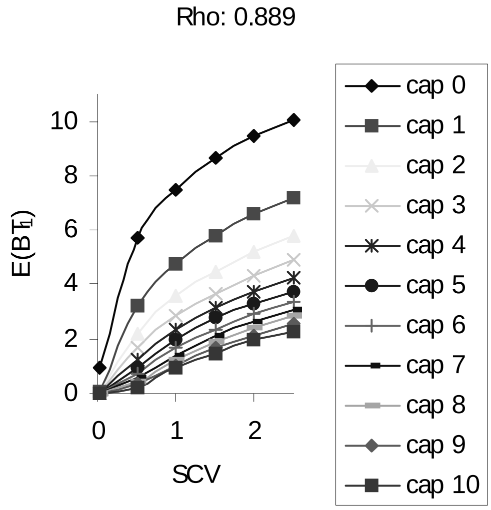


FIGURE 7b

A cross-section of the average waiting time at the first supplier for Rho equal to 0.889 (right-hand side)

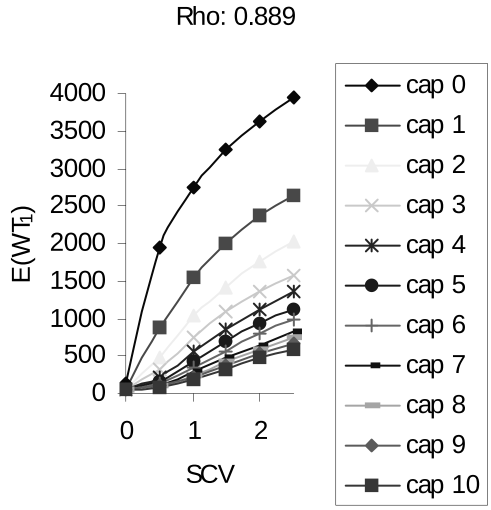


FIGURE 8a

A cross-section of the average total lead time for Rho equal to 0.889
(left-hand side)

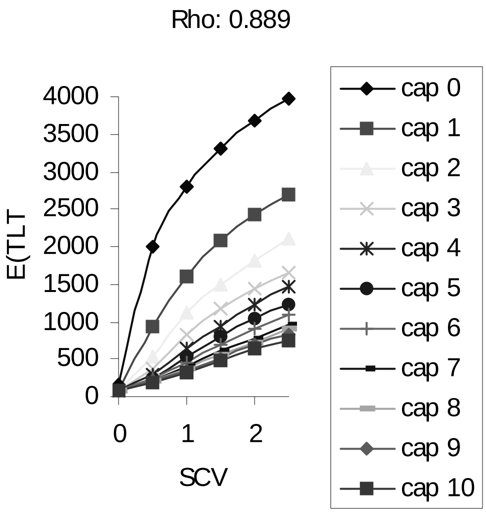
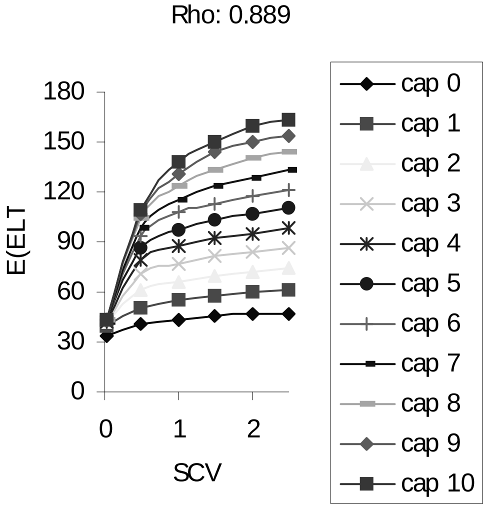


FIGURE 8b

A cross-section of the average effective lead time for Rho equal to 0.889
(right-hand side)



All cross-sections have a positive slope. The difference between the cross-sections for Cap equal to zero and the other values is relatively large, and the curves become closer to each other as the value for Cap rises.

Figures for the average value and the squared coefficient of variation of the blocking time at the second supplier, the waiting times at the second and third supplier, the inter-departure time at the last supplier, the squared coefficient of variation of the waiting time at the first supplier, and the total lead time are not presented because they show the same trends as Figures 2a to 8b. These results are discussed below.

The average blocking time at the first and the second supplier increases when Rho as well as SCV rises in contrast to an increasing Cap; this leads to lower blocking times. This is logical as more warehouse capacity lowers the blocking probability. For the cross-sections with SCV equal to 1, the blocking times for different values of Rho follow a less concave curve for smaller values of Cap and a more concave curve for higher values of Cap. The cross-sections with Rho equal to 0.889 for different values of SCV are more convex for smaller values of Cap and less convex for larger values of Cap. In addition, the average blocking time at the first supplier is smaller than the average blocking time at the second supplier, *ceteris paribus*.

Similar trends are observed for the *average waiting time at the first supplier*. Because limited capacity is restricted to the second and third supplier warehouses, it is logical that the waiting time at the first supplier increases with lower capacities of these warehouses, since there is less storage in between, thus pushing the inventory upstream. For the cross-sections with SCV equal to 1 for all the values of Rho, the curves are all concave, but this effect decreases for higher values of Cap. For the cross-sections with Rho equal to 0.889 for all the values of SCV, the curves are more convex for smaller values of Cap and less convex for larger values of Cap.

Similar observations concerning Rho and SCV occur for the *average waiting time at the second and third supplier*. However, the waiting time increases as the warehouse capacity rises because, when waiting is required in between, the waiting time will increase if the capacity to wait exists. For the cross-sections with SCV equal to 1 for all values of Rho, a less concave curve arises for lower values of Cap and a more concave curve arises for larger values of Cap. For the cross-sections with Rho equal to 0.889 for different values of SCV, the curves are all convex, but this effect increases for higher values of Cap. The average waiting time at the second supplier is always higher than the average waiting time at the third supplier, ceteris paribus. For a value of Cap equal to zero, a value of zero is obtained for the average waiting time at both suppliers.

Similar conclusions can be drawn for the average inter-departure time at the third supplier as for the average waiting time at the first supplier because these changes in the input parameters lead to increased blocking, which lowers the output rate. For the cross-sections with SCV equal to 1 for all values of Rho, the curves are all concave, but this effect decreases for higher values of Cap. For the cross-sections with Rho equal to 0.889 for all values of SCV, the curves are more convex for smaller values of Cap and less convex for larger values of Cap.

The average total lead time changes along the same lines as for the average waiting time at the first supplier, while the *average effective lead time* is totally in line with the plots of the average waiting times at the second and third supplier. This is because the greatest part of the total lead time is the waiting time at the first supplier, while this waiting time is not present in the formula of the effective lead time. For all cross-sections, the same plots are obtained for the average total lead time as for the average waiting time at the first supplier. All cross-sections for the average effective lead time follow the same pattern as for the average waiting time at the second and third supplier.

The *squared coefficient of variation of all performance measures* is much more difficult to explain. One of the reasons can be traced back to the way it was calculated, as an average value of the squared coefficient of variations of 501 entities over all replications of each scenario. There is no longer one direction in which the squared coefficient of variation moves when the input parameters change. It is known from the literature (Hopp and Spearman (2000)) that the variance, and hence the squared coefficient of variation, is a parameter that is very difficult to quantify. As the simulation results clearly prove this finding, cross-sections are not shown for the squared coefficient of variation of the performance measures.

B. Rel*ation between the average lead time and the output rate*

The relation between the average inter-departure time and the average total effective lead time is plotted in Figures 9a, 9b, and 9c.

FIGURE 9a

The relationship between the average inter-departure time and the average total lead time (top left-hand figure)

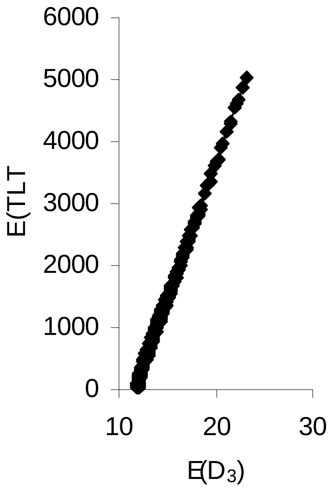


FIGURE 9b

The relationship between the average inter-departure time and the average effective lead time for all values of Cap (bottom left-hand figure)

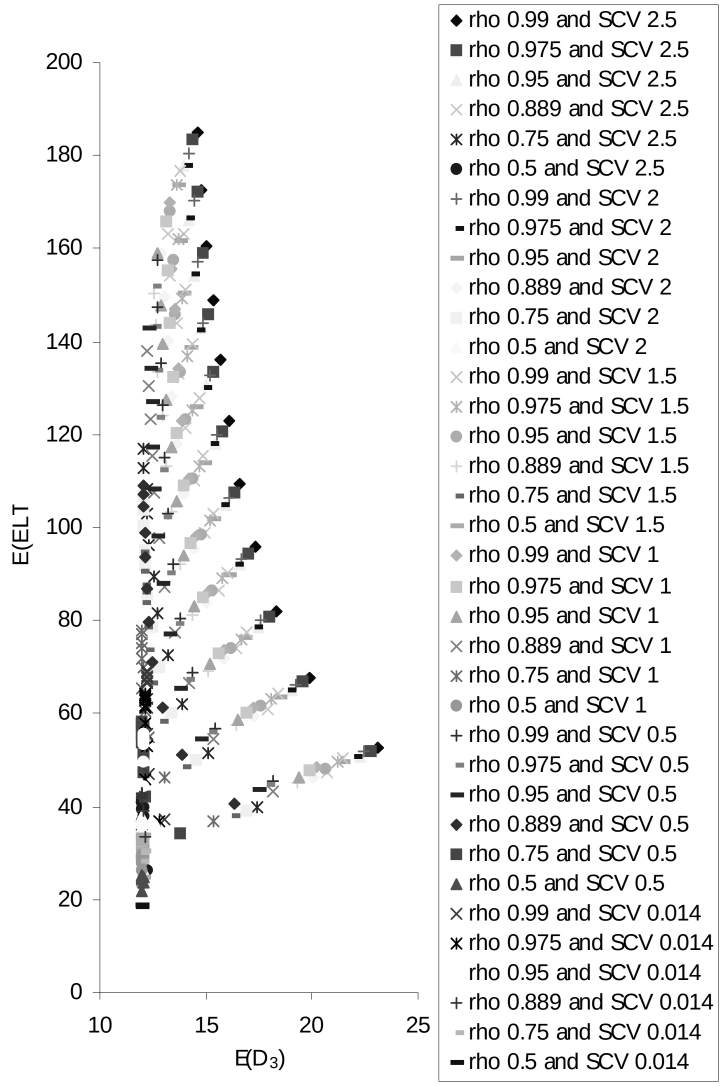
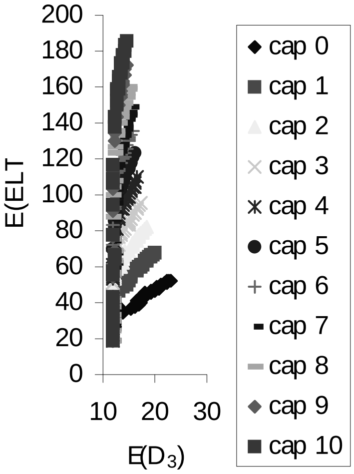


FIGURE 9c

The relationship between the average inter-departure time and the average effective lead time for all values of Rho and SCV (right-hand side figure)



There is an almost perfect positive linear relationship between the average inter-departure time and the average total lead time (see Figure 9a). The higher the inter-departure time, the higher the total lead time. This lead time increases with higher values for Rho and SCV and lower values for Cap. The lower warehouse capacity causes more blocking and prevents the arriving products from leaving the system at the same rate as they entered. This explains why the average inter-departure time rises and the output rate decreases. This relationship is also confirmed by the correlation coefficient, which equals 0.998.

At first sight there does not appear to be any relation between the average inter-departure time and the average effective lead time (see Figures 9b and 9c) (the correlation coefficient equals 0.096). However, if this graph is plotted for the different warehouse capacities, there is a clear relationship. The correlation coefficients for the different warehouse capacities range from 0.94 (for lower capacities) to 0.83 (for higher capacities). Each figure starts with a vertical part that turns outward in an almost linear positive line, which is flatter for lower warehouse capacities. The vertical part is larger for higher warehouse capacities since the inter-departure time decreases when the warehouse capacity rises. That is because the probability of blocking decreases. When the warehouse capacity is high enough to prevent blocking, the curve will be a vertical line where the higher points correspond to higher values of Rho and SCV. Higher average inter-departure times (or lower output rates) and higher average effective lead times are observed for all curves as Rho and SCV increase.

It was noted previously that the average total lead time moves in the same direction as the average inter-departure time at the third supplier. In other words, a lower output rate corresponds to a higher average total lead time. Therefore, it is not possible to find a balance between *the output rate and the average total lead time*. The only possibility is to change the input parameters so as to lower the average inter-departure time at the third supplier and the average total lead time; i.e., increasing Cap and decreasing Rho and SCV.

We now consider the relation between *the output rate and the average effective lead time*. A higher average inter-departure time and a lower average effective lead time for lower warehouse capacities is noted in Figures 9b and 9c for a specific Rho and SCV. Since a negative relation is denoted in function of the warehouse capacities (higher effective lead times and higher output rates for higher values of Cap and lower effective lead times and lower output rates for lower values of Cap), a value for Cap for a specific Rho and SCV can be determined, for which the average effective lead time per unit for a batch of U units is minimized. A batch of U units refers to the successive individual processing of U units. After the processing of the Uth unit of each batch, the system is totally emptied before a new batch of U units is processed. Natural events such as working in shifts or new batch setup times determine the value of U. The average effective lead time per unit for a batch of U units equals the following expression for a specific scenario:

$$E(ELT_U) = \frac{E(ELT) + (U - 1)\, E(D_3)}{U} \ .$$

Calculating this expression for all Cap for a specific value for U, Rho, and SCV provides a value of Cap for which the expression is minimized. Note that if the batch size rises above a specific threshold value, the highest warehouse capacity will always return the minimum average effective lead time per unit for a specific Rho and SCV because $E(D_3)$ is lower for higher warehouse capacities. As the share of $E(D_3)$ in $E(ELT_U)$ rises as U grows, $E(ELT_U)$ will decrease for higher warehouse capacities. Note that the value of U for which this occurs depends on the relative sizes of E(ELT) and $E(D_3)$.

This behaviour is graphically shown in Figures 10a and 10b for Rho equal to 0.5 and 0.75 and for all values of SCV where U equals 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, 210, and 220 units, respectively.

FIGURE 10a

The average effective lead time per unit for a batch of U units for Rho equal to 0.50 (left-hand side)

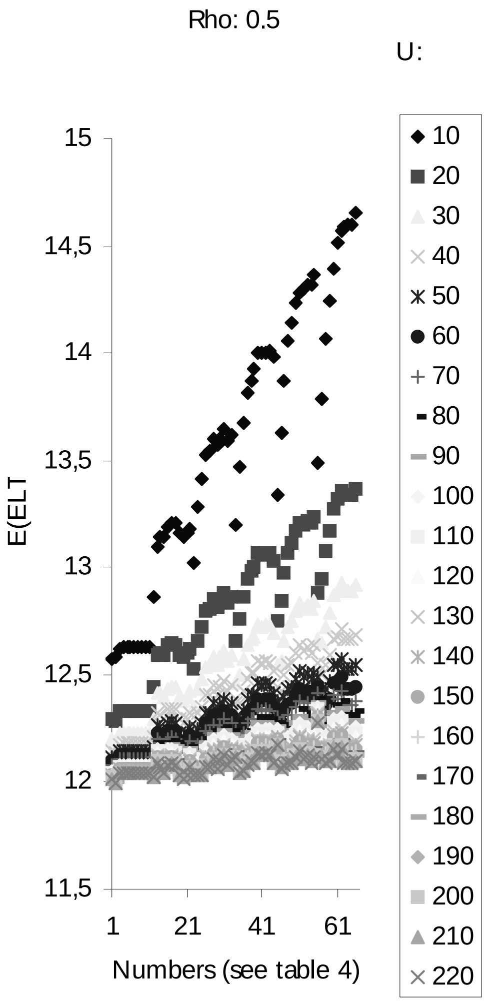
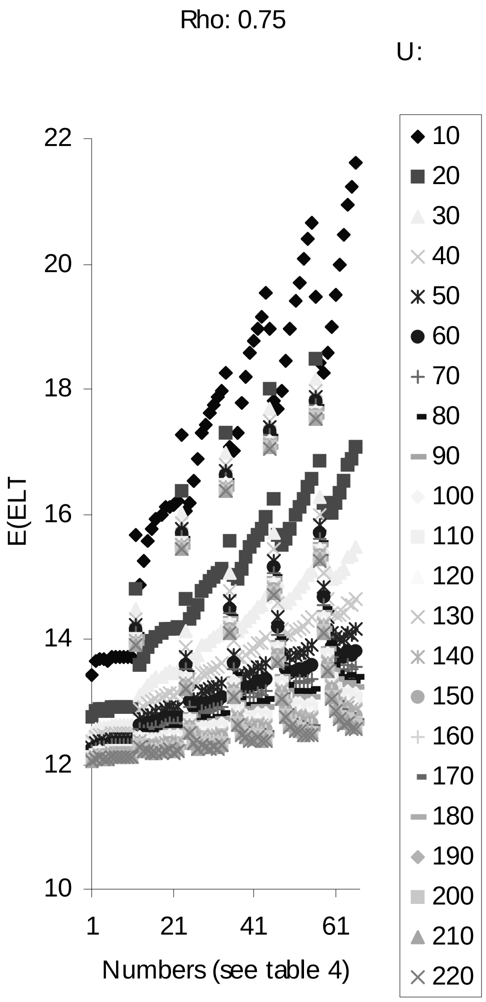


FIGURE 10b

The average effective lead time per unit for a batch of U units for Rho equal to 0.75 (right-hand side)



Since the graphs for Rho equal to 0.889, 0.95, 0.975, and 0.99 show the same trends as the graph for Rho equal to 0.75, they are not shown. The numbers on the X axis correspond to the values of SCV and Cap as represented in Table 4.

TABLE 4

The numbering on the X axis of Figures 10a and 10b

	Numbers
SCV 0.014	1–11
SCV 0.5	12–22
SCV 1	23–33
SCV 1.5	34–44
SCV 2	45–55
SCV 2.5	56–66
Cap 0	1, 12, 23, 34, 45, 56
Cap 1	2, 13, 24, 35, 46, 57
Cap 2	3, 14, 25, 36, 47, 58
Cap 3	4, 15, 26, 37, 48, 59
Cap 4	5, 16, 27, 38, 49, 60
Cap 5	6, 17, 28, 39, 50, 61
Cap 6	7, 18, 29, 40, 51, 62
Cap 7	8, 19, 30, 41, 52, 63
Cap 8	9, 20, 31, 42, 53, 64
Cap 9	10, 21, 32, 43, 54, 65
Cap 10	11, 22, 33, 44, 55, 66

For each of Figures 10a and 10b, we can clearly identify six parts:

- the first part (X-axis numbers 1–11)—SCV equal to 0.014;
- the second part (X-axis numbers 12–22)—SCV equal to 0.5;
- the third part (X-axis numbers 23–33)—SCV equal to 1;
- the fourth part (X-axis numbers 34–44)—SCV equal to 1.5;
- the fifth part (X-axis numbers 45–55)—SCV equal to 2;
- the sixth part (X-axis numbers 56–66)—SCV equal to 2.5.

Specific shapes, which are determined by the different warehouse capacities, are observed for each part. Moreover, we note the switch from a U-shaped curve for smaller values of U to a decreasing concave curve for larger values of U, keeping the same SCV. The only exceptions are the scenarios with Rho equal to 0.50 and all scenarios with SCV equal to 0.014. In these scenarios, the curves are convex. Note also that the curves are on a higher level for increasing SCV.

If Rho and SCV are sufficiently high (in our case from Rho, respectively from SCV equal to 0.75, 0.5 on), there exists a capacity for which the average effective lead time per unit for a batch of U units is minimized. The value of this capacity depends on the input parameters of the system. Hence, to minimize the time from the moment the product arrives at the processing part of the first supplier until the moment it leaves the system, the warehouse capacity of the system should be determined from a specific value for U, Rho, and SCV.

Note that in our case, there is no sense in optimizing (minimizing) the average effective lead time per unit for a specific Rho, SCV, and Cap in function of U because the average effective lead time per unit will always be smaller if U increases. Expressed as a formula, we have:

$$\frac{\mathrm{E(ELT)} + (\mathrm{U} - 1)\,\mathrm{E(D_3)}}{\mathrm{U}} > \frac{\mathrm{E(ELT)} + \mathrm{U}\,\mathrm{E(D_3)}}{\mathrm{U} + 1}.$$

If we simplify this expression, we obtain:

$$\mathrm{E(ELT)} > \mathrm{E(D_3)},$$

which is true in all our scenarios.

VI. CONCLUSION

In this paper, we have tried to reach general conclusions for the performance of a specific supply chain type consisting of successive suppliers operating in a make-to-order environment. The customer demand is immediately fed back to the first supplier. All suppliers work along the same lines and produce their goods according to the first-come–first-served discipline. Once each supplier finishes its production, the goods are immediately sent to the next supplier.

Two types of results were considered: firstly, the impact of the input parameters (warehouse capacity; Cap; initial utilization of the suppliers; Rho; and the squared coefficient of variation of the supplier lead time, SCV) on the average value and on the squared coefficient of variation of different performance measures (blocking time and waiting time at all suppliers, total lead time, effective lead time, and inter-departure time at the last supplier); and secondly, the relation between the average total and effective lead time and the output rate.

For the impact of the input parameters on the average value of all performance measures, we can draw the following framework:

	Cap	Rho (given a specific SCV)		SCV (given a specific Rho)	
		Smaller Cap	Higher Cap	Smaller Cap	Higher Cap
$E(BT_1)$ ($< E(BT_2)$)	–	+ less CC	+ more CC	+ more CV	+ less CV
$E(BT_2)$	–	+ less CC	+ more CC	+ more CV	+ less CV
$E(WT_1)$ ($>> E(WT_2)$)	–	+ more CC	+ less CC	+ more CV	+ less CV
$E(WT_2)$ ($> E(WT_3)$)	+	+ less CC	+ more CC	+ less CV	+ more CV
$E(WT_3)$	+	+ less CC	+ more CC	+ less CV	+ more CV
$E(D_3)$	–	+ more CC	+ less CC	+ more CV	+ less CV
E(TLT)	–	+ more CC	+ less CC	+ more CV	+ less CV
E(ELT)	+	+ less CC A	+ more CC	+ less CV	+ more CV

The following symbols are used in this framework:

- –: negative relation;
- +: positive relation;
- CC: concave (‘more CC’ means a higher positive slope); and
- CV: convex (‘more CV’ means a higher positive slope).

This framework shows that higher SCV and Rho have a negative impact on the average value of *all* performance measures. In contrast, a higher warehouse capacity results in lower average values for the blocking time at all suppliers, the waiting time at the first supplier, the inter-departure time at the last supplier, and the total lead time. A lower warehouse capacity induces lower averages for the waiting times at all but the first supplier and the effective lead time. The cross-sections in function of the different values for Rho are all concave whereas the cross-sections in function of the different values for SCV are all convex.

No straightforward conclusions could be drawn for the *impact of the input parameters on the squared coefficient of variation of all performance measures*. This proves the general idea that the squared coefficient of variation is difficult to capture.

The following managerial conclusions can be summarized. A lower total lead time for this supply chain can be accomplished by decreasing Rho, decreasing SCV, and increasing Cap. A decrease in Rho, respectively SCV will have relatively more effect if Rho is high, respectively SCV is low since all the cross-sections have a positive slope and are concave for Rho (given a specific value for SCV), respectively convex for SCV (given a specific value for Rho). For a decrease in the total effective lead time, the same reasoning can be followed except for Cap. Here, the effective lead time will decrease when the warehouse capacity decreases.

We can construct the following framework for the relation between the average total and effective lead time and the average inter-departure time (the output rate),

	E(D$_3$)
E(TLT)	+
E(ELT°	o + for a specific Cap o - for a specific Rho and SCV: o higher E(D$_3$) and lower E(ELT) for lower values of Cap o lower E(D$_3$) and higher E(ELT) for higher values of Cap

The framework shows that higher average total lead times correspond to lower output rates. The only possibility in this case is to implement changes to reach low output rates and low average total lead times. Therefore, Rho and SCV should be decreased and Cap should be increased. Rho can be decreased by lowering the input rate, by lowering the supplier lead time, or by increasing the warehouse capacity (to avoid blocking). SCV can be decreased by reducing the variability in the system.

The framework demonstrates that the average effective lead time also shows a negative relationship with the output rate for a specific value of Cap. However, when keeping Rho and SCV constant, a positive relationship between the average effective lead time and the output rate is observed. Therefore, it follows that a Cap can be found for a specific value of Rho and SCV, for which the average effective lead time per unit for a batch of U units is minimized. This minimization could only be performed if Rho and SCV were sufficiently 'large' (in our example Rho from 0.75 and SCV from 0.5). These values depend on the input parameters and concern those graphs for a specific value of Rho and SCV for which the relation between the average inter-departure time (on the X axis) and the average effective lead time (on the Y axis) is almost a straight vertical line. Note that this straight line can only appear if, for that specific value of Rho and SCV, the maximum warehouse capacity for all values of Cap is rarely reached.

The above results are obtained for a supply chain with three suppliers. However, we are convinced that the frameworks also account for the same type of supply chain with more than three suppliers because we can apply the findings about extensions that exist for queueing theory (see, e.g., Suri, Sanders and Kamath (1993)); the logic will not change by adding more suppliers when keeping all assumptions of the supply chain the same, as discussed at the beginning of this paper. Hence, the above frameworks are also valid for the same type of supply chains with S suppliers.

REFERENCES

Hopp, W.J. and Spearman, M.L., 2000, Factory Physics, (McGraw-Hill).

Kleinrock, L., 1975, Queueing Systems: Vol. I: Theory, (Wiley & Sons).

Law, A.M. and Kelton, W.D., 2000, Simulation Modeling & Analysis, (McGraw-Hill).

Shanthikumar, J.G., Yao, D.D. and Zijm, W.H.M., 2003, Stochastic Modeling and Optimization of Manufacturing Systems and Supply Chains, (Kluwer Academic Publishers).

Simchi-Levi, D., Kaminsky, P. and Simchi-Levi, E., 2000, Designing and Managing the Supply Chain, (McGraw-Hill).

Stadtler, H. and Kilger, S., 2000, Supply Chain Management and Advanced Planning, (Springer-Verlag).

Suri, R., Sanders, J.L. and Kamath, M., 1993, Performance Evaluation of Production Networks, in Graves, S.C., Rinnooy Kan, A.H.G. and Zipkin, P.H., eds., Handbooks in Operations Research and Management Science Vol. 4: Logistics of Production and Inventory, (North-Holland), 199–286.

Vandaele, N., 1996, The Impact of Lot Sizing on Queueing Delays: Multi Product, Multi Machine Models, PhD thesis, (Katholieke Universiteit Leuven).

Vandaele, N., De Boeck, L. and Callewier, D., 2002, An Open Queueing Network for Lead Time Analysis, *IIE Transactions* 32, 1, 1–9.

Walrand, J., 1988, An Introduction to Queueing Networks, (Prentice-Hall).