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The horn torus model in light and context of division by zero calculus

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Abstract

To anticipate it: the horn torus model, I want to talk about here, is not a mathematical or physical theory, it is not science at all. But it is an important and helpful method to improve and mastermind abstract thinking and to direct scientific views away from entrenched habits of interpreting nature to a completely different idea of fundamental relationships concerning space, time, physical objects and their mathematical description by numbers and by a figurative geometry, which our innate capability of imagination is able to manage in some degree. I use to call the model an intellectual game to reveal engrams of dimensional thinking and a thought experiment as an exercise for abstraction ability.

Here, in this present article, we first want to show the relevance of the horn torus model in relation to the **division by zero calculus**, but furthermore – only for those, interested in physical interpretation – explain the new thoughts as an allegoric tool which is apt to visualize fundamental properties of nature by a geometrically describable dynamic, aiming at a better understanding of quantum-physical and relativistic coherencies by new alternate interpretations of known facts and recognitions.

Until now the horn torus model is a stand-alone topic and thus, I do not refer to other publications, only to my own websites, e.g.: www.horntorus.com, which contains many explanatory graphics and animations. The actual article primarily includes excerpts from text, already published on this website (2006...2021).

Keywords

horn torus, horn torus model, Daeumler model, Puha model, Riemann sphere, stereographic projection, conformal mapping, complex manifold, complex analysis, topology, infinity, **division by zero calculus**.

1. Riemannian stereographic projection

Mathematicians know the Riemann sphere as compactification of the Gaussian complex plane, as metric space, as complex manifold, and the relevance in complex analysis and topology is sufficiently known. Physicists use the Riemann sphere in quantum mechanics, for description of spin and polarization e.g., and also in string and twistor theories. Wherever complex numbers occur, the Riemann sphere always plays a role.

For readers of this journal there surely is no need to discuss the method and properties of mapping points, lines and curves (functions) from the complex plane onto the Riemannian unit sphere. The stereographic projection with all its properties and with all the associated mathematical laws are familiar armamentarium. Here we only designate the following elements for figure 1:

- M: centre of the Riemann sphere
- 0: touch point of plane and sphere (zero point of complex plane)
- N: ‘north pole’ of sphere on extended line 0M (opposite 0)
- Z: point on complex plane, assigned to the complex number z
- |z|: absolute value of z *and* length of line 0Z
- P: intersection point of line NZ on surface of sphere
- α: central angle 0MP for chord 0P

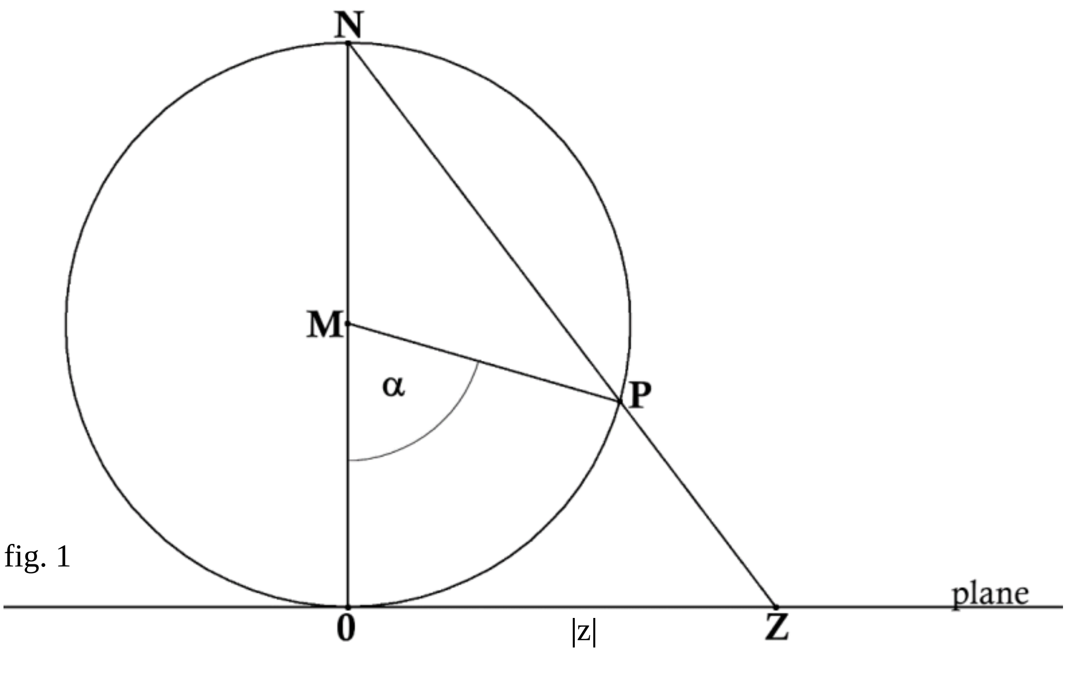

fig. 1

Because of angle 0NZ = $\alpha/2$ (it is circumferential angle for chord 0P) and 0N = 1 (by definition of Riemann sphere) we have for the mappings from plane to sphere and vice versa:

$$\alpha = 2 \tan^{-1}(|z|) \tag{1.1}$$

$$|z| = \tan(\alpha/2) \tag{1.2}$$

Point N is not constructed by the stereographic projection, but added artificially as so called Alexandroff compactification, making the set of all points on the sphere to a compact manifold. More in-depth topological aspects concerning the Riemann sphere however, are not subject of the present discussion.

2. Puha's stereographic projection onto the horn torus

In June 2018, Vyacheslav Puha (Kola Science Center, Russian Academy of Sciences, Murmansk, Russia) proposed during correspondence with Professor Saburou Saitoh about the topic **division by zero calculus** an extension of the Riemannian stereographic projection with the objective to combine north pole N (infinity point) and south pole 0 (zero point) in one single point, namely the centre of a horn torus, inscribed into the Riemann sphere. His method is purely geometric: intersect line MP with the surface of the horn torus to get the projected point P′. All points, lines and curves can be mapped between plane and horn torus easily this way. Mapping is bijective, except for centre M, which has two possible mappings: 0 and N, zero and infinity.

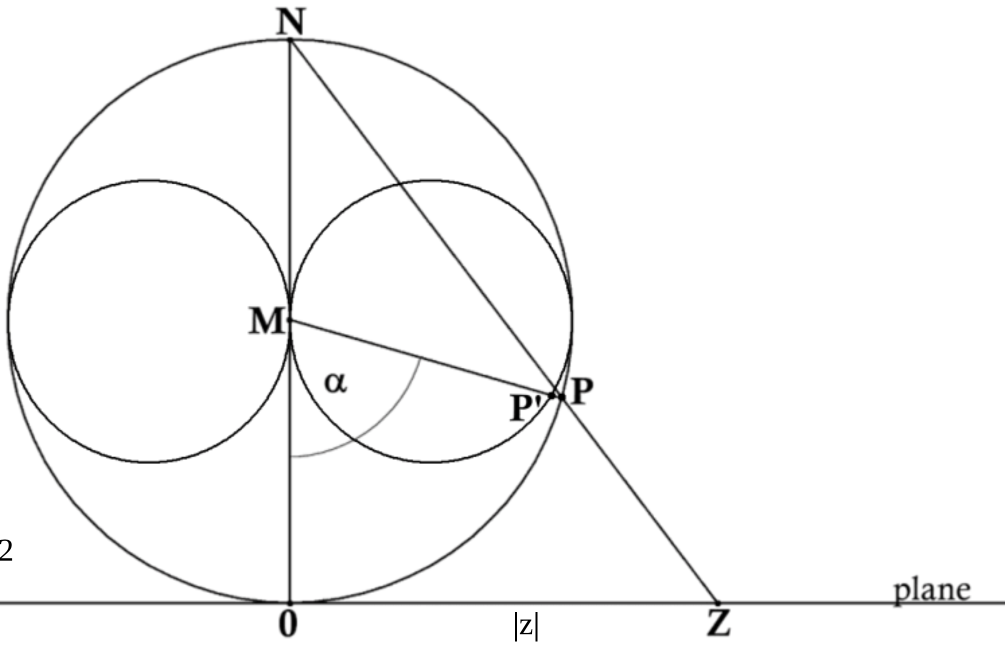

fig. 2

This mapping from the (x, y) plane to the (ξ, η, ζ) horn torus can be expressed as

$$\xi = \frac{2x\sqrt{x^2+y^2}}{(x^2+y^2+1)^2}$$

$$\eta = \frac{2y\sqrt{x^2+y^2}}{(x^2+y^2+1)^2}$$

$$\zeta = \frac{(x^2+y^2-1)\sqrt{x^2+y^2}}{(x^2+y^2+1)^2} + \frac{1}{2}$$

Vyacheslav Puha's analytical description including many of his informative depictions are available from Prof. Saitoh and are published in various related papers by him.

3. Transformation of a sphere to a horn torus

To analyse more properties of the Puha mapping, we now choose a very special construction of the horn torus: bend a half longitude continuously to a circle, preserving its length, as shown in fig. 3 for two opposite ones. In the finally obtained two circles the original angles are doubled (from π to 2π). The transformation of both half longitudes to circles and vice versa, i.e., mapping of every point on the longitudes by this method is bijective. But is the mapping conformal, when the circles are cross sections of a sphere resp. a horn torus?

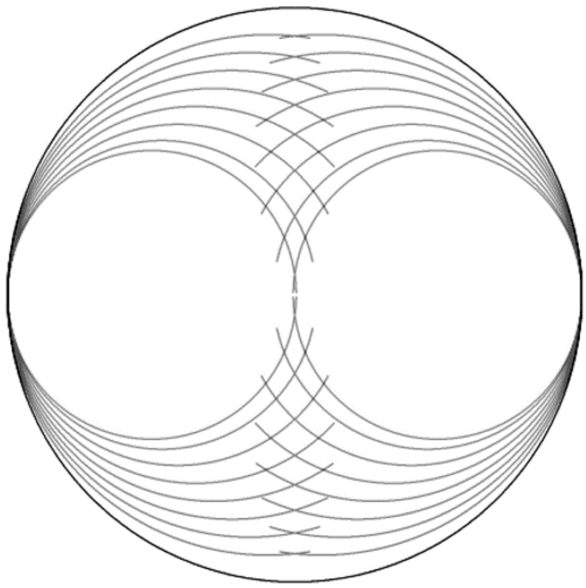

fig. 3

We treat all half longitudes in the described manner simultaneously and recognize that they pass through a series of different spindle tori to finally form a horn torus. We conclude from this construction (see figure 4): to project a point P' from the Riemann sphere onto the horn torus, first double the angle α between closer pole (here N) and the point's radius, then draw the perpendicular line from the horn torus bulge (!) centre M (note the changed designation) to this leg of the doubled angle and intersect with horn torus (choose the intersection P'' on the same 'hemisphere' on which the original point P' is located on the Riemann sphere). It is the same point as the intersection with the original radius! That was proposed by Vyacheslav Puha already. Now consider and compare the lengths of corresponding latitudes on Riemann sphere and on horn torus. From figure 4 we take following length:

SN = 1/2 (N is 'north pole', S is centre, not south pole!)

$$P'Q' = 1/2 \cdot \sin(\alpha) \qquad 3.1)$$

$$P''M'' = 1/4 \cdot \cos(2\alpha)$$

$$P''Q'' = 1/4 \cdot (1 - \cos(2\alpha)) \qquad (3.2)$$

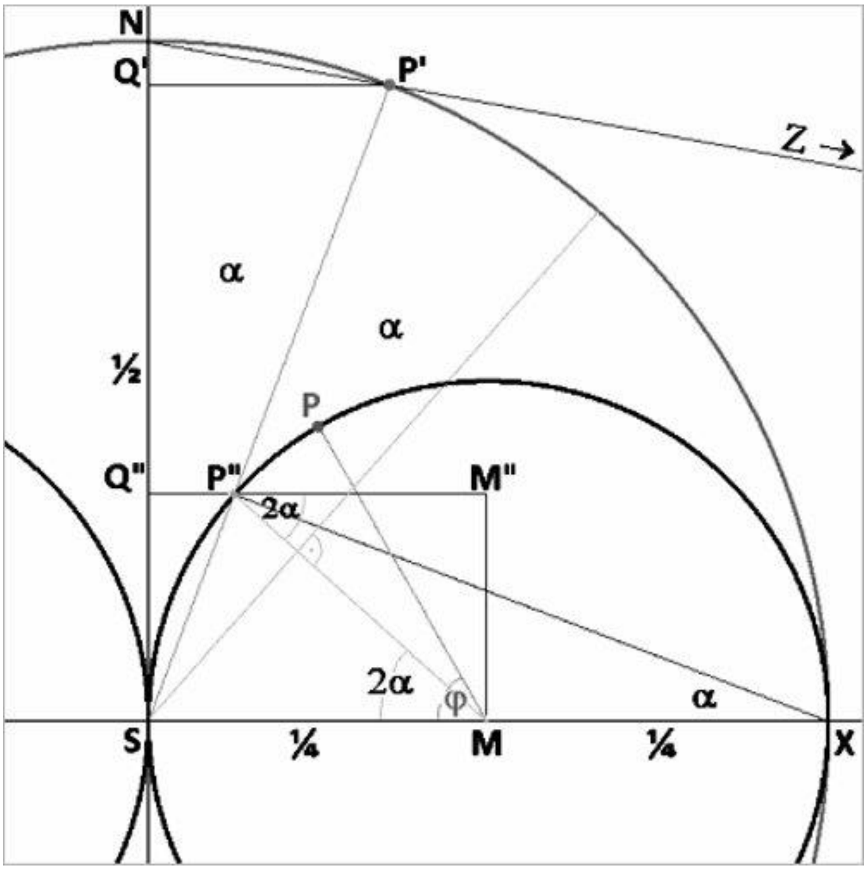

fig. 4

Length of latitude α through P' on sphere (insert 3.1):

$$l_{sph}(\alpha) = 2\pi \cdot P'Q' = \pi \cdot \sin(\alpha) \tag{3.3}$$

Length of latitude α through P'' on horn torus (insert 3.2):

$$l_{ht}(\alpha) = 2\pi \cdot P''Q'' = \pi/2 \cdot (1 - \cos(2\alpha)) = \pi \cdot \sin^2(\alpha) \tag{3.4}$$

On the sphere we get for the latitude length $l_{sph}(\alpha) = \pi\cdot\sin(\alpha)$, on the horn torus however $l_{ht}(\alpha) = \pi\cdot\sin^2(\alpha)$, the ratio of both ($l_{ht} : l_{sph}$) thus is $\sin(\alpha)$, i.e., length of latitude φ decreases faster than angles α and φ, when these diminish, what means that this projected point P'' cannot be the correct one on the horn torus longitude, when we require conformality for the mapping.

So, unfortunately the Puha method fails to obtain a conformal mapping for the horn torus as whole, and any other straightforward stereographic projection P' ↔ P, equivalent $\alpha \leftrightarrow \varphi$, is not in sight, most likely not possible, but, because we do not attempt to square the circle, we at least will search for an analytically generated solution. First some words about the topology:

4. Topological questions

It is easily detectable that point N, which represents infinity, and point 0 (zero) on the sphere combine after the described transformation in the centre of the original sphere to one single point S, as the rotational centre of the horn torus, where all longitudes (meridians) of the horn torus touch the symmetry axis through N and 0. The topology of this figure is very intricate: during transformation the sphere loses its compact property, because N and 0 only remain on one single half longitude each during bending. Compactification then is re-established, as soon as N and 0 combine to one single point (S), but now one of the points has disappeared. Perhaps it makes more sense to revoke the Alexandroff compactification first, what means, point N has to be removed before transformation, and we regain the compactification with point 0 alone. Infinity then is replaced by Zero! This notation seems to be a very helpful aspect in the **division by zero calculus**.

The topology however has to be reviewed thoroughly. We are confronted with a strange, nearly contradictory topology: the horn torus appears as a simply connected 'clopen' set and as differentiable manifold with Riemannian metric, but that occasionally is discussed rather controversially: how are the neighbourhoods of point S interpreted? are they disjoint or connected? is the horn torus an open set? or closed, maybe? clopen, as indicated? is it connected at all? simply connected then?

or multiply connected? Even for the static figure there is no clear consensus, and for the dynamized horn torus then, where rotations around the axis and revolutions around the bulge occur, the situation gets totally intricate and weird.

As an example of weirdness, consider that a line which surrounds the horn torus bulge only once (a longitude e.g.) is not a closed curve, it corresponds to a line on the Gaussian plane, with zero as boundary point, but when circling around twice or more (an even number times!), it is closed without border and can be contracted to a point, however, a certain problem with compactification remains, because lines lack one point, either on the north or on the zero side, again depending on interpretation of point S and its neighbourhood.

All our considerations about topology of the horn torus so far are restricted to the static figure. The properly meaning of the horn torus model with physical interpretations however will be completely different, as explained later, and the static topological results have no relevance for the dynamic properties there. All lines which appear on the horn torus surface, likewise are dynamical and, as such, they pass point S from ‘zero’ to ‘infinity’ and vice versa equicontinuously.

Topology gets maximally intricate when we incorporate inverse figures (as solids and as numbers) into our reflections (nice ambiguity!), e.g. by simply interchanging longitudes and latitudes. In the dynamic horn torus model (see rough description in the addendum) we then change from an infinite number of particles within one universe to an infinite number of universes within one particle or ‘entity’, but that's another, exciting story ...

For now, we remain with the static horn torus. To decide, whether it can replace the Riemann sphere, then with much more properties, we have to show that a *conformal* bijective mapping exists between both figures. In the following section we are on the point of doing that:

5. Conformal mapping from sphere to horn torus and vice versa

A similar simple stereographic projection like the Riemannian probably does not exist for these mappings, when conformality likewise is stipulated, and so we study sphere and horn torus separately, not as representation of complex numbers only, but for all cases and generally valid.

Therefor we don't declare any appropriate stereographic projection and try to proof the conformality afterwards, but we use the conditions of conformality instead to compile and establish the wanted mapping analytically:

One condition for conformal mapping is, that small circles on the origin are mapped as small circles on the target surface, and therefore we construct small circles on both surfaces that only are dependent on the position of points P on the surface, i.e., only dependent on the angles $\alpha = \angle NMP$ for the sphere (see fig. 5) resp. $\varphi = \angle SMP$ for the horn torus (fig. 6). Due to rotation symmetry of both figures, the rotation angle ω doesn't play any role and is arbitrary, and likewise, because of mirror symmetry, angle α can be exchanged by $\pi - \alpha$ (what corresponds to the exchange of north and south pole of the sphere) and φ can be exchanged by $2\pi - \varphi$. Even the radius turns out to be arbitrary and independent, in both figures.

For the stated purpose we consider a small circle around any point P and set the condition that the radii dm (in direction of meridians) and dl (parallel to latitudes) have to be equal (compare figure 5 for the sphere and figure 6 for the horn torus).

Following drawings (fig. 5 and 6) show longitudes, spacing 10°, and latitudes, spacing 20°, points P (not yet as mappings!) are located at angle $\alpha = 50°$ (fig. 5) resp. angle $\varphi = 140°$ (fig. 6) and $\omega = 35°$ on both figures.

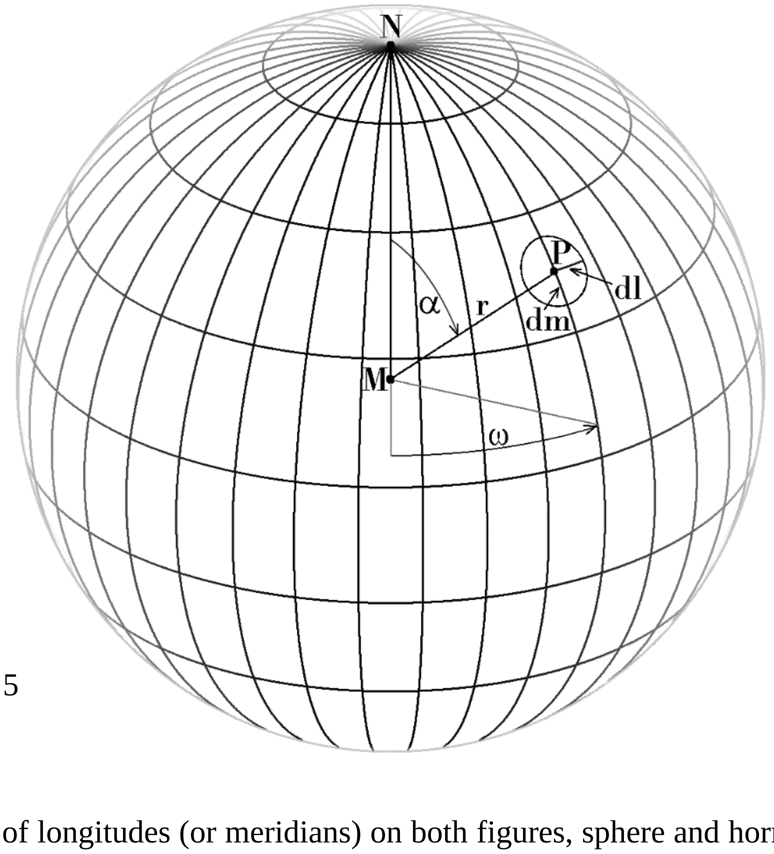

fig. 5

Lengths m of longitudes (or meridians) on both figures, sphere and horn torus, are $m = 2\pi \cdot r$ and lengths l of latitudes (computed differently for both figures, refer to section 8 for derivation) are:

$$l = 2\pi \cdot r \cdot \sin(\alpha) \qquad (5.1)$$

on the sphere, and (from 8.1)

$$l = 2\pi \cdot r \cdot (1 - \cos(\varphi)) \qquad (5.2)$$

on the horn torus.

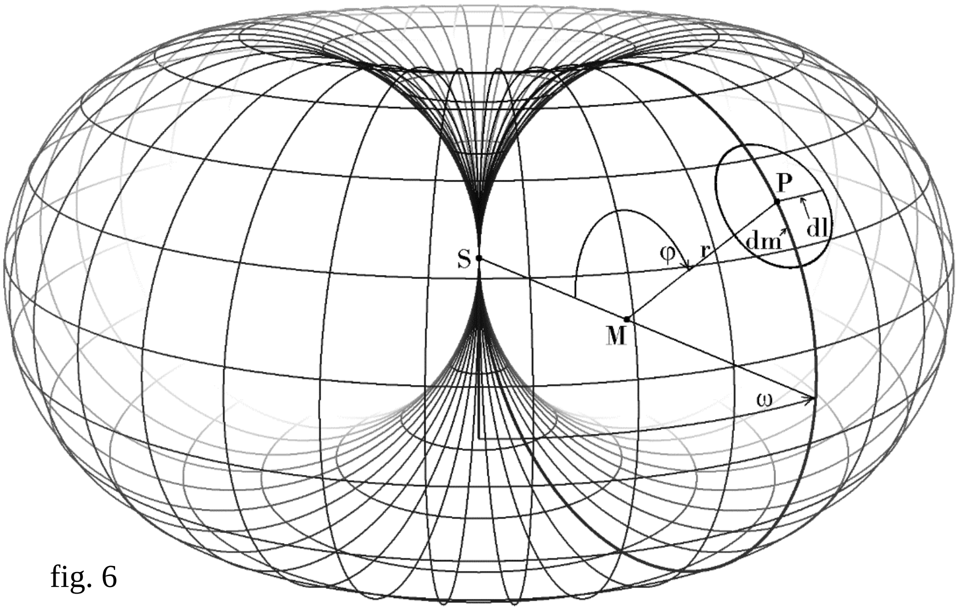

fig. 6

The differentials dm – radii of the small circles on the longitude – are

$dm = r \cdot d\alpha$ on the sphere and
$dm = r \cdot d\varphi$ on the horn torus,

differentials dl – radii of the respective small circles on the latitude – are

$dl = d\omega \cdot r \cdot \sin(\alpha)$ on the sphere and
$dl = d\omega \cdot r \cdot (1 - \cos(\varphi))$ on the horn torus.

After equalling dm and dl in both figures separately and cancelling r we have

$d\alpha = d\omega \cdot \sin(\alpha)$ for the sphere,
$d\varphi = d\omega \cdot (1 - \cos(\varphi))$ for the horn torus.

By solving both equations to $d\omega$ and equalling we get the differential equation:

$$d\alpha/\sin(\alpha) = d\varphi/(1 - \cos(\varphi)) \qquad (5.3)$$

We now obtain the condition for conformal mapping by integration:

$$\int (1/\sin(\alpha))\, d\alpha \;=\; \int (1/(1-\cos(\varphi)))\, d\varphi \tag{5.4}$$

$$\ln(|\tan(\alpha/2)|) \;=\; -\cot(\varphi/2) + C \tag{5.5}$$

Finally, the conformal mapping from sphere to horn torus and vice versa is expressed by a relationship between angles α and φ only, as defined in figures 5 and 6:

$$\varphi \;=\; 2\cot^{-1}(-\ln(|\tan(\alpha/2)|) - C) \tag{5.6}$$

$$\alpha \;=\; 2\tan^{-1}(e^{-\cot(\varphi/2) + C}) \tag{5.7}$$

with $0 < \alpha < \pi$, $0 < \varphi < 2\pi$, C any real number.

C is a kind of 'zoom/diminishing factor' for the mapped figures and shifts them:
case $\alpha \rightarrow \varphi$: φ moves towards 2π with increasing $C > 0$, towards 0 with $C < 0$,
case $\varphi \rightarrow \alpha$: α moves towards π with increasing $C > 0$, towards 0 with $C < 0$.

Conformality is given for $C \neq 0$ as well, but the mappings are no longer bijective, when the constant $C \neq 0$ shall be the same in the inverse mapping.

Btw.: Vyacheslav Puha derived a nice property from the formulas and spotlighted that Euler's number *e* lies 'on top' of the horn torus, 1/*e* 'on bottom'.

6. Generalized Riemannian conformal mapping

The familiar Riemannian stereographic projection (plane ↔ sphere) likewise is a special case amongst others, the generalized formulas for conformal mapping, replacing (1.1) and (1.2), but then with loss of bijectivity when $C \neq 1$, are

$$\alpha = 2\tan^{-1}(C\cdot|z|) \tag{6.1}$$

$$|z| = \tan(\alpha/2) / C \tag{6.2}$$

Real number $C > 0$ again is a kind of 'zoom resp. diminishing factor' for the mapped figures. Maybe, theoretical physicists will find an interpretation for factor C in these generalized formulas.

7. Mapping direct from plane to horn torus and vice versa

When we combine (1.2) with (5.6) resp. (5.7) we get the formulas for conformal mapping direct from plane to horn torus and vice versa:

$$\varphi = 2 \cot^{-1}(-\ln(|z|) - C) \tag{7.1}$$

$$|z| = e^{-\cot(\varphi/2) + C} \tag{7.2}$$

The mapping is quite intricate and has primarily no practical use. But the general validity and the existence of a conformal bijective mapping shows that the horn torus is equivalent to the Riemann sphere and can replace it, whenever it seems to be useful, in complex analysis and in theoretical physics as well. As soon as you introduce the horn torus instead applying the Riemann sphere, the described object instantaneously gains a lot more properties, due to the complex geometry of the horn torus, especially the direct neighbourhood of infinitesimal small and infinitely large values, caused by the coincidence of zero and infinity in one point.

Meaning and importance of these statements depend on the context. When you interpret the horn torus geometry as a symbolic, allegoric visualisation, it serves as an explanatory description of mathematical rules only, but if you take it real, embed it into the Euclidian space, accept the related axioms, then it enables you to establish an extension of conventional mathematics, first and foremost the novel **division by zero calculus**. New insights, concerning laws of nature, are bound to occur ...

8. Supplement: length of horn torus latitude

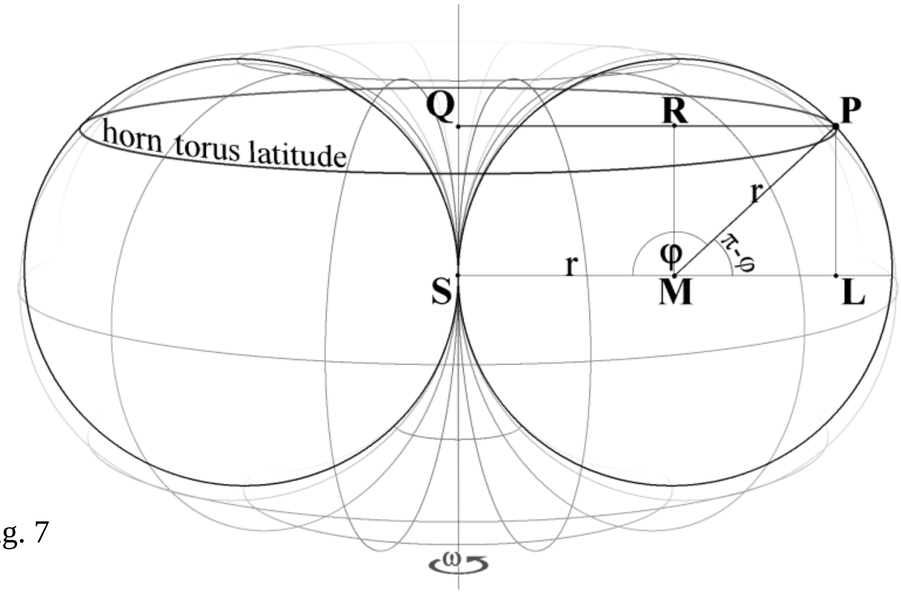

fig. 7

Figure 7 shows details of a horn torus cross section, embedded in a slightly tilted perspective view. By means of the depicted points, lines and angles we calculate length l of the latitude with radius QP as follows (result is valid for all values of φ):

$$QP = QR + RP = SM + ML = r + r\cdot\cos(\pi - \varphi) = r - r\cdot\cos(\varphi) = r\cdot(1 - \cos(\varphi))$$

and with $l = 2\pi\cdot QP$ we get

$$l = 2\pi\cdot r\cdot(1 - \cos(\varphi)) \tag{8.1}$$

9. Properties of the horn torus

As static figure, the horn torus is a rather simple object with only little properties, e.g., one can establish the parametric form in three dimensions

$$x = r\cdot(1 - \cos(\varphi))\cdot\sin(\omega)$$
$$y = r\cdot(1 - \cos(\varphi))\cdot\cos(\omega)$$
$$z = r\cdot\sin(\varphi)$$

or similar, depending on particular definition of axes and angles (compare fig. 6), and the surface area resp. the volume can be computed as

$$A = 4\pi^2 r^2$$
$$V = 2\pi^2 r^3$$

As the horn torus is a special case of ring torus which has two determining radii, R and r, all findings concerning this figure as mathematical or real physical object can be taken for the horn torus by equalling both radii, as a matter of course.

The usual geometrical construction of a horn torus – not the sophisticated method of bending longitudes as described above – is trivial as well: choose any point on a circle, draw the tangent to the circle in this point, rotate the circle around this tangent (as axis of revolution) and get as result a solid of revolution, the horn torus, as three-dimensional object, embedded in the Euclidian space.

All that, honestly, is not really notable. But: horn torus properties get most thrilling when we add ***dynamic*** to the figure! We let it turn, around the main symmetry axis (we call that *'rotation'*) and around the torus bulge (new designation: *'revolution'*), let it change its size during the turns, combine all these motions, and finally, we interlace two, more or even infinitely many such dynamic horn tori of different sizes by nesting them into each other, with the common point S. We then are confronted with a complexity which we never experienced before, only to recognize then, that our conventional mathematics seems to be totally inapt to describe these conditions.

Addendum

The dynamic horn torus model as an allegoric visualisation of fundamental physical entities

10. Relevance of the horn torus in interpretations of 'reality'

To emphasize it again: the horn torus model is neither a mathematical nor a physical theory, it is not science at all, it is rather an allegoric, symbolic representation only, which playfully aims for different interpretations of fundamental processes, for an exciting creation of novel insights and for an amazing enhancement of existing explanations. It remains on an ontological, epistemological, philosophical level.

After science for centuries ignored the existence of the geometric figure horn torus completely or neglected its relevance unduly − nearly nobody realized, described and applied the exceptional topology, maximum symmetry, high complexity and creative capabilities of this unique object – it now, in recent years, increasingly appears in publications, mostly in context of particle and quantum physics and in connection to cosmological questions. The subject is developing and promises to stay exciting for a good while.

The horn torus is not only a mathematical object, not only a geometrical entity, embedded in the familiar Euclidian space, not only a solid figure, which by the way is impossible to be manufactured exactly, but it also is a visual aid, that might help to understand some secrets, mysteries and contradictions of our 'real world' in a different way, not only by use of ad hoc assumptions and abstract physical formulas and equations, but by means of analogously figurative visualisations.

We indeed possess mathematical descriptions for all realized physical processes and apply them very successfully, without any doubt, but no actual model of nature describes all in one big idea, no one comprises all aspects of experienceable 'reality', and furthermore – most important – there is a lack of appropriate *images* which illustrate objects and processes in such a way that they match the human thought structure and mental capability adequately.

Here starts the relevance of horn tori. The associated model tries to fill the lack of imagery, what most of us need for a good understanding. Dynamization of horn tori, illustrated in an easily intelligible pictorial way, generates – only after intense familiarization of course – multifarious and sophisticated new properties, and there will open a completely new mathematical world which bears the possibility to see and describe the fundaments of our 'real world' in a quite different and potentially most comprehensive way.

Dynamic of space is the magic word. Not in the sense of geometrodynamic, where our familiar three- or more-dimensional space somehow is distorted dynamically. The space, we head for with the horn tori, is not spanned by dimensions, it is neither a Euclidean nor non-Euclidean space, nor any vector space, not even in the sense of the abstract Hilbert space – it is spanned instead by continuously and dynamically changing coordinates.

We dynamize the primarily static figure horn torus – I repeat that again and again, to emphasise the underlying principle – by introducing rotation around the main symmetry axis and revolution of the torus bulge around itself, detect the then emerging trajectories on the surface, observe their paths, which depend on the ratio of turns, let the horn torus change its size according to the 'unrolled distance' by the revolution, what again changes the mentioned ratio and the trajectories, then interlace different horn tori by nesting them into one another with common centre, interpret the superposition or mutual interference of trajectories as particle interaction, finally abstract from the three-dimensional space of imagination to the effect that we renunciate dimensions at all, which in common view span a physical space as container for physical objects, and – as quintessence of all previous mental efforts – detect that our new space is *'dimensionless'*.

That all sounds extremely intricate and artificial, and to realise, that the three-dimensional space (likewise all linear vector spaces!) shall be a man-made construct and not an a priori existing entity, seems to be difficult to place within the scientific community. But an open-minded readiness for broad familiarization with the allegoric images and processes, leading to a far-reaching abstraction from most traditional ideas, soon enables you to revel in new interpretations, perceptions, cognitions, and lots of aha experiences.

The dynamic horn torus model highlights and perhaps reveals big mysteries in our comprehension of 'reality', at least in an allegorical, descriptive sense for issues concerning intrinsicality of time, space and physical objects, continuum vs. discrete nature of space, metrisation of space, origin and cause for quantization, minimum values, constancy and maximum value of lightspeed, non-locality of quantum processes, entanglement, arrow of time, determinism, causality, …

In our intellectual game we reduce the many (too many!) necessary components of the standard model to one single principle, which underlies all physical objects and processes as a unified mathematical structure or rule, by introducing graphically describable and imaginable all-embracing fundamental entities. But again: I want to emphasize, that the horn torus model is neither physics nor mathematics, and least of all it is esotericism or anything like 'holy' or 'sacred' geometry! It is a pure intellectual game to realize generally, that fundamental entities as representation of physical objects, processes, elementary particles, and space *are* imaginable and constructable.

11. Principle of the horn torus model

Origin of the idea (in 1988) was the endeavour to find an analogously figurative visualisation of complex numbers, different from the well-known Riemann sphere. Target was to illustrate *dynamic* physical processes which very often are described mathematically by means of complex numbers – be it in electrodynamic, relativity or in quantum mechanics. As proposal then emerged that the real part shall be represented by the rotation of a horn torus around the main symmetry axis, the imaginary part by the revolution of the torus bulge around itself, measured in degrees of rotation respectively revolution, not as localisation on the horn torus surface. Values of numbers thus are represented and visualised *dynamically*.

Main principle of the dynamic horn torus is that it rolls along an imaginary axis $i{\cdot}$t with *constant* 'speed' *c* and changes its size according to the unrolled 'distance' on this axis. Without rotation around the axis, the difference of horn torus longitudes before and after a motion corresponds to the unrolled distance during 'locomotion'. Circumferential speed is constant, but angular velocity of revolution depends on size: these latter quantities are inversely proportional. When we now add rotation, the situation gets more intricate: the place of the unrolling longitudes takes a trajectory, which figuratively unrolls without 'slippage' on the axis.

These trajectories, 'unrolling lines' or cycloids show a particular pattern on the horn torus surface, according to the ratio of revolution and rotation, more precisely, to the ratio of their angular velocities. Here, in fig. 8, we show some trajectories for different ratios in a perspective view:

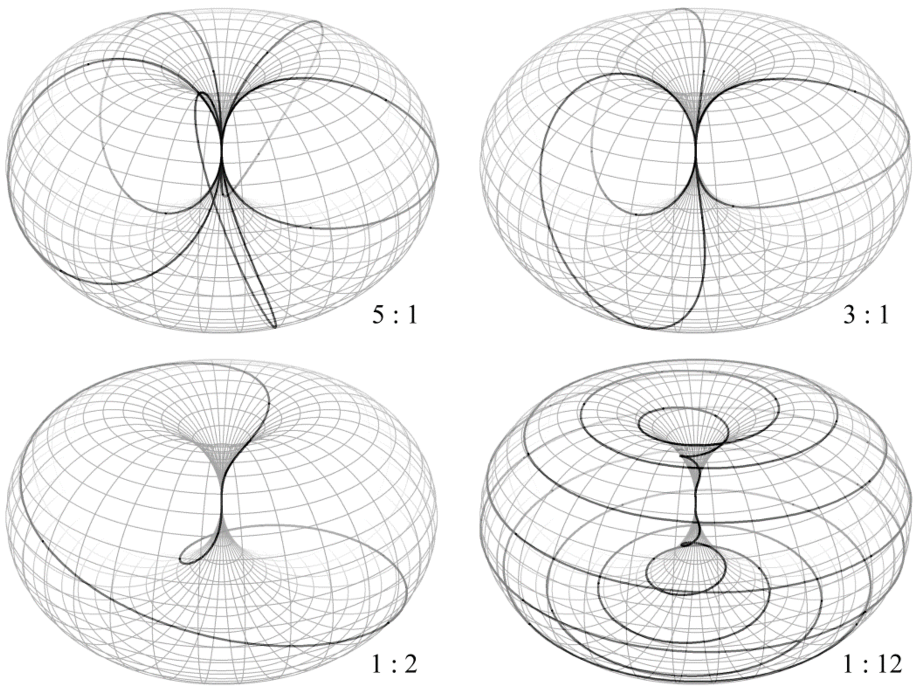

fig. 8

The examples in fig. 8 show trajectories for rational ratios (revolution : rotation). For ratios ≥ 1 they loop after one rotation, for ratios ≤ 1 after one revolution. I call them resonances or *Lissajous* figures on horn torus surfaces. During unrolling along the axis, a horn torus changes its size and therefore passes all possible ratios and all resonances as well. For small horn tori we see many revolutions per rotation and for big horn tori many rotations per revolution. A kind of mathematical mirror is the ratio 1:1, and in the physical interpretations it will appear to have a very special meaning. The related figurative object is called ‘standard dynamic horn torus’.

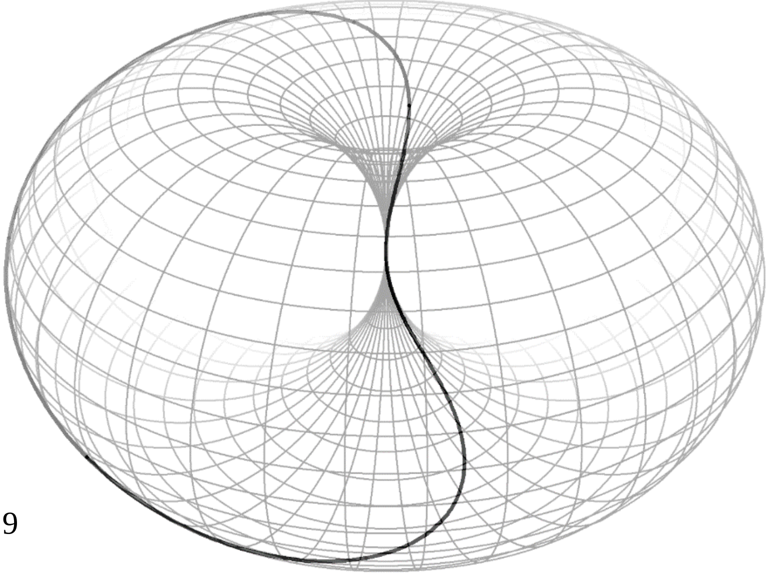

fig. 9

To anticipate later considerations: the standard dynamic horn torus already *symbolises* some of the most fundamental quantities. For this *unit* horn torus, we determine that *one* revolution represents Planck time t_P and the circumference of a longitude (meridian) Planck length l_P (in a different interpretation: length of the 1:1-trajectory), while the circumferential speed of revolution is light speed c, and *one* rotation represents Planck’s reduced constant $\hbar$.

The unit horn torus divides the set of all (infinitely many) possible horn tori into two parts. Those with ratio revolution : rotation > 1 have a smaller (down to infinitesimal) size, compared with this unit, and we see trajectories with more than one ‘loop’ around the torus bulge per one rotation, while tori with ratio < 1 are bigger (up to infinitely large) with more than one ‘spiral’ around the rotational axis per one revolution of the bulge (compare examples in fig. 8) .

The complete set comprises all horn tori, all sizes, and all ratios (but note, size and ratio are not independent, they are inversely proportional!), and we designate it as ‘fundamental entity’. A particular horn torus occurs exactly *once* in the set.

To give an impression of the complexity, we now draw a simplified graph of this entity, represented by the 'unrolling line' in dependence of an increasing r, selected between ratios $v(\varphi) : v(\omega) = 10 : 1$ and $v(\varphi) : v(\omega) = 1 : 1000$, but not to scale and using rather rough differentials dr for the single steps (otherwise the lines would lay very densely). r is horn torus radius, v angular velocity, φ revolution, ω rotation.

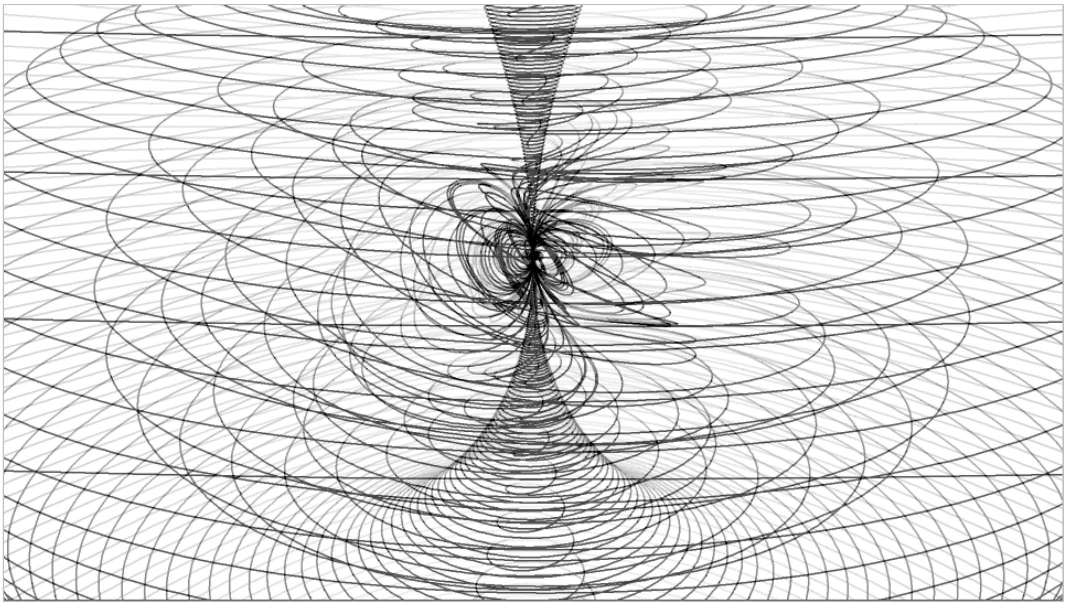

fig. 10

The related parametric form must be established for both parts of the entity separately in the following way (setting $r = 1$ for ratio $v(\varphi) : v(\omega) = 1 : 1$)

For case $r > 1$, r and ω increase with φ, starting with $\varphi_1 = 2\pi$,
and according to $r = \varphi / 2\pi$ ($\leadsto r_1 = 1$), $\omega = r\cdot\varphi = \varphi^2 / 2\pi$ ($\leadsto \omega_1 = 2\pi$), we have

$x = (1 - \cos(\varphi))\cdot\cos(\varphi^2/2\pi)\cdot\varphi/2\pi$
$y = (1 - \cos(\varphi))\cdot \sin(\varphi^2/2\pi)\cdot\varphi/2\pi$
$z = \sin(\varphi) \cdot \varphi/2\pi$

For case $r < 1$, $1/r$ and φ increase with ω, starting with $\omega_1 = 2\pi$,
and according to $1/r = \omega / 2\pi$ ($\leadsto r_1 = 1$), $\varphi = \omega / r = \omega^2 / 2\pi$ ($\leadsto \varphi_1 = 2\pi$), we have

$x = (1 - \cos(\omega^2 / 2\pi))\cdot\cos(\omega)\cdot 2\pi/\omega$
$y = (1 - \cos(\omega^2 / 2\pi))\cdot \sin(\omega)\cdot 2\pi/\omega$
$z = \sin(\omega^2 / 2\pi)\cdot 2\pi/\omega$

The parametric forms show mirrored conditions, so, as already mentioned, the unit standard dynamic horn torus indeed is a kind of mirror within the new mathematical world, and likewise, as physical interpretation, it separates the big 'outer' world, consisting of electrons and photons from the small 'inner' atomic and nuclear realm.

Next step should be, that we interlace entities into one another, maintaining a common point for all, namely the point S (S for symmetry or singularity), the centre of all nested horn tori. But alone the complexity of one single entity indicates that this endeavour appears to have no prospect of success, when we apply common mathematics only. But it could be worse. For our aim, to get an idea of the model, it is enough to understand the principle of nesting two or a few horn tori into one another. As first impression here two (fig. 11) and 12 (fig. 12) nested horn tori:

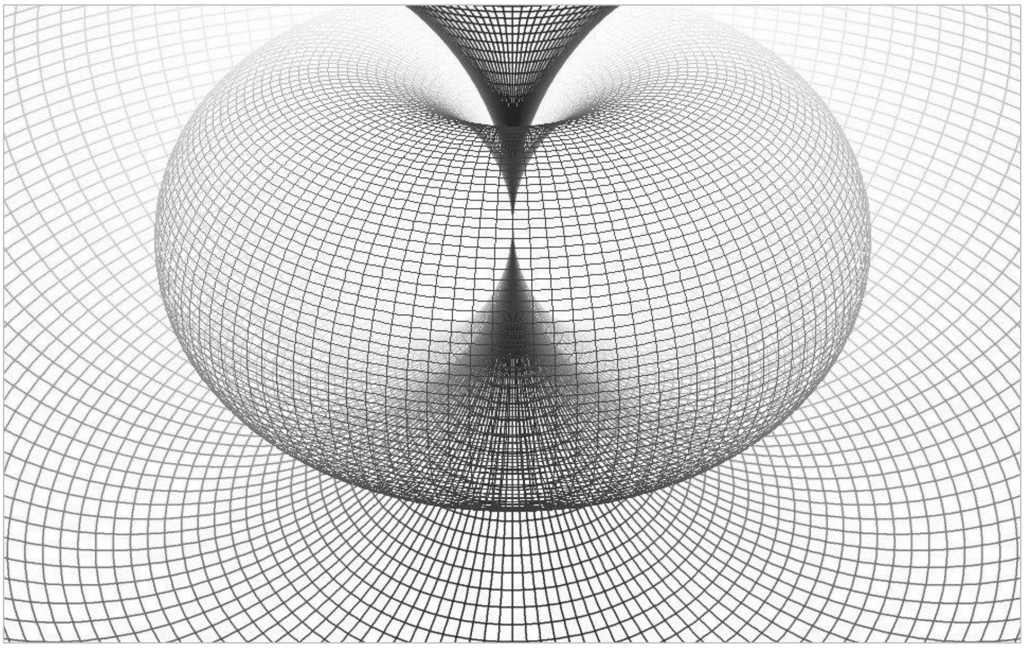

fig. 11

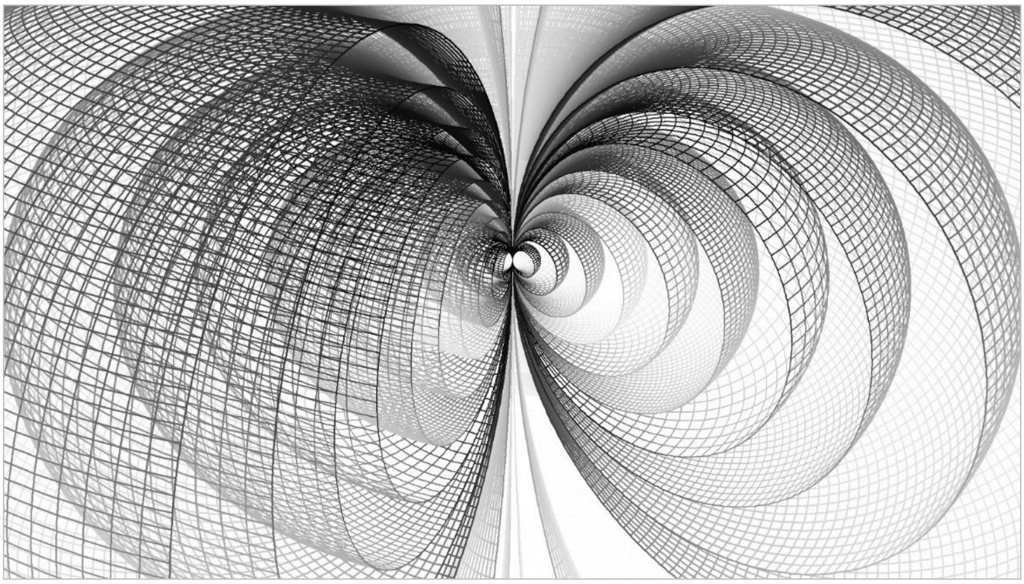

fig. 12 (a 65° wedge is removed to highlight the nesting)

The dynamic, however, as most important intrinsic property of the model, cannot be described properly and comprehensively by static depictions like the above ones. We need moving pictures, animations, simulations for an adequate presentation of dynamic. In this printed publication we have no choice but to provide links to respective webpages, where such animations can be viewed. Here I cite websites only that are created by myself:

https://www.horntorus.com/enter/
https://www.horntorus.com/sitemap.html
http://www.mathematical-universe.com/
http://www.bighiss.video/
https://www.horntorus.com/illustration/URLdetail.html

The latter shows a wide section of a single entity in a coarse, simplified depiction of the dynamically uncoiling horn torus coordinate. The ‘unrolling line’ starts parallel to longitudes on a very small horn torus, forms Lissajous figures, when passing rational ratios revolution : rotation, which are shown as ‘resonances’ with short stops during increase of size, and approaches infinite size with parallels to latitudes, without resonances. The animation possibly might lead to the following:

12. Physical interpretation

Sharp resonances represent fermions, the sections between them bosons, and the lines without any resonances on big horn tori (ratio << 1) are photons. In this way all kinds of elementary particles are beaded on one thread, within one single coordinate, forming a unique ‘fundamental entity’. What appears to be a ‘particle’ is only the ‘local’ property of the much more comprehensive entity at a specific ‘location’ in the horn torus space. What we describe as a nucleon here, is an electron there, farther away a photon, and simultaneously a completely different ‘particle’ of the same entity elsewhere.

Now we combine another or more entities with a singled-out entity by nesting them in their common point S. Place of ‘interaction’, where a horn torus unrolls on a trajectory, always remains this centre. It is the ‘spatial point’, to which any ‘particle’ in the universe is connected as one portion of its individual ‘entity’. All infinite many ‘particles’ of the universe are represented in every ‘spatial point’. Every horn torus shares the common tangent with all other horn tori, when they are nested into one another at their centres. Size of a horn torus symbolises the ‘distance’ to the location, where the associated entity (the dynamic coordinate) converges to size zero. Different ‘spatial points’ differ in the combination of horn torus sizes. All combinations with *natural numbers* (explication for that see links above) of sizes are possible, forming an infinite-dimensional regular pattern. All paths through this *discrete* pattern (space) are equally possible, and *every* horn torus can unroll at a trajectory formed by *any* other horn torus as interaction at *any* ‘spatial point’.

This space is by its definition a multiverse (due to infinite many paths) with *non-local* correlations between ‘particles’. A limited neighbourhood of every ‘spatial point’ contains the complete information about one selected universe. With this property one – by the way – immediately recognizes the possibility that big bang has not started out of one tiny spot but rather took place all-over a pre-existing just one-dimensional infinity. Much more information you will find on linked websites.

The (allegoric or analogously figurative) horn torus model primarily only provides ontologically relevant interpretations. These can be used to justify and substantiate a philosophical worldview that can be assigned to subjective idealism or constructivism, and on the basis of which both axiomatic mathematics and phenomenological physics are regarded as *not* real (independent of consciousness). As first conclusions, among others, one perhaps could state these:

- There is *no* empty space in which physics takes place. Complex dynamic processes *generate* the space, an idle state (statics) does *not* occur.
- There are *no* ‘dimensions’, neither a Euclidean nor non-Euclidean space, nor any vector spaces, not even in the sense of the abstract Hilbert space.
- Time is *not* a basic physical quantity, it is included and identifiable in the model, but is redundant and reducible to more fundamental quantities.
- Without a consciousness that describes them, there are *no* individual, independent ‘physical objects’, especially *no* isolated or *isolatable* particles.
- *All* entities, i.e., all ‘particles’ too, are connected (linked, entangled) with *all* other entities or ‘particles’ of the universe and ‘interact’ with each other.
- Interacting means being dynamically interlaced, constantly measuring and comparing each other and *deterministically* integrating into an entirety.
- The universe is an orderly, mathematically describable network, a regularly structured pattern, the mere structure of an *a priori existing mathematics*.
- A consciousness uses its special senses to pick out only very small parts of this abstract mathematical pattern from the comprehensive total of processes.
- All described properties of what is perceived are *interpretations* by the perceiver, generated by subjective mental processing.

Postulation as a consistent theory, based on the horn torus model, will be an ambiguous project for physicists in the future, after mathematicians and computer programmers will have established methods to handle that dynamic. I'm afraid we need a different, perhaps disruptive, ‘fundamental’ mathematics.

In retrospect, with a view onto history of science, horn tori could be associated with the ‘ur-alternatives’ by C.F.v. Weizsäcker, but also with spinors, Penrose’s twistors, Lie groups and all related − also current − symmetry considerations.

* * *

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