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SOME NOTES ON SOFT DIMONOIDS

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ABSTRACT. In this paper an idea of soft dimonoids which is a more general structure than soft monoids is presented and some of their structural properties are examined. Besides, the restricted (extended) intersection and \wedge -intersection of the family of soft dimonoids are studied. At the end of the paper, the category of soft dimonoids is proposed and the definition of soft subdimonoids is given with examples.

1. Introduction

Jean-Louis Loday proposed the definition of a dimonoid and constructed the structure of free dimonoid [1]. Dimonoids have some applications in the dialgebra theory [2]. Every dimonoid is also isomorphic to some transformation dimonoid in Cayley's theorem for dimonoids [3]. They are closely connected with trialgebras and trioids defined by Loday and Ronco in the context of algebraic topology [5]. For a main introduction and general theory see [4,6].

In different fields of mathematical sciences, there are many intricate problems involving various uncertainties. To solve these problems, the theories as theory of probability, theory of fuzzy sets and theory of rough sets is introduced. Although these theories are successfull to tackle, they have complexities which reason from the inadequacy of the parametrization tool. In this point, soft set theory is presented by Molodstov as a powerful mathematical tool which be free of all such impediments [8]. It is rapidly getting popularity among the researchers and a good number of studies is being published every year [9,14]. In addition to engineering and computer sciences, the applications of in areas such as algebra, topology, analysis and geometry has carried this theory to a different point [10-13, 15-18].

Recently, many connections between soft sets and some algebraic structures has been investigated. In this perspective, the concept of soft dimonoids is defined as a soft approach to dimonoids. Examples of this concept are given and some important properties are examined. Also, the category of soft dimonoids is establish and the definition of soft subdimonoids is proposed.

2. Preliminaries

This section provides some main concepts and results about soft sets and dimonoids which will be needed in the sequel.

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Let \mathcal{V} be an initial universe set and M be a set of parameters. Also, let $P(\mathcal{V})$ denotes the power set of \mathcal{V} and $K \subset M$. The description of a soft set proposed by Molodtsov is as follows:

Definition 2.1. [8] A pair (Ω, K) is called a soft set over \mathcal{V} , where Ω is a mapping defined by

$$\Omega: K \longrightarrow P(\mathcal{V})$$

Notice that a soft set over \mathcal{V} can be considered as a parametrized family of subsets of the universe \mathcal{V} .

Definition 2.2. [16] The support of a soft set (Ω, K) is defined as a set

$$Supp(\Omega, K) = \{ \alpha \in K : \Omega(\alpha) \neq \emptyset \}$$

If $Supp(\Omega, K)$ is not equal to the empty set, then (Ω, K) is said to be non-null.

We state below some general characterizations for the nonempty family $\{(\Omega_i, K_i) \mid i \in \mathcal{I}\}$ of soft sets over the common universe \mathcal{O} such that \mathcal{I} is an index set.

Definition 2.3. [12] The restricted intersection of the family $\{(\Omega_i, K_i) \mid i \in \mathcal{I}\}$ is defined by a soft set $(\Omega, K) = \bigcap_{i \in \mathcal{I}} (\Omega_i, K_i)$ such that $K = \bigcap_{i \in \mathcal{I}} \mathcal{K}_i \neq \emptyset$ and $\Omega(\alpha) = \bigcap_{i \in \mathcal{I}} \Omega_i(\alpha)$ for all $\alpha \in K_i$.

Definition 2.4. [12] The extended intersection of the family $\{(\Omega_i, K_i) \mid i \in \mathcal{I}\}$ is a soft set $(\Omega, K) = (\bigcap_{\Omega_i \in \mathcal{I}} (\Omega_i, K_i) \text{ such that } K = \bigcup_{i \in \mathcal{I}} K_i \text{ and } \Omega(\alpha) = \bigcap_{i \in \mathcal{I}(\alpha)} \Omega_i(\alpha), \mathcal{I}(\alpha) = \{i \in \mathcal{I} \mid \alpha \in K_i\} \text{ for all } \alpha \in K_i.$

Definition 2.5. [12] The \wedge -intersection of the family $\{(\Omega_i, K_i) \mid i \in \mathcal{I}\}$ is defined by a soft set $(\Omega, K) = \bigwedge_{i \in \mathcal{I}} (\Omega_i, K_i)$ such that $K = \prod_{i \in \mathcal{I}} K_i$ and $\Omega((\alpha_i)_{i \in \mathcal{I}}) = \bigcap_{i \in \mathcal{I}} \Omega_i(\alpha_i)$ for all $(\alpha_i)_{i \in \mathcal{I}} \in K_i$.

Here, the concepts of dimonoid, dimonoids homomorphism and subdimonoid will be reviewed.

Definition 2.6. [1] A \mathcal{D} dimonoid is a triple $(\mathcal{D}, \circ, \diamond)$ together with two binary operations \circ and \diamond that satisfies the following properties:

$$\begin{aligned} a \circ (b \diamond c) &= (a \circ b) \diamond c \\ a \diamond (b \circ c) &= (a \diamond b) \circ c \\ a \diamond (b \diamond c) &= (a \circ b) \diamond c \\ a \circ (b \diamond c) &= (a \circ b) \diamond c \\ a \diamond (b \diamond c) &= (a \diamond b) \diamond c \end{aligned}$$

for all $a, b, c \in \mathcal{D}$. Also, a dimonoid \mathcal{D} is be called commutative, if its both binary operations are commutative.

Example 2.7. [4] Consider a semigroup \mathcal{D} and an idempotent endomorphism $\phi : \mathcal{D} \longrightarrow \mathcal{D}$. Define the operations \circ and \diamond on \mathcal{D} as follows:

$$a \circ b = a\phi(b), \ a \diamond b = \phi(a)b$$

for all $a, b \in \mathcal{D}$. Together with these operations it is clear that $(\mathcal{D}, \circ, \diamond)$ is a dimonoid.

Definition 2.8. [4] A dimonoid \mathcal{D} is called idempotent dimonoid or diband, if $a \circ a = a = a \diamond a$ for all $a \in \mathcal{D}$.

Example 2.9. [4] Choose $\mathcal{D} = \{a, b\}$. Define a semigroup $(\mathcal{D}, \diamond, \diamond)$ with the binary operation as follows:

$$\begin{array}{c|ccc} \diamond & a & b \\ \hline a & a & b \\ b & b & b \end{array}$$

Then, it is easy to see that $(\mathcal{D},\diamond,\diamond)$ is a commutative idempotent dimonoid.

Theorem 2.10. [4] Let \mathcal{D} be a semigroup. Define operations \circ and \diamond on $\mathcal{D} \times \mathcal{D}$ by

$$(a,b)\circ(c,d)=(ac,bc),\quad (a,b)\diamond(c,d)=(ac,ad)$$

for all $(a, b), (c, d) \in \mathcal{D} \times \mathcal{D}$. Then $\mathcal{D}' = (\mathcal{D} \times \mathcal{D}, \circ, \diamond)$ is a dimonoid. Conversely, for an arbitrary dimonoid \mathcal{D}' , there exists a semigroup \mathcal{D} such that \mathcal{D}' is isomorphically embedded into \mathcal{D} .

Definition 2.11. [4] A subset S of a dimonoid D is called a subdimonoid of D if it forms a dimonoid under the same binary operations as that of D.

Definition 2.12. [4] A homomorphism $\psi : \mathcal{D}_1 \longrightarrow \mathcal{D}_2$ between two dimonoids \mathcal{D}_1 and \mathcal{D}_2 is a mapping from \mathcal{D}_1 to \mathcal{D}_2 such that $\psi(a \circ b) = \psi(a) \circ \psi(b)$ and $\psi(a \diamond b) = \psi(a) \diamond \psi(b)$ for all $a, b \in \mathcal{D}_1$.

Example 2.13. [4] Take $\mathcal{D}_1 = \mathcal{D}_2 = \{a, b\}$. Consider a dimonoid $(\mathcal{D}_1, \circ, \circ)$ with the binary operation as follows:

$$\begin{array}{c|cc} \circ & a & b \\ \hline a & a & a \\ b & b & b \end{array}$$

Also, choose a dimonoid $(\mathcal{D}_2, \circ, \diamond)$ with the binary operations as follows:

0	a	b	<	>	a	b
a	a	a	0	ı	a	b
b	b	b	ł)	a	b

Now define the mapping $\psi : \mathcal{D}_1 \longrightarrow \mathcal{D}_2$ letting $\psi(x) = a$ for all $x \in \mathcal{D}_1$. It is straightforward to check that ψ is a homomorphism of dimonoids.

Definition 2.14. [4] $(\mathcal{D}, \circ, \diamond)$ is called a diband of subdimonoids \mathcal{D}_i for $i \in I$ if if the following axioms are satisfied: $\mathcal{D}_i = [\mathcal{D}_i] = \mathcal{D}_i$

$$D = \bigcup_{i \in \mathcal{I}} \mathcal{D}_i$$

ii.
$$\mathcal{D}_i \cap \mathcal{D}_j = \emptyset$$
 for all $i \neq j$.

iii. For all $i, j \in I$, there is any one $k, r \in I$ such that $\mathcal{D}_i \circ \mathcal{D}_j \subseteq \mathcal{D}_k$ and $\mathcal{D}_i \diamond \mathcal{D}_j \subseteq \mathcal{D}_r$.

3. Soft Dimonoids

In this section, the notion of soft dimonoids is introduced and the related structural properties are studied.

Definition 3.1. Let \mathcal{D} be a dimonoid and let $P(\mathcal{D})$ denotes the set of all subdimonoids of \mathcal{D} . A pair (Ω, K) is called a soft dimonoid over \mathcal{D} , where Ω is a mapping given by

$$\Omega: K \longrightarrow P(\mathcal{D})$$

and K is a set of parameters, if F(a) is a subdimonoid of \mathcal{D} for all $a \in K$.

It is worse pointing out that if each $\Omega(a)$ is commutative as a dimonoid, the soft dimonoid \mathcal{D} is said to be commutative. Also, a soft dimonoid \mathcal{D} can be regarded as a parameterized family of subdimonoids of the dimonoid \mathcal{D} . In what follows, (\mathcal{D}, Ω, K) is refer to the soft dimonoid (Ω, K) over the dimonoid \mathcal{D} .

Example 3.2. Let (S, \diamond) be a semigroup and (Ω, K) be a soft semigroup over S. Then, the triblet (S, \diamond, \diamond) is a dimonoid such that $\Omega(a)$ is a subdimonoid of S for all $a \in K$. Thus, (Ω, K) becomes a soft dimonoid over S. So, every soft semigroup can be considered as a soft dimonoid.

Example 3.3. Let (Ω, K) be an soft dimonoid over \mathcal{D} and $\mathcal{S} \subseteq \mathcal{D}$. Then, $(\Omega|_{\mathcal{S}}, K)$ is a soft dimonoid over \mathcal{S} if it is non-null.

Definition 3.4. Let (\mathcal{D}, Ω, K) and $(\mathcal{F}, \Omega', R)$ be two soft dimonoids. The product of them is described as $(\mathcal{D}, \Omega, K) \times (\mathcal{F}, \Omega', R) = (\mathcal{D} \times \mathcal{F}, \Omega'', K \times R)$, where $\Omega''(a, b) = \Omega(a) \times \Omega'(b)$ for all $(a, b) \in K \times R$.

From this definition, we have the following proposition.

Proposition 3.5. The product of any two soft dimonoids is also a soft dimonoid.

Proof. Consider the soft dimonoids (\mathcal{D}, Ω, K) and $(\mathcal{F}, \Omega', R)$. Then,

$$\begin{array}{rccc} \Omega: K & \longrightarrow & P(\mathcal{D}) \\ a & \mapsto & \Omega(a) \end{array}$$

and

$$\begin{array}{rccc} \Omega': R & \longrightarrow & P(\mathcal{F}) \\ a & \mapsto & \Omega'(a) \end{array}$$

such that $\Omega(a)$ is a subdimonoid of the dimonoid \mathcal{D} for all $a \in K$ and $\Omega'(b)$ is a subdimonoid of the dimonoid \mathcal{E} for all $b \in R$. Using these mappings, we describe Ω'' by

$$\begin{array}{rcl} \Omega'': K \times R & \longrightarrow & P(\mathcal{D} \times \mathcal{F}) \\ (a,b) & \mapsto & \Omega''(a,b) = \Omega(a) \times \Omega'(b) \end{array}$$

From the product of dimonoids, observe that $\Omega(a) \times \Omega'(b)$ is also a subdimonoid of the product dimonoid $\mathcal{D} \times \mathcal{F}$ for all $(a, b) \in K \times R$ which implies that $(\mathcal{D} \times \mathcal{F}, \Omega'', K \times R)$ is a soft dimonoid.

The following theorems relate some generalizations for a nonempty family of soft dimonoids.

Theorem 3.6. Let $\{(\Omega_i, \mathcal{K}_i) \mid i \in \mathcal{I}\}$ be a non-empty family of soft dimonoids over \mathcal{D} .

ii. The restricted intersection of the family $\{(\Omega_i, \mathcal{K}_i) \mid i \in \mathcal{I}\}$ with $\bigcap_{i \in \mathcal{I}} \mathcal{K}_i \neq \emptyset$ is a soft dimonoid over \mathcal{D} if it is non-null.

iii. The extended intersection of the family $\{(\Omega_i, \mathcal{K}_i) \mid i \in \mathcal{I}\}$ is a soft dimonoid over \mathcal{D} if it is non-null.

Proof. i. The restricted intersection of the family $\{(\Omega_i, \mathcal{K}_i) \mid i \in \mathcal{I}\}$ with $\bigcap_{i \in \mathcal{I}} \mathcal{K}_i \neq \emptyset$ given by the soft set $\widetilde{\bigcap}_{i \in \mathcal{I}} (\Omega_i, \mathcal{K}_i) = (\Omega, \mathcal{K})$ such that $\bigcap_{i \in I} \Omega_i(\alpha)$ for all $\alpha \in \mathcal{K}$ from Definition 2.3. Take $\alpha \in Supp(\Omega, \mathcal{K})$. Together with the hypothesis, $\bigcap_{i \in \mathcal{I}} \Omega_i(\alpha) \neq \emptyset$, implies that $\Omega_i(\alpha) \neq \emptyset$ for all $i \in \mathcal{I}$. Since $\{(\Omega_i, \mathcal{K}_i \mid i \in \mathcal{I}\} \text{ is a non-empty family of soft dimonoids over } \mathcal{D}$, it is then easy to see that $\Omega_i(\alpha)$ is a subdimonoid of \mathcal{D} for all $i \in \mathcal{I}$. Morever, $\bigcap_{i \in \mathcal{I}} \Omega_i(\alpha)$ is a subdimonoid of \mathcal{D} too. Consequently, (Ω, \mathcal{K}) is a soft dimonoid over \mathcal{D} .

ii. It is similar to the proof of previous case.

Similarly, it follows that

Theorem 3.7. Let $\{(\Omega_i, \mathcal{K}_i) \mid i \in \mathcal{I}\}$ be a non-empty family of soft dimonoids over \mathcal{D} . Then the \wedge -intersection $\widetilde{\bigwedge}_{i \in \mathcal{I}}(\Omega_i, \mathcal{K}_i)$ is a soft dimonoid over \mathcal{D} if it is non-null.

Proof. Assume $(\Omega, \mathcal{K} = \bigwedge_{i \in \mathcal{I}} (\Omega_i, \mathcal{K}_i, \tau)$ for a non-empty family $\{(\Omega_i, \mathcal{K}_i) \mid i \in \mathcal{I}\}$ of soft dimonoids over \mathcal{D} . Choose $\alpha \in Supp(\Omega, \mathcal{K})$. It holds from the the assumption $\bigcap_{i \in \mathcal{I}} \Omega_i(\alpha_i) \neq \emptyset$ that $\Omega_i(\alpha_i) \neq \emptyset$ for all $i \in \mathcal{I}$ and $(\alpha_i)_{i \in \mathcal{I}} \in \mathcal{K}_i$. Hence, $\Omega_i(\alpha_i)$ is a subdimonoid of \mathcal{D} for all $i \in \mathcal{I}$ so that their intersection is a subdimonoid of \mathcal{D} too. Hence, (Ω, \mathcal{K}) is a soft dimonoid over \mathcal{D} . The proof is complete.

4. Soft Subdimonoids

In this section, we study the soft subdimonoids and describe the notion of soft dimonoid homomorphisms.

Definition 4.1. Let (Ω, K) and (Ω', R) be any two soft dimonoids over \mathcal{D} and \mathcal{F} , respectively. Then, (Ω', R) is said to be a soft subdimonoid of (Ω, K) if $R \subset K$ and $\Omega'(b)$ is a subdimonoid of $\Omega(b)$ for all $b \in R$.

Example 4.2. For the soft dimonoid (Ω, K) over \mathcal{D} , assume (Ω', R) be a soft dimonoid over the idempotent dimonoid \mathcal{F} and $R \subset K$. It is easy to check that (Ω', R) is a subdimonoid of (Ω, K) .

From the above definition, it is straightforward to see that

Theorem 4.3. Let (Ω, K) , (Ω', R) and (Ω'', S) be any three soft dimonoids over \mathcal{D} , \mathcal{F} and \mathcal{T} , respectively. If (Ω'', S) is a soft subdimonoid of (Ω', R) and (Ω', R) is a soft subdimonoid of (Ω, K) , then (Ω'', S) is a soft subdimonoid of (Ω, K) .

Proof. It is immediate.

Theorem 4.4. Let (Ω, K) and (Ω', R) be two soft dimonoids over \mathcal{D} . Then (Ω', R) is a soft subdimonoid of (Ω, K) if (Ω', R) is a soft subset of (Ω, K) .

Proof. Assume that (Ω, K) and (Ω', R) are two soft dimonoids over \mathcal{D} . In this case, if (Ω', R) is a soft subset of (Ω, K) , we obtain $R \subseteq K$ and $\Omega'(b) \subseteq \Omega(b)$ for all $b \in Supp(\Omega', R)$. It follows that $\Omega'(b)$ is a subdimonoid of $\Omega(b)$. Therefore, (Ω', R) is a soft subdimonoid of (Ω, K) .

Theorem 4.5. Let (Ω, K) be a soft dimonoid over \mathcal{D} and (Ω', R) be a soft subdimonoid of (Ω, K) .

i. The restricted intersection of (Ω, K) and (Ω', R) is a soft subdimonoid of (Ω, K) if it is non-null.

ii. The restricted union of (Ω, K) and (Ω', R) is a soft subdimonoid of (Ω, K) if it is non-null.

Proof. i. Suppose (Ω', R) be a soft subdimonoid of (Ω, K) . If it is non-null, then $R \subseteq K$ and $\Omega'(a)$ is a subdimonoid of $\Omega(a)$ for all $a \in Supp(\Omega', R)$. Thus, it is immediate to check that $R \cap K \subseteq K$ and $\Omega'(a) \cap \Omega(a)$ is a subdimonoid of $\Omega(a)$ for all $a \in Supp(\Omega', R)$. So, the restricted intersection $(\Omega, K) \cap (\Omega', R)$ is a soft subdimonoid of (Ω, K) .

ii. It is similar to previous proof.

Definition 4.6. Let (Ω, K) be a soft dimonoid over \mathcal{D} and (Ω_i, K_i) be soft subdimonoids of (Ω, K) for $\in \mathcal{I}$. Then, (Ω, K) is said to be a soft diband of soft subdimonoids (Ω_i, K_i) for $i \in I$ if the following conditions hold:

i. $(\Omega, K) = \bigcup_{i \in \mathcal{I}} (\Omega_i, K_i)$

ii. $(\Omega_i, K_i) \widetilde{\bigcap} (\Omega_j, K_j) = \widetilde{\emptyset}$ for all $i \neq j$.

iii. For all $i, j \in I$, there is any one $n, m \in I$ such that $(\Omega_i, K_i) \circ (\Omega_j, K_j) \subseteq (\Omega_n, K_n)$ and $(\Omega_i, K_i) \diamond (\Omega_j, K_j) \subseteq (\Omega_m, K_m)$.

Now we would like to study the soft homomorphism between soft dimonoids.

Definition 4.7. Let (Ω, K) and (Ω', R) be soft dimonoids over \mathcal{D} and \mathcal{D}' , respectively. Let $\delta : K \longrightarrow R$ and $\lambda : \mathcal{D} \longrightarrow \mathcal{D}'$ be two mappings. In this case, the pair (δ, λ) is called a soft homomorphism if the following conditions are satisfied:

i. δ is a dimonoid homomorphism;

ii. $\lambda(\Omega(a)) = \Omega'(\delta(a))$ for all $a \in Supp(\Omega, K)$.

Notice that a soft homomorphism (δ, λ) is a mapping of soft dimonoids. Also, we establish a new category whose objects are soft dimonoids and whose arrows are soft homomorphisms.

Example 4.8. Assume (Ω', R) be a soft subdimonoid of (Ω, K) over \mathcal{D} . Considering the inclusion map $\varrho: R \longrightarrow K$ and the identity mapping $\mathcal{I} \upharpoonright : \mathcal{D} \longrightarrow \mathcal{D}$, it follows that pair $(\mathcal{I} \upharpoonright, \varrho)$ is a soft homomorphism from (Ω', R) to (Ω, K) .

From the above definition, we have directly the following consequence:

Theorem 4.9. Let (Ω, K) , (Ω', R) and (Ω'', S) be soft topological dimonoids over $\mathcal{D}, \mathcal{D}'$ and \mathcal{D}'' , respectively. If $(\delta, \lambda) : (\Omega, K) \longrightarrow (\Omega', R)$ and $(\delta', \lambda') : (\Omega', R) \longrightarrow (\Omega'', S)$ are two soft homomorphisms, then $(\delta' \circ \delta, \lambda' \circ \lambda) : (\Omega, K) \longrightarrow (\Omega'', S)$ is a soft homomorphism.

Let us draw the final corollary.

Theorem 4.10. Let the pair (δ, λ) be a soft homomorphism from the soft dimonoids (Ω, K) and (Ω', R) over \mathcal{D} and \mathcal{D}' , respectively. Then $(\delta^{-1}(\Omega'), K)$ is a soft dimonoid over \mathcal{D} if it is non-null.

Proof. Let (Ω, K) and (Ω', R) be two soft dimonoids over \mathcal{D} and \mathcal{D}' , respectively. Then it is easy to check that

$$\lambda(Supp(\delta^{-1}(\Omega', R) = \lambda^{-1}(Supp(\Omega', R)))$$

for all $b \in Supp(\Omega', R)$. Taking $a \in Supp(\delta^{-1}(\Omega', K))$, we get $\lambda(a) \in Supp(\Omega', R)$. So, the nonempty set $\Omega'(\lambda(a))$ is a subdimonoid of \mathcal{D}' . Morever, since δ is a dimonoid homomorphism, we conclude that $\delta^{-1}(\Omega'(\lambda(a))) = \delta^{-1}(\Omega'(a))$ is a subdimonoid of \mathcal{D} . This implies that $(\delta^{-1}(\Omega'), K)$ is a soft dimonoid over \mathcal{D} . \Box

5. Conclusion

Soft sets play a very important role in mathematics and are now the research topic of many mathematicians around the world. In particular, soft algebraic concepts are introduced by combining soft sets and algebraic concepts. The motivation of the current paper is to define and study a new concept called as soft dimonoid which is a generalization of the soft monoids. Morever, the concepts of soft subdimonoids and soft homomorphism between soft dimonoids are introduced. Characterizations and properties of these concepts have been obtained. It is hoped that the results of this study may help in the investigation and extend of soft algebraic concepts.

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